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AN ALGORITHM FOR RECOGNITION OF CURVE TRACKS DETECTED BY DRIFT TUBES OF THE MUON SPECTROMETER

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1 Introduction

A conventional track recognition problem can be reduced to the search for a "sufficient" number of data points, which must satisfy conditions of a "sufficient" smoothness of their alignment along a straight line or a higher-order curve. The notion "sufficient" depends on the statistical efficiency of the track recognition problem for a given experiment.

In cases, when the experimental data are 2D- or 3D-coordinates registered by a track chamber, the track recognition problem is usually solved by an exhaustive sorting of all data point into subsets (track candidates). Then the smoothness of the data point alignment for each subset is to be estimated by some criterion (usually by fitting a second order curve to a of 2D projection of these points and then applying a χ^2– criterion).

The efficiency of the track reconstruction algorithm depends on reasonability of a clustering method applied to group data points into track candidates, i.e. on the maximum possible reducing of the search trials made by the used method over all points. As examples of such reasonable algorithms, one can point out well known methods like variable slope histogramming or stringing (track following) methods [1, 2], as well as relatively new approaches like Hopfield neural networks [3, 4].

One of the detector systems widely used in modern experiments of high energy physics (ATLAS, EVA/E850) are high pressure drift tubes (HPDT). Each time, when a passing particle track hits a tube, it registers two data: its own center coordinate and the drift radius, i.e. the drift distance between the particle tracks and the anode wire placed in the center of this tube. Thus, a track passing the HPDT provides a set of anode wire coordinates and corresponding drift radii. Unfortunately, some of these data can be lost due to the straw tube inefficiency, besides a number of noise coordinates is also registered additionally. However, the main problem, which hinders applications of above mentioned conventional track recognition methods, is so-called left-right-ambiguity of drift radii. They do not contain the information on which side of the anode wire the track passed. Anode wire coordinates themselves are very rough indicators of particle locations. So if one would even recognize a subset of these points belonging to a concrete track and would then approximate it by a second-order curve (circle or parabola), the resulting parameter accuracy will not be satisfactory.

In this report the algorithm of track recognition in an uniform magnetic field is proposed for the HPDT in the ATLAS design for the muon spectrometer experiment. A solution of the problem is given for (x,y) plane perpendicular to the magnetic field and anodes of drift straw tubes. Our algorithm is elaborated on the basis of modifications of the Hough transform and deformable template methods.

2 Formulation of the Problem

The HPDT system for the muon spectrometer consists of the modules formed by several layers of straw tubes arranged in honeycomb order (see Fig.1). In the middle of every tube there is an anode wire with known XY-coordinates. All tracks of an event passing these layers produce N signals, i.e. set $M = \{x_i, y_i; r_i, \quad i = \overline{1,N}\}$, where (x_i, y_i) are coordinates of the hitt tube centers, r_i are drift radii. Let us suppose, first, that the recognition problem is solved, i.e. from the set M a subset S was extracted of triplets $(x_i, y_i; r_i)$ produced by one of tracks and, probably, also by some extra noise tubes. For the sake of simplicity let's keep for S the same notation, as for M, i.e. $S = \{x_i, y_i; r_i, \quad i = \overline{1,N}\}$. Geometrically the set S can be considered as the set of circles on the plain with centers (x_i, y_i) and radii r_i.

Thus, **the mathematical formulation of the problem** is to draw the track line as a parabola $y = Ax^2 + Bx + C$ tangential to the maximum number of these little circles from S.

Let us introduce as a measure of tangency a circle $(x_i, y_i; r_i)$ and a parabola $y = Ax^2 + Bx + C$ the difference between the distance from the center of the circle to the parabola and the radius r_i. If the circle and parabola are tangential, their tangency measure is, obviously, equal to zero. Then our problem above formulated can be reformulated as the following: **to find such a parabola (A, B, C) that minimizes the sum of its tangency measures with all circles from the set S.**

Let us denote by $D_i(A, B, C)$ the distance from the center of the circle $(x_i, y_i; r_i)$ to parabola (A, B, C)

$$D_i(A, B, C) = min_{\{x,y\}}\{\sqrt{(x_i - x)^2 + (y_i - y)^2}\}$$

where $(x, y) \in Ax^2 + Bx + C$.

In this form the distance $D_i(A, B, C)$ does not satisfy the solving of the problem mentioned above. Replacement the part of the parabola near the drift tube on the straigt line and then for this distance obtained the next form:

$$D_i(A, B, C) \approx \frac{|A(x_i^2 + R_{tub}^2/4) + Bx_i + C - y_i|}{\sqrt{(2Ax_i + B)^2 + 1}}$$

where R_{tub} - radius of the drift tube.

This variable can take both positive and negative values. Therefore, the tangency measure square of those circles $(x_i, y_i; r_i)$ and parabola (A, B, C) is twofold:

if $D_i(A, B, C) > 0$, then

$$d_i^- = (D_i(A, B, C) - r_i)^2,$$

otherwise

$$d_i^+ = (D_i(A, B, C) + r_i)^2.$$

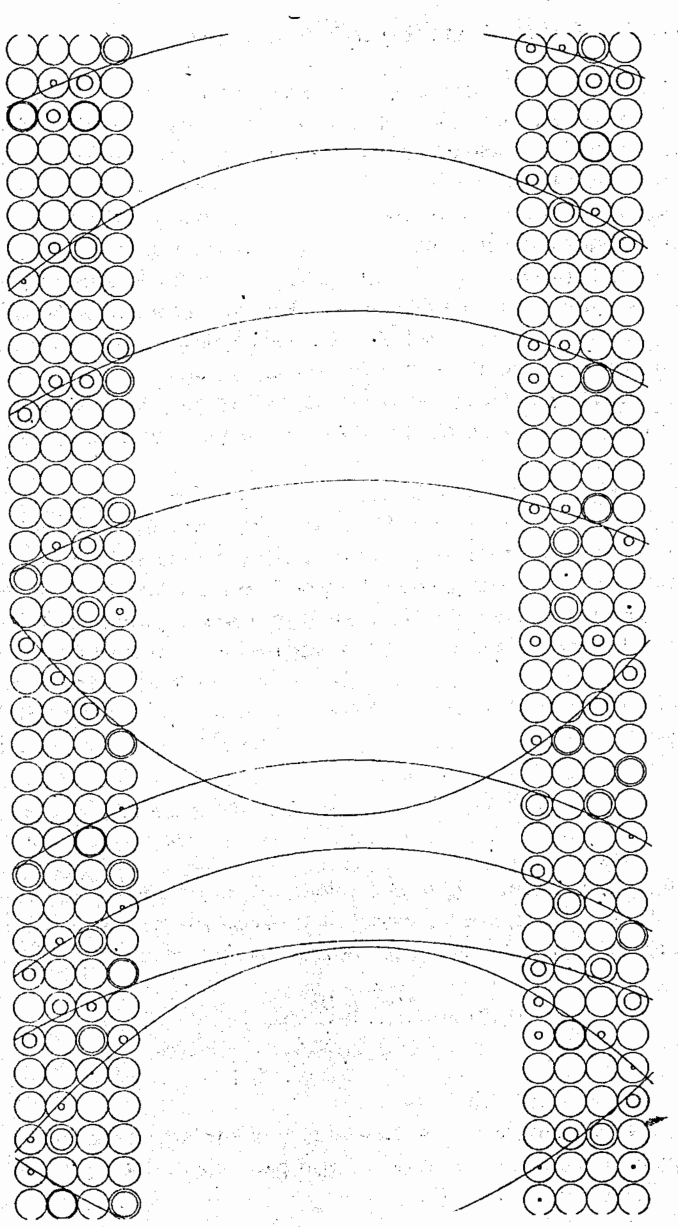

Fig. 1. Example of the event.

As in [5, 6] we define the two-dimensional vector $\vec{s}_i = (s_i^+, s_i^-)$ with admissible values $(1,0), (0,1), (0,0)$. Let us denote by λ the measurement error of the drift radius and define a functional L depending of five parameters (a, b, R, s_i^-, s_i^+) :

$$L = \sum_{i=1}^{N} \left\{ d_i^- s_i^- + d_i^+ s_i^+ + \lambda((s_i^- + s_i^+) - 1)^2 \right\}. \tag{1}$$

It's obvious that the parabola parameters (A, B, C) corresponding to a track in question would define a point in the parameter space, where our functional L reaches its global minimum with the conditions that $\vec{s}_i = (0,0)$ means i-th tube for the given track is the noise tube and the combination $\vec{s}_i = (1,1)$ is forbidden, i.e.

$$s_i^+ + s_i^- \leq 1. \tag{2}$$

Thus, to recognize a track one has to:

1. from the set of all measurement M extract a subset S, which as much as possible contains all data for one of the tracks;

2. find the L global minimum (although it would be enough to reach its close vicinity).

To solve the first problem, we modify the Hough transform method [7], which we following to [8] call as a method of sequential histogramming by parameters (SHPM). Besides of extracting of a subset S SHPM provides also starting values of the parabola (A_0, B_0, C_0) needed to solve the problem on the next step. The second problem is solved by the deformable template method (DTM) with the special correction of parameters of obtained tracks.

3 A sequential histogramming method

Let $\Omega = \{X_i, Y_i, i = \overline{1, N}\}$ be a set of coordinates X_i, Y_i measured in the process of registering an event. So to Ω belong both: coordinates of track points as well as noise coordinates. A parabola arch is supposed to be a good approximation of any track .

Let us consider all triplets of points of the Ω set. If these three points do not belong to a straight line, one can draw a parabola through them. As a result a set of such parabola parameters is obtained $W = \{A_j, B_j, C_j, j = \overline{1, C_N^3}\}$. One could imagine a 3-D histogram constructed on that W-set as a hilly surface, where hills should most likely correspond to tracks. This idea together with the so-called sequential histogramming approach [8] gives us the following algorithm for finding initial track parameters:

1. Parabolas are drawn through all admissible point triplets. Then the parameter C_j of each parabola is histogrammed.

2. The value C_m is obtained corresponding to the maximum of this histogram.

3. With the fixed C_m parabola are drawn through all admissible pair of points from Ω. Then the second parameter B_j of each parabola is histogrammed.

4. The value B_m is obtained corresponding to the maximum of this second histogram.

5. With the fixed parameters C_m, B_m parabola are drawn through all admissible points from Ω. Then the third parameter A_j are histogrammed.

6. The value A_m is obtained corresponding to the maximum of this third histogram.

Then the obtained parameters (A_m, B_m, C_m) are subjected to more sophisticated tests and a more precise definition. If results are positive. i.e parameters (A_m, B_m, C_m) are accepted as a true track, all measurements corresponding to it are eliminated from the set Ω and the whole procedure is repeated starting from the step 1. If the parabola $(A_m, B_m; C_m)$ is rejected by testing, then the maximum A_m of the third A_j-histogram is eliminated and the procedure is repeated starting from the step 6. If there are no more peaks in the A_j-histogram, then the peak B_m of the second histogram is eliminated and the procedure is repeated starting from the step 4 and so on unless the procedure would find a true parabola or all peaks in the second histogramm would be eliminated. In this case the peak C_m of the first histogram is eliminated and the procedure is repeated starting from the step 2. It's clear, that this method of sequential histogramming by parameters (SHPM) gives us a possibility to "capture" an area where tracks are likely situated and provides us by initial parameters of these tracks. In order to apply SHPM the results of measurements must have a format of the Ω-set, i.e. to be a set of track point coordinates. However, we have instead the set M of little circles $\{x_i, y_i; r_i, \quad i = \overline{1, N}\}$, so we have to determine on each of these circles a point associated with some of tracks. It would not restrain us in applying of the SHPM, but it should be kept in mind that the left-and-right uncertainty factor doubles the elements number of the set $\Omega = \{X_i, Y_i, i = \overline{1, 2N}\}$ in a comparison with the number of elements in the original set $M = \{x_i, y_i; r_i, i = \overline{1, N}\}$, where $X_i = x_i$ *and* $Y_i = y_i + r_i$ *or* $Y_i = y_i - r_i$.

To decrease the histogramming search domain of the Ω-set it is necessary to use the maximum of *a priori* information.

The SHPM-description of the stresses given above an importance of the way used to extract a histogram peak from a background. Our experience shows that it is useless to look for an universal peak-background threshold common for all events of a given experimental run, since this threshold strongly depends on the informative load of a given event. Aiming a statistical efficiency of our method, we elaborated the following heuristical formula for the peak-background threshold of a particular event:

$$N_{bound} = 5\frac{H_{max}}{12} + H_{mean}, \tag{3}$$

where H_{max} – is the maximum value of the histogram, H_{mean} – its mean value.

Choosing the bin size, one should find a reasonable compromise between either too small or too big size. The first could lead to the loss of a histogram peak, i.e. one of tracks, while a big size decreases the accuracy.

4 Deformable template method

After obtaining by SHPM initial values of track parameters and choosing an area where this track could lie, we proceed to look for the global minimum of the functional L (1). One of the main problems here is how to avoid local minima of L provoked by a stepwise character of the vector $\vec{s}_i = (s_i^+, s_i^-)$ behaviour. One of the known way to avoid this obstacle is the standard mean field theory (MFT) approach leads to *the simulated annealing schedule* [9]. Our system is considered as a thermostat with the current temperature T [10]. Then as it was shown in [5, 6], parameters s_i^+ и s_i^- of the functional L with fixed (A, B, C) can be calculated by the following formulae, where the stepwise behaviour of the vector $\vec{s}_i$ is replaced in fact onto sigmoidal one:

$$s_i^- = \frac{1}{1 + e^{\frac{d_i^- - \lambda}{T}} + e^{\frac{d_i^- - d_i^+}{T}}}, \tag{4}$$

$$s_i^+ = \frac{1}{1 + e^{\frac{d_i^+ - \lambda}{T}} + e^{\frac{d_i^+ - d_i^-}{T}}}. \tag{5}$$

The L global minimum is calculated according to the following scheme:

1. Three temperature values are taken: high, middle and a temperature in a vicinity of zero, as well as three noise levels corresponding to them [5, 10].

2. According to the simulated annealing schedule, our scheme is started from the high temperature. With initial parabola values (A_0, B_0, C_0) parameters s_i^+, s_i^- are calculated by formulae (4), (5).

3. For obtained s_i^+, s_i^- new parabola parameters A, B, C are calculated by a modification of the standard gradient descent method. This modification consists of individual updating of L parameters and of holding a condition

$$L(a_k, b_k, R_k) < L(a_{k+1}, b_{k+1}, R_{k+1}). \tag{6}$$

4. The ending rule is as follows: either

$$|L(a_k, b_k, R_k) - L(a_{k+1}, b_{k+1}, R_{k+1})| < \epsilon \tag{7}$$

holds or the iteration number exceeds a prescribed number $k = const$.

5. If the conditions of the step 4 are not satisfied, then with the new parabola parameters $(A_{k+1}, B_{k+1}, C_{k+1})$ next values of s_i^+, s_i^- are again calculated by (4),(5) and we go to the step 3.
6. After converging the process with the given temperature, it is changed (system is cooled), the values of (A, B, C) achieved with the previous temperature are taken as starting values and we go to the step 2 again.
7. With each temperature value after completing step 5 the condition

$$L < L_{cut}, \tag{8}$$

is tested. If it satisfied, our scheme is completed and the algorithm proceeds the next stage of correcting of obtained track parameters (A, B, C). Otherwise, if with the temperature in a vicinity of zero we obtain

$$L > L_{cut}, \tag{9}$$

then a diagnostic is provided that the track finding scheme failed.

5 Procedure of the track parameter correction

The deformable template method provides us with track parameters (A, B, C). However, these parameters, even if they satisfy (8), could appear rather apart of the L global minimum. Therefore, we have to elaborate an extra stage for the track parameter correction.

On each circle of the set $S = \{x_i, y_i; r_i, i = \overline{1, N}\}$ taking in account the corresponding values of $\vec{s}_i$, a point is found nearest to the track-candidate. Then all these points are approximated by a parabola and χ^2 value is calculated as a criterion of their smoothness and fitness quality.

If it is hold

$$\chi^2 < \chi^2_{cut},$$

then the approximating parameters (A_{ap}, B_{ap}, C_{ap}) are accepted as true. Otherwise the track-candidate is rejected.

While statistical testing of our algorithm efficiency it was found useful to apply this procedure yet before the deformable templates to track-candidate parameters obtained on the SHPM-stage. The only difference is if one would obtain $\chi^2 > \chi^2_{cut}$, as the result of this preliminary testing, then the process does not stop, but passes to the stage of deformable templates.

6 Results

The proposed track finding algorithm of the tracks detected by the HPDT system in a magnetic field was tested on different series of simulated events. The number

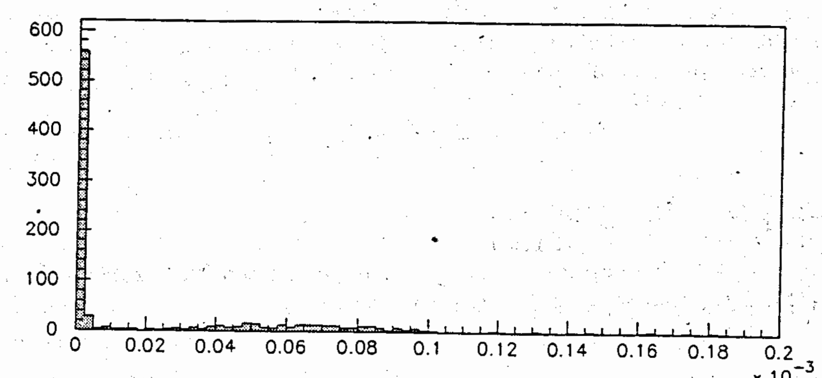

Fig.2 Distribution of the error for the A parameter

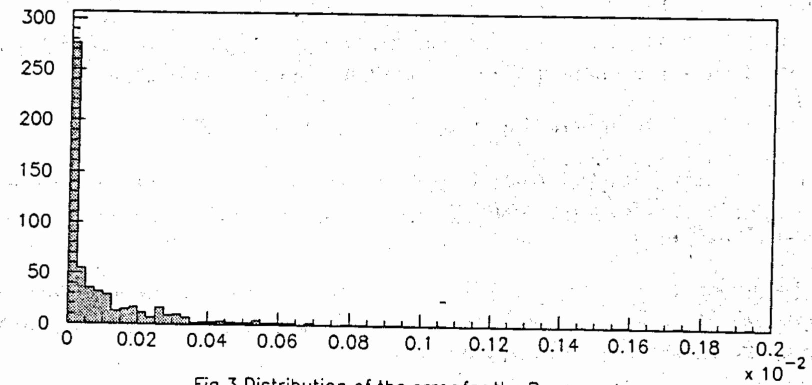

Fig.3 Distribution of the error for the B parameter

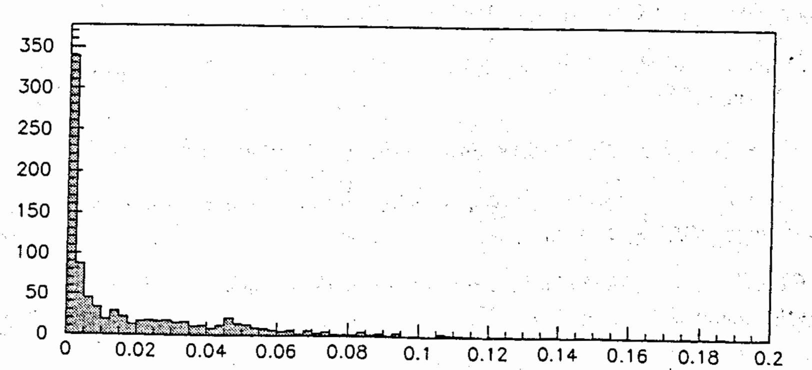

Fig.4 Distribution of the error for the C parameter

of the tested events ≈ 1000. Example of the tested events you can see on the Fig. 1. The efficiency of correctly recognized events kept in the range 94%–96%. Fig.2, Fig.3 and Fig.4 shou the distribution of the error for the parameter A, B, C, i. e. ;

$$|A_{find} - A_{model}|,\ |B_{find} - B_{model}|,\ |C_{find} - C_{model}|.$$

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