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ALIGNMENT OF SDD WITHIN SVT SYSTEM

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Introduction

In this note the problem of alignment (accurate positioning) of Silicon Drift Detector's (SDD) wa-fers within Silicon Vertex Tracker (SVT) system is addressed. Most of the SVT physics involves precise determination of the impact parameter (extrapolation accuracy to a vertex), which is significantly affected by misalignment caused by numerous physical effects (thermal expansion, stress during insertion, magnetic field effects, etc.) Taking the SDD wafer as a rigid object, the misalignments break down into 4 categories (6 parameters):

- planar translation (2 parameters)
- planar rotation (1 parameter)
- radial translation (1 parameter)
- out-of-plane rotation (2 parameters)

To compensate for these deviations the data-driven alignment is required [1]. We propose and investigate a method for such alignment using a simple model consisting of three parallel SDD wafers.

By data-driven alignment one means a determination of wafer displacements using a given set of coordinates of the points, where the tracks intersect the three SDD wafers. It is easy to demonstrate the impossibility of reconstructing all 18 parameters (6 for each wafer) using the data only from these three wafers, without any additional information from other

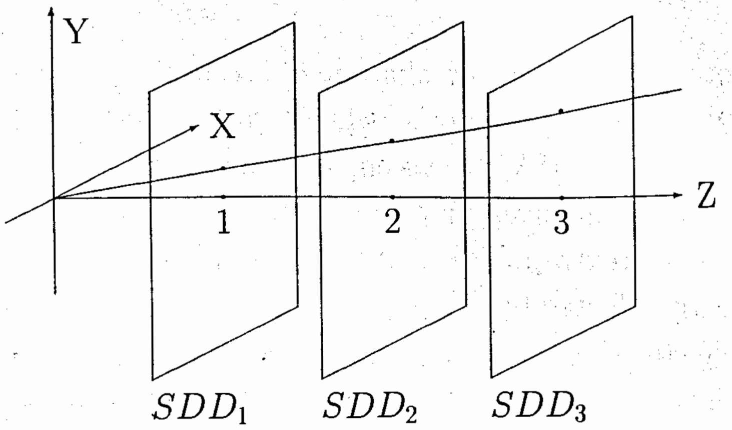

Figure 1: Simple model of SVT system

parts of detector. Let Δx_i, Δy_i and Δz_i ($i = 1, 2, 3$) denote the i-th wafer shift along the corresponding coordinate axes, α_i - the angle of i-th wafer rotation in xy-plane (the direction from y to x taken to be positive), β_i and γ_i - the angles of rotation in xz and yz planes (directions from x to z and from y to z are taken to be positive). Let us consider a track coming from the beginning of coordinates and having the slopes (a, b) in the xz and yz planes, correspondingly. Then for the ideal wafer positions the coordinates of the intersection points would be $(a, b, 1)$, $(2a, 2b, 2)$ and $(3a, 3b, 3)$ (see Fig.1).

Supposing the angles and shifts to be sufficiently small one can write down transformations which take place in case

of misalignment in the following way:

$$(a, b, 1) \rightarrow (a + \Delta x_1 + \alpha_1 b, b + \delta y_1 - \alpha_1 a, 1 + \\ + \Delta z_1 + a\beta_1 + b\gamma_1)$$

$$(2a, 2b, 2) \rightarrow (2a + \Delta x_2 + 2\alpha_2 b, 2b + \Delta y_2 - \\ -2\alpha_2 a, 2 + \Delta z_2 + 2a\beta_2 + 2b\gamma_2)$$

$$(3a, 3b, 3) \rightarrow (3a + \Delta x_3 + 3\alpha_3 b, 3b + \Delta y_3 - \\ -3\alpha_3 a, 3 + \Delta z_3 + 3a\beta_3 + 3b\gamma_3)$$

Let's investigate the conditions that should be kept for the track to be linear, i.e, that the straight line going through the points on the first and second wafers also passes the point on the third one:

$$x_1 + \frac{x_2 - x_1}{z_2 - z_1}(z_3 - z_1) = x_3,$$
$$y_1 + \frac{y_2 - y_1}{z_2 - z_1}(z_3 - z_1) = y_3,$$

where (x_i, y_i, z_i) are coordinates of the intersection points for the i-th wafer. Using the parametrization given above one obtains:

$$\frac{(a + (\Delta x_2 - \Delta x_1) + b(2\alpha_2 - \alpha_1))}{(1 + (\Delta z_2 - \Delta z_1) + a(2\beta_2 - \beta_1) + b(2\gamma_2 - \gamma_1))} \cdot$$

$$(2 + (\Delta z_3 - \Delta z_1) + a(3\beta_3 - \beta_1) + b(3\gamma_3 - \gamma_1)) + \\ +(a + \Delta x_1 + \alpha_1 b) = 3a + \Delta x_3 + 3\alpha_3$$

$$\frac{(b+(\Delta y_2-\Delta y_1)+a(2\alpha_2-\alpha_1))}{(1+(\Delta z_2-\Delta z_1)+a(2\beta_2-\beta_1)+b(2\gamma_2-\gamma_1))}\cdot$$

$$(2+(\delta z_3-\Delta z_1)+a(3\beta_3-\beta_1)+b(3\gamma_3-\gamma_1))+ \\ +(b+\Delta y_1-\alpha_1 a)=3b+\Delta y_3-3\alpha_3$$

Using the fact that all the deviations are small and the approximate formula $1/(1+a)\approx 1-a$ (where a is small) one has:

$$(-\Delta x_1+2\Delta x_2-\Delta x_3)+b(\alpha_1-4\alpha_2-3\alpha_3)+ \\ +a(\Delta z_3-2\Delta z_2+\Delta z_1)+a^2(3\beta_3+4\beta_2-\beta_1)+ \\ +ab(3\gamma_3-4\gamma_2+\gamma_1)=0$$

$$(-\Delta y_1+2\Delta y_2-\Delta y_3)+a(\alpha_1-4\alpha_2-3\alpha_3)+ \\ +b(\Delta z_3-2\Delta z_2+\Delta z_1)+ab(3\beta_3-4\beta_2+\beta_1)+ \\ +a^2(3\gamma_3-4\gamma_2+\gamma_1)=0$$

As we are interested in the case when all the tracks remain to be linear, these equations should be valid for any a and b, so

$$\begin{cases}\Delta x_3-2\Delta x_2+\Delta x_1=0\\ \Delta y_3-2\Delta y_2+\Delta y_1=0\\ \Delta z_3-2\delta z_2+\Delta z_1=0\end{cases}\begin{cases}3\alpha_3-4\alpha_2+\alpha_1=0\\ 3\beta_3-4\beta_2+\beta_1=0\\ 3\gamma_3-4\gamma_2+\gamma_1=0\end{cases}$$

This means that the requirement of track linearity can be used only for the reconstruction of one wafer position with respect to the other two. However, we can use for alignment the assumption that all the tracks originate from the common vertex. Obviously, this assumption will not affect shifts,

so for the sake of simplicity let us put $\Delta x_i=\Delta y_i=\Delta z_i=0$. If x_v, y_v, z_v denote the vertex coordinates, then the condition of common vertex can be written down as follows:

$$(a+\alpha_1 b)+\frac{(a+b(2\alpha_2-\alpha_1))}{(1+a(2\beta_2-\beta_1)+b(2\gamma_2-\gamma_1))}\cdot \\ (z_v-1-a\beta_1-b\gamma_1)=x_v.$$

$$(b+\alpha_1 a)+\frac{(b+a(2\alpha_2-\alpha_1))}{(1+a(2\beta_2-\beta_1)+b(2\gamma_2-\gamma_1))}\cdot \\ (z_v-1-a\beta_1-b\gamma_1)=y_v.$$

Again using the fact that angles have small values and supposing the vertex to be near the beam pipe centre (i.e., x_v and y_v are small) we have :

$$2az_v+2b(\alpha_1-\alpha_2)+a^2(2\beta_2-2\beta_1)+ab(2\gamma_2-2\gamma_1)\approx 0$$

$$2bz_v+2a(\alpha_1-\alpha_2)+b^2(2\gamma_2-2\gamma_1)+ab(2\beta_2-2\beta_1)\approx 0$$

$$\Rightarrow\begin{cases}\alpha_1=\alpha_2\\ \beta_1=\beta_2\\ \gamma_1=\gamma_2\end{cases}$$

Taking into account the expressions obtained earlier this means:

$$\begin{cases}\alpha_3=\alpha_1\\ \beta_3=\beta_1\\ \gamma_3=\gamma_1\end{cases}$$

So we can reconstruct the rotation of two wafers with respect to the third one. Thus, data from wafers allow one to reconstruct only 9 out of 18 required parameters and we need some additional information to reconstruct the rest of the parameters.

In this note we propose an algorithm for reconstruction of all 9 possible parameters (3 translations and 6 rotations). The following parameters were chosen : translations along x, y and z axes of the 1st wafer with respect to the other two (Δx, Δy and Δz) and all possible rotations of the first and second wafers with respect to the third one (α_i, β_i, γ_i, where $i = 1, 2$). To carry out this task some preliminary work should be done, namely:

- to develop a computational model as close as possible to the real experimental data,
- to choose the method for optimal track reconstruction and vertex finding, and to investigate its accuracy, resistance to a contamination, etc., with the help of the developed model.

1 Computational model

To estimate the precision of vertex and detector position reconstruction by means of Monte-Carlo method the following model was used. The distance between detectors, their size and also the distance between the beam pipe centre and SDD_1 were taken in accordance with the real SVT. Vertex coordinates were randomly chosen inside the beam pipe. Then 30 direct tracks were simulated from this vertex in such a way that each of them passed through all the three

SDD's. For this purpose two slopes a and b were chosen for every track (projection slopes in xz and yz planes, where z - beam direction, xy - detector plane). a and b are random numbers uniformly distributed on the interval (a_{min}, a_{max}), where a_{min} is the slope of the line passing through the chosen vertex and the lower bound of SDD_3, and a_{max} - through the vertex and the upper bound of SDD_3 (see Fig.2a).

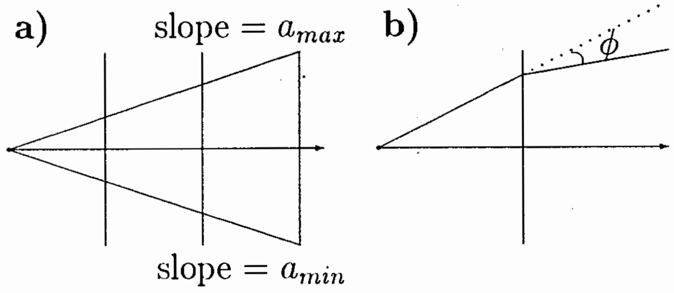

Figure 2: Definition of slopes (a) and scattering angle (b)

To get closer to the experimental data a normally distributed random shifts were added to the coordinates of points in which the simulated tracks crossed the detectors. The mean square deviation (distribution width) σ of this displacements was taken to be equal to 10, 20, or 30 μm when vertex reconstruction precision was analysed. While investigating the precision of detector position reconstruction σ was taken to be 20 μm. Besides, the multiple Coulomb scattering [2] was simulated for SDD_2 and SDD_3. For this purpose the slope of the line simulating the track was changed by some angle ϕ after it crossed SDD_1 and SDD_2 (see Fig.2b). The value of this angle was determined by a normally distributed random number with some σ_ϕ [3]. The analysis of the vertex reconstruction precision was done with different values of σ_ϕ (1, 2 and 3 mrad). When the precision of the detector position reconstruction was investigated the scattering angles were simulated with $\sigma_\phi = 1$ mrad. The azimuth direction of scattering (in xy plane) was determined by some angle Θ uniformly distributed on the interval $(0, 2\pi)$. After that a number of uniformly distributed points were added to each of the detectors as a background simulation. This number for our model was chosen at the 30% level.

2 Vertex reconstruction

The search for all possible tracks was done before vertex fitting. For this purpose the following method was used. For each pair of points p_i, p_j (where p_i is a point from the data set of SDD_1, and p_j - SDD_2) the point p_k (from the data set of SDD_3) was searched for in the vicinity of the crossing of SDD_3 and the line passing through p_i and p_j. If such a point was found, then a line was drawn through these 3 points with the help of the least square method. The following parametrization was used for this line:

$$x = x_0 + az \qquad y = y_0 + bz$$

Let (x_i, y_i, z_i), $i = (1, 2, 3)$ be the coordinates of the three points, and $(x_0+az_i, y_0+bz_i, z_i)$ the coordinates of the crossing of the approximating line with i-th plane. To determine

track parameters a, x_0, b, y_0 the following functional was minimized:

$$L = \Sigma(w_i[(x_0 + az_i - x_i)^2 + (y_0 + bz_i - y_i)^2]).$$

Thus, we choose for minimization a distance between points and track line in the wafer planes because it leads to the simple linear equations on the track parameters and allows one to determine these parameters by a simple one step procedure without any time consuming iterations. Having found the partial derivatives we get two uncoupled pairs of equations:

$$\begin{cases} n_p x_0 + \Sigma(z_i a w_i) = \Sigma(w_i x_i) \\ \Sigma(z_i w_i x_0) + \Sigma(z_i^2 a w_i) = \Sigma(x_i z_i w_i) \end{cases}$$

$$\begin{cases} n_p y_0 + \Sigma(z_i b w_i) = \Sigma(w_i y_i) \\ \Sigma(z_i w_i y_0) + \Sigma(z_i^2 b w_i) = \Sigma(y_i z_i w_i) \end{cases}$$

Where n_p is the number of the points which determine every track (in our case $n_p = 3$), w_i is a weight function, chosen in the following way :

$$w_i = \begin{cases} 1, & if\ (x_i, y_i, z_i)\ are\ coordinates \\ & of\ SDD_1\ point \\ 1/2, & if\ (x_i, y_i, z_i)\ are\ coordinates \\ & of\ SDD_2\ point \\ 1/3, & if\ (x_i, y_i, z_i)\ are\ coordinates \\ & of\ SDD_3\ point \end{cases}$$

We insert such a weight function because the data from the SDD_2 and SDD_3 are more strongly deviated from the real values due to the multiple Coulomb scattering. This function weakens the influence of these points. Thus the line is drown closer to the SDD_1 point, whose distribution width is smaller than that of SDD_2 and SDD_3 points. This method of tracks reconstruction can be used for any number of wafers.

After all the possible tracks had been found the preliminary vertex fit was accomplished. For this purpose the region inside the beam pipe where the vertex might be situated was divided into cubes with the side $a = 600\mu m$. Then the number of the tracks passing through each cube was counted. Then this procedure was repeated for a cube that was crossed by the biggest number of the tracks with $a = 150\mu m$. After that the vertex position was determined precisely. It was done by finding the point whose sum of distance squares to each track was minimal. Let (x_v,y_v,z_v) be the vertex coordinates, and ($x_{0i} + a_i z$, $y_{0i} + b_i z$, z) coordinates of some point on i-th track. Let (x^i_{min}, y^i_{min}, z^i_{min}) be the coordinates of crossing of i-th track with the perpendicular drawn from (x_v,y_v,z_v). The ortogonality condition may be written as $\vec{A}\cdot\vec{B} = 0$, where $\vec{A} = \{a_i, b_i, 1\}$ - vector determining the i-th track direction, and $\vec{B} = \{x_{0i}+a_i z-x_v, y_{0i}+b_i z-y_v, z-z_v\}$ - vector determining the perpendicular direction. So, it is easy to get for the coordinates of the required point

$$\begin{cases} z^i_{min} = \frac{(a_i(x_v-x_{0i})+b_i(y_v-y_{0i})+z_v)}{(a_i^2+b_i^2+1)} \\ x^i_{min} = x_{0i} + a_i z^i_{min} \\ y^i_{min} = y_{0i} + b_i z^i_{min} \end{cases}$$

Then in oder to reconstruct vertex position it is necessary

to minimize the functional

$$L1 = \sum_{n_t}((x^i_{min} - x_v)^2 + (y^i_{min} - y_v)^2 + (z^i_{min} - z_v)^2),$$

where n_t - the number of tracks. Substituting the expressions for x_{min}, y_{min} and z_{min} and finding the partial derivatives we have :

$$\sum\frac{a_i x_v}{d_i} + \sum\frac{b_i y_v}{d_i} - \sum\frac{(a_i^2+b_i^2)z_v}{d_i} = \sum\frac{a_i x_{0i} + b_i y_{0i}}{d_i}$$

$$\sum\frac{(1+b_i^2)x_v}{d_i} - \sum\frac{a_i b_i y_v}{d_i} - \sum\frac{a_i^2 z_v}{d_i} = $$
$$= \sum\frac{(1+b_i^2)x_{0i} - a_i b_i y_{0i}}{d_i}$$

$$\sum\frac{a_i b_i x_v}{d_i} - \sum\frac{(1+a_i^2)y_v}{d_i} + \sum\frac{b_i^2 z_v}{d_i} = $$
$$= \sum\frac{(1+b_i^2)y_{0i} - a_i b_i x_{0i}}{d_i},$$

where $d_i = 1 + a_i^2 + b_i^2$. As one can see the functional is quadratic on the x_v, y_v and z_v so the solution of this system of equations gives us the required values and such algorithm turns out to be a very fast one.

To determine the precision of vertex reconstruction a thousand of tests were carried out for each value of σ and σ_ϕ parameters. Figure 3 demonstrates the distribution of vertex reconstruction errors for 20 μm and 1 mrad. Figure 4 shows the dependence of vertex reconstruction precision $\sigma_{vertex} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$ on values of σ and σ_ϕ. The results obtained show a good precision of the method proposed.

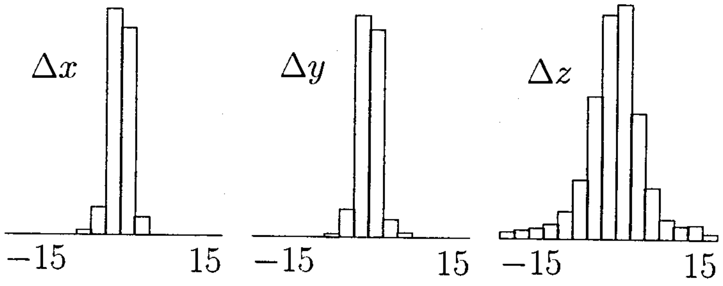

Figure 3: Vertex reconstruction errors (μm)

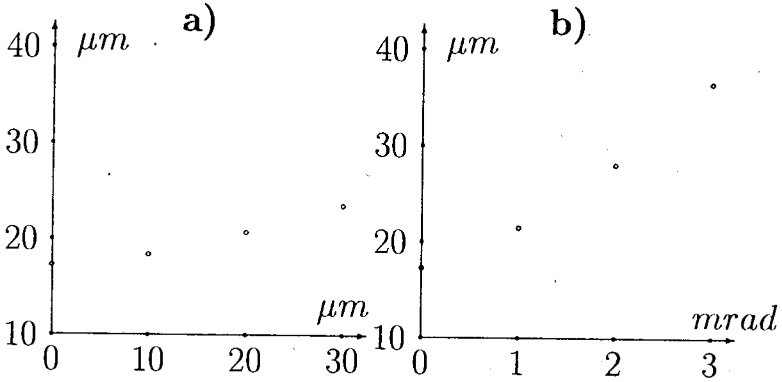

Figure 4: σ_{vertex} dependencies on σ (a) and σ_ϕ (b)

That allowed us to use this method for wafer position reconstruction (alignment).

3 Alignment

As we have seen, the condition of common vertex for all tracks is essential for the possibility to determine as much as 9 parameters. So for each track χ^2 we take the distance between the track and the points of each detector as well

as the distance between the track line and the found vertex. The minimization of the following functional was used for the alignment:

$$M = \frac{\chi^2}{nt} = (\sum_{nt}((z_{min} - z_v)^2 + (x_{0i} + a_i z_{min} - x_v)^2 +$$
$$+(y_{0i} + b_i z_{min} - y_v)^2 + \sum_{n=1}^{3}((x_{0i} + a_i z_{ni} - x_{ni})^2 +$$
$$+(y_{0i} + b_i z_{ni} - y_{ni})^2))/nt,$$

where x_{ni}, y_{ni}, z_{ni} - i-th track coordinates on n-th plane. We worked with χ^2/nt because the number of the found tracks may be different. It is accounted for by presence of background and mostly by the loss of some tracks due to the misalignment.

The possibility of existence of such a great misalignment that can lead to the loss of considerable number of tracks made us to carry out the alignment in 2 stages. At first the selection of primary approximation for some of the parameters was done. The investigation showed that to find primary approximation for all 3 translations and for planar rotation is quite enough, because the influence of out-of-plane rotation on χ^2 is very small. The true detector position can be taken as the primary approximation for these 4 rotation parameters (i.e., they were assumed to be equal to zero). The approximation for Δx and Δy was searched in the interval (-600 μm, +600 μm) with a step 150 μm, and for Δz with a step 300 μm. For planar rotation the α approximation was searched for in the interval (-20 mrad, +20 mrad) with a step 8 mrad. For each combination of parameter values the track and vertex reconstructions were done and the functional value was found. As the primary approximation and parameters values we chose those that gave the minimal value of M. Certainly, such a search for primary approximation is a bit time consuming, but it is absolutely unavoidable, because the dependence $M = M(\Delta x, \Delta y, \Delta z, \alpha_i, \beta_i, \gamma_i)$ is a complicated function with a great number of local minima and the selection of primary approximation helps to escape from finding one of them. To make the algorithm work faster the primary approximation was determined in the above-mentioned way for the first 10 events only, then for all other events

$$par_0^n[i] = \frac{\Sigma_{m=1}^{n-1}(par^m[i])}{n-1}$$

was chosen as the primary approximation. And this substantially decreased the time of program work.

After the selection of primary approximation the alignment was carried out. For this purpose the values of all the partial derivatives were determined

$$m[i] = \frac{\partial M}{\partial par[i]} = \frac{M(par_0[i] + st) - M(par_0[i] - st)}{2st},$$

where st - is some step, and par - array of parameters. Then all the parameters were changed according to:

$$par[i] = par[i] - v_i \frac{m[i]st}{\sqrt{\Sigma_j m[j]^2}},$$

where v_i are some constants accounting for different influence of the misalignment parameters on total χ^2 [1] and different physical dimensions of shifts and rotations. Then M was calculated with the new values of the parameters $par[i]$, $i = 1, 2..9$ and the result was compared with the previous value of M. In case the functional value had become smaller, then parameter values were taken as new primary approximation after that the procedure was repeated. If the value of M turned out to be greater the previous one, then the step variable st was reduced by a factor 1/2 and the procedure was continued. The whole fitting procedure was finished when the value of step variable became less than the value of $step_min$, which was determined by the required precision [4].

4 Results

The precision estimation of the alignment method was done in the following way. A thousand of events was simulated in the above described way. The values of distortion parameters were randomly chosen and coordinates of SDD point were recalculated taking into consideration translations and rotations. After that the alignment was done and the obtained results were compared with the initial misalignment parameters. For each reconstructed parameter the histogramming of estimation errors was done

$$\delta[i] = par[i] - par_{real}[i],$$

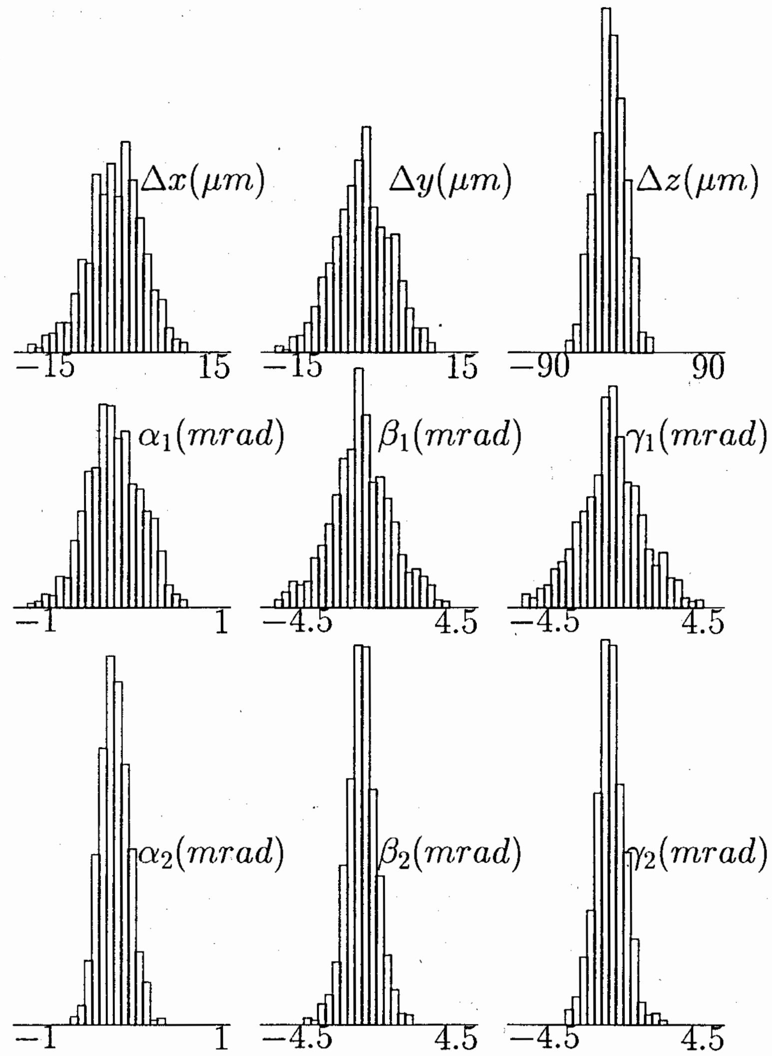

Figure 5: Detector reconstruction errors

Table. Precision of the wafers position determination

σ_x	σ_y	σ_z	σ_{α_1}	σ_{β_1}	σ_{γ_1}	σ_{α_2}	σ_{β_2}	σ_{γ_2}
4.4	4.3	12.2	0.26	1.30	1.35	0.13	0.63	0.60
μm			$mrad$					

where $par_{real}[i]$ - a true value of i-th parameter. The obtained histogramms are shown in Figure 5. All the distributions turned out to be unbiased, i.e., the mean value of parameter reconstruction errors $\bar{\delta}[i] \approx 0$. The obtained results of mean square deviations for all parameters are given in the Table.

5 Conclusion

Using a simplified SVT system we proposed the fast algorithms for the tracks and vertex reconstruction and investigated the method of detector position reconstruction. The results obtained show the good precision of the method proposed, that allows us to offer it for a Monte-Carlo test on the model with the real SVT geometry and then for processing of real experimental data.

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