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METHOD AND PROGRAM
FOR DECAY SCHEME RECONSTRUCTION
BASED ON FORMAL LOGICAL ANALYSIS
OF GAMMA-GAMMA COINCIDENCE MATRIX

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1. Introduction

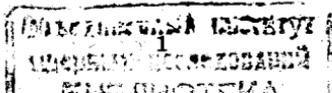
The importance of the γ - γ coincidence method for constructing a scheme of excited nuclear levels is difficult to overestimate. Recently the amount of the information on γ - γ coincidences has been increasing owing to many-detector facilities for on-line and off-line investigations of angular correlations and for measurements of γ - γ coincidence spectra in various cases including the study of nuclear reactions in accelerated beams of protons and heavy ions. The total number of coincidences recorded in one experiment has increased from 10^6 to 10^8 .

Anti-Compton spectrometers used to measure γ - γ coincidences considerably (by more than an order of magnitude) improved the ratio of useful coincidences (peak-peak) to ballast coincidences (peak-Compton, Compton-Compton). Many-detector facilities not only appreciably improve the main characteristics of γ - γ coincidences but also, owing to n-fold coincidences ($n > 2$), radically improve the resolving power R of measurements [1]:

$$R = \left| \frac{SE_{\gamma} P}{dE_{\gamma} T} \right|^n,$$

where SE_{γ} is the average difference in energy between γ -quanta (60 keV for deformed nuclei), dE_{γ} is the energy resolution of the Ge detector ($dE_{\gamma} = 2$ keV for ^{60}Co); P/T is the ratio of the full absorption peak intensity (P) to the total photopeak intensity (T).

A possibility of obtaining extensive information on coincidences sets a task to develop appropriate software for accumulation, processing and analysis of the information. There are effective programs to sort data, to



get integral spectra and "cross section" spectra, programs for automatic and semiautomatic processing of coincidence spectra with evaluation of intensities by volumes of coincidence peaks [2, 3, 4, 16] or from analysis of "cross section" spectra [5], and programs that allow reliable subtraction of background in two-dimensional distribution of γ - γ coincidences [6].

Thus, the methods of gaining coincidence information show tangible progress. On the other hand, the methods [7] of processing this information to construct or revise decay scheme have hardly changed in the recent years. Actually, these methods and programs based on them use little, if any, information on coincidences themselves. The result is either solution of a single specific problem or "raw" data requiring time-consuming analysis.

In [8] a fundamentally different approach was proposed in co-operation with some of the authors of the present paper. It is based on direct analysis of the γ - γ coincidence matrix and allows reconstruction of the decay scheme by formal logical operations with the matrix if it is sufficiently complete and reliable. Naturally, without additional information this involved inevitable "top-bottom" ambiguities for the entire system and for transitions to intermediate levels not connected with the scheme via other transitions.

Then a software to apply the "ideal" version of the method was reported [9, 10].

Yet, processing of real experimental data with a software for the "pure" version of the method revealed great difficulties arising from incompleteness of the data and false coincidences of different origin. This stimulated a search for a possibility of using the method with imperfect experimental data.

Now we have developed and tested a program COIN based on the above-mentioned method. It includes several

essential supplements and extensions to ensure successful use of real experimental data for construction of decay schemes.

2. Method of decay scheme construction based on formal logical analysis of γ - γ coincidences

2.1. Initial points

Before discussing the program COIN, we should describe the basic points of the method underlying it to make clear its principle of operation. Let us introduce definitions according to [8].

The initial level in the decay scheme is a level with at least one γ -transition from it and none to it (levels 5 and 6 in Fig. 1a). The final level in the decay scheme is the ground state and metastable levels (isomers) with no outgoing γ -transitions within the resolution limits of the coincidence scheme (level 1 in Fig. 1). A nucleus may have several initial and final levels depending on the way of its excitation and the presence of metastable levels. All other levels are intermediate (levels 2, 3, 4 in Fig. 1a). The method employs a natural and basic characteristic of a decaying nucleus called "decay mode", i.e. a particular succession of transitions of a nucleus from the initial to the final level.

The complete k -th mode $\{E_m\}_k$ is a full set of transitions taking place at the transition of a nucleus from a given initial level to a given final one, m is the index of transitions belonging to the mode, k is the number of the mode.

The incomplete mode is any subset of transitions from the complete mode. Transitions $\{E_m\}$ not belonging to any mode with a transition E_n shall be called transitions parallel to E_n .

To make consideration of the method rigorous and simple, we assume that the coincidence matrix contains all possible true γ - γ coincidences and does not contain false or stray ones. The matrix is symmetrical about the main diagonal, its rows and columns correspond to γ -transitions which are labelled by energies E_i and can be arranged randomly but identically in rows and columns. A coincidence at the intersection of a row and a column is labelled by "1", its absence by "0". Each row (column) shows transitions with which the given row (column) "labelling" transition coincides (see Fig. 1b).

Let us designate a matrix row by "line E_i " following the energy of its transition. If in the above-defined matrix zeros of the principal diagonal are replaced by unities, it is possible to formulate and prove the following three theorems underlying the decay scheme construction from the formal logical analysis of the γ - γ coincidence matrix.

Theorem 1.

If there exists a set of k coincidence matrix rows $\{\text{line}E_i\}_k$ with a set of indices of its component rows $\{i\}_k$ such that logical multiplication of its component rows yields a row with unities over the chosen set $\{i\}_k$ of indices, the transitions with labels from this set $\{E_i\}_k$ make up at least an incomplete mode (in a particular case there can be a complete mode). The opposite is valid too.

Theorem 2.

If there is a set of coincidence matrix rows $\{\text{line}E_i\}_k$ with indices of its component rows $\{i\}_k$ such that logical multiplication of all its component rows yields a row with unities and only unities over the entire set of indices and no unities in another places, the transitions of this set $\{E_i\}_k$ make a complete mode (CM). The opposite is valid too.

Theorem 3. Let E_j be an arbitrary transition, $\{PE_j\}$ a set of transitions parallel to it. If intersection of this set $\{PE_j\}$ with an arbitrary complete mode $\{E_a\}_b$ consists of more than one element, the part of this complete mode that fully belongs to the intersection is a continuity region (CR), i.e. consists of a continuous succession of transitions of this complete mode. The theorems are proved in Appendix 1.

2.2. Decay scheme construction procedure

The above theorems allow reconstruction of a decay scheme if all complete modes are found and then consistently arranged into order. Theorems 1 and 2 are used to search for all complete modes.

We find all incomplete modes of two elements by logical multiplication of all possible row pairs (theorem 1). Using theorem 2 we separate complete double modes and "throw them out of the game". The remaining two-element incomplete modes are multiplied by another row to test them for triple complete or incomplete modes. Complete triple modes are thrown out of the game and incomplete ones continue participating in the process of searching for complete and incomplete modes of higher order, and so on. This simple algorithm of direct "exhaustion" of all possible logical products of rows ensures finding all complete modes up to the highest existing in the scheme. The exhaustive search ends with a set of only complete modes of maximum multiplicity.

We divide the process of reconstructing the decay scheme from the complete modes (CM) into two stages: ordering of transitions in modes (here and below complete modes are meant) and mutual matching of modes. Ordering of transitions in modes (with allowance made for the above-

mentioned "top-bottom" ambiguities) can be performed by analysis of continuity regions - CR (theorem 3). To this end, one must find all these regions and indicate their presence in the modes. The process of ordering the transitions and matching the modes may depend on the type of nucleus and decay scheme. For example, several double modes may have a common transition. This suggests that the common transition is final for a nucleus which has one final state and it is the ground state. In modes of a higher rank these transitions, if any, are also extreme and final. This greatly simplifies further analysis.

Thus, some structure is revealed in modes. It includes continuity regions and extreme final and initial transitions or "elements" consisting of groups of transitions. Consideration of transition energies appreciably facilitates the process of ordering the transitions and matching the modes. If a total energy is given for each mode, one can isolate groups of modes of different rank but of identical total energy. It means that these modes are all between the same initial and final energy levels. The task is greatly simplified, a whole group of modes is easily ordered, and a decay scheme fragment is constructed.

Let us consider an example. The scheme in Fig. 1a corresponds to the coincidence matrix in Fig. 1b but with the principal diagonal of unities in conformity with the initial point of subsection 2.1. Two-element modes are E_1E_2 , E_1E_3 , E_1E_4 , E_1E_5 , E_2E_3 , E_2E_4 , E_2E_6 , E_3E_4 , E_3E_6 , E_4E_5 , E_5E_6 . Of them a complete mode is E_5E_6 because

$$\text{line}_{E_5} \& \text{line}_{E_6} = (100111) \& (011011) = 000011$$

(in conformity with theorem 2, since unities are only in positions 5 and 6). Combinations E_1E_6 , E_2E_5 , E_3E_5 , E_4E_6 are not modes because multiplication of the appropriate rows

does not yield unities over the chosen set of indices. At the next stage we find complete modes $E_2E_3E_6$, $E_1E_4E_5$, and at the fourth stage we get a four-element complete mode $E_1E_2E_3E_4$. Further search does not reveal any complete and incomplete modes. Let make a matrix of complete modes

$$\begin{aligned} & \{ E_5 \quad E_6 \} \\ & \{ E_2 \quad E_3 \quad E_6 \} \\ & \{ E_1 \quad E_4 \quad E_5 \} \\ & \{ E_1 \quad E_2 \quad E_3 \quad E_4 \} \end{aligned} \quad (1)$$

Now we have to solve an inverse problem of reconstructing the decay scheme from the complete modes obtained. The problem can be solved by comparing identical parts of the modes (which means following the same levels of the scheme) and succession of transitions in the modes.

If only the coincidence data are used, this process features two types of ambiguity. One is impossibility to distinguish between groups of upper and lower levels. Without additional information one cannot know whether the direct or "reverse" scheme is valid. The other ambiguity arises when successive transitions proceed via such intermediate levels that are connected only by two, upper and lower, transitions.

Neither of the ambiguities can be removed without additional information on total intensities of transitions ($I_\gamma + ICE$). The intensity balance must be fulfilled.

In a general case the mode matching process can be formalized if any transition is assumed to be parallel to itself. For the extreme (top-bottom) elements of one mode $\{E_a\}_i$ we find parallel transitions among those belonging to the mode $\{E_b\}_j$. This is always possible because, if the extreme elements of the modes coincide, it is enough to match them. If the extreme parts of the modes do not coincide, there are parallel sections which allow matching

their structures. By convention, we take one of the chosen extreme elements of the mode $\{E_a\}_1$ to be initial (top) and the other final (bottom). Then continuous transitions from $\{E_b\}_j$ parallel to the initial one from $\{E_a\}_1$ are above the continuous group of transitions from the mode $\{E_b\}_j$ parallel to the final transition from $\{E_a\}_1$. The operation performed with the two extreme elements is repeated with the neighbouring ones, successively treated as extreme, until the two modes are fully matched. Then sequences of the remaining modes are matched with the "ordered" ones.

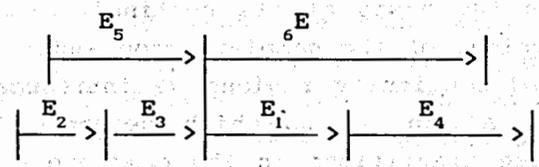
Let us apply the procedure to the example in Fig. 1. Each of the transitions E_1, E_2, E_3, E_4 is parallel to only one transition, which gives no information on continuity regions. The transition E_5 has two parallel transitions E_2, E_3 and the transitions parallel to E_6 are E_1, E_4 . To formalize the transition ordering process, we put the continuity regions in modes in parentheses and rewrite matrix (1) as

$$\begin{pmatrix} (E_5, E_6) \\ (E_1, E_4), E_5 \\ (E_2, E_3), E_6 \\ (E_1, E_4)(E_2, E_3) \end{pmatrix} \quad (2)$$

Ambiguity of extreme elements allows us to choose, for example, E_5 as the upper and E_6 as the lower transition in the first mode. The presence of identical transitions in different modes means that these modes go through the same level. Therefore in the course of ordering we put identical groups of transitions above one another:

$$\begin{pmatrix} \{ E_5, E_6 \} \\ \{ E_5, (E_1, E_4) \} \\ \{ (E_2, E_3), E_6 \} \\ \{ (E_2, E_3), (E_1, E_4) \} \end{pmatrix} \quad (3)$$

This representation is equivalent to the following decay scheme:



Using the relation $\varepsilon_6 = \varepsilon_1 + \varepsilon_4$ (where ε_1 are the energies of transitions E_1) we reduce the number of levels in the scheme. However this does not remove the ambiguity in placement of E_2, E_3, E_1, E_4 and in establishment the top and the bottom of our conventional scheme.

We have knowingly chosen and considered a very simple example to make it all clear and obvious. In a general case we deal with quite large matrices (e.g. 198 x 198), a great number of transitions in complete modes and continuity regions. In this case these processes must all be strictly formalized.

As a data representation version, it is convenient to arrange transitions in a matrix and in modes (before ordering) in order of increasing energy. This facilitates further check of and search for required transitions.

Theorem 3 is used to search for continuity regions in a given complete mode. The algorithm is quite simple. Transitions parallel to the given transition E_1 are designated by "0" in the row line E_1 . Continuity regions in the mode based on the transitions parallel to E_1 (i.e. the set $\{PE_1\}$) can be derived from logical products of the row $\overline{\text{line}E_1}$ the inverse of $\text{line}E_1$, and the row which is a complete mode derived in the above process of search for it. This procedure should be carried out for all E_1 and their associated sets of parallel transitions until all possible continuity regions are revealed in the complete

mode. In this way continuity regions are found for all complete modes.

The next stage is ordering of transitions in a complete mode on the basis of its continuity regions. To designate transitions of the complete mode and transitions of its associated continuity regions we introduce indices (parameters) m_1, a_1, b_1, \dots , which determine possible positions of these transitions in the complete mode or in continuity region. We shall write down the complete mode as

$$\{ E_{\mu_1}(m_1), E_{\mu_2}(m_2), \dots, E_{\mu_N}(m_N) \},$$

where μ_j are the indices of the transition belonging to the mode, m_1 are the indices of the transition position in the mode. Then the continuity region associated with the mode will be represented as

$$[E_{\alpha_1}(a_1), E_{\alpha_2}(a_2), \dots, E_{\alpha_k}(a_k)],$$

where the meaning of the Greek and Latin symbols α_j and a_j is the same as for the complete mode. These symbols have obvious properties:

$$\mu_1 \neq \mu_2 \neq \dots \neq \mu_N -$$

i.e. all transitions in the mode are different,

$$\alpha_1 \neq \alpha_2 \neq \dots \neq \alpha_k -$$

all transitions in the continuity regions are also different. For each α_j one can find only one μ_i for which $\alpha_j = \mu_i$. Any index m_1 has a value in the interval (1, N), and all indices differ from one another, i.e.

$$m_1 \neq m_2 \neq \dots \neq m_N -$$

Similarly, for the continuity region:

$$a_1 \neq a_2 \neq \dots \neq a_k$$

and, considering that the continuity region is in a complete mode, any index a_1 is in the interval (1+j, 1+j), where j can take on any of the values 0, 1, 2, ..., (N-1). Ordering of transitions in the complete mode begins with their comparison with transitions of the longest continuity region belonging to this mode. Comparison of transitions consists in assignment of identical position indices to the transitions in the mode and position indices to the continuity region transitions identical with those mode transition. In a particular case relationships between the identical transitions can be for example as follows:

$$\mu_3 = \alpha_1, \mu_5 = \alpha_2, \dots, \mu_k = \alpha_1$$

and then we shall have

$$m_3 = a_1, m_5 = a_2, \dots, m_k = a_1$$

Thus a continuity region is built in the mode. Transitions beyond the continuity region must have position indices that do not fall into the interval (1+j, 1+j) belonging to the continuity region. Thus the interval of possible position indices of complete mode transitions is limited. Then we compare possible (shortened) intervals of positions indices of complete mode transitions with the next continuity region (CR):

$$[E_{\beta_1}(b_1), E_{\beta_2}(b_2), \dots, E_{\beta_k}(b_k)]$$

Comparison with each next CR goes on up to the last CR of the given mode.

This procedure results in full (or partial) ordering of transitions in the complete mode at hand, i.e. in unambiguous (or not quite unambiguous because of insufficient coupling conditions) determination of values of position parameters m_1 .

The next step in constructing a scheme of excited levels is full (or partial) matching of ordered complete modes. Noteworthy is that fully ordered modes allow unambiguous placement of transitions. Using a recurrence histogram of transitions in complete modes we choose all modes with a transition appearing in a majority of complete modes. Obviously, it is reasonable to place this transition at the bottom of the decay scheme. For the complete modes which were not included in the first set we find a transition that appears in a majority of the remaining complete modes. These modes make up a second set analysed like the first one. As the above procedure is repeated several times, all complete modes are analysed and construction of the scheme of excited levels is finished.

3. Adaptation of formal logical method to real experimental data

The method and the program on its basis [8, 9, 10] work perfectly with ideal experimental data, i.e. with a coincidence matrix that contains all true coincidences and no false ones. However experimental limitations do not normally allow information on all true coincidences of γ -transitions, and errors in processing and analysis may yield false coincidences as well. As a result, there are inevitable errors in the matrix and a lot of false CM's and CR's appear. For example, in our analysed coincidences only 35-50% of CM's and 10-30% CR's turn out to be "correct". Therefore it is necessary to find a possibility of selecting "correct" CM's and CR's to deal with, otherwise intractable contradictions arise.

To select true, i.e. "correct", CM's and CR's we demand that the sum of transition energies in the derived CM's and CR's should be in a certain balance [11]. Thus, in

the further analysis we use only those CM's and CR's for which the sum of transition energies in them is equal to the sum of energies of all transitions in another CM or CR or to the energy of a single transition which may not be observed in coincidences:

$$\left| \sum_{i=1}^a E_i - \sum_{j=1}^b E_j \right| \leq 2 \sqrt{\sum (dE_i)^2 + \sum (dE_j)^2} \leq r \quad (4)$$

where r is the cut-off parameter of inaccurate data, which can be set for example equal to 0.2 keV.

It turned out to be reasonable to unite all CM's and CR's selected in this way to a single array of "correct" CM's and CR's. We shall call it an array of "generalized continuity regions", GCR (complete modes are a particular, limiting case of continuity regions).

This allows compensation for inevitable loss of CM's and CR's arising from the energy selection. Some CR's from "incorrect" CM's turn out to be "correct" CM's and are added to them. Besides, by comparing elements of the GCR array (the details will be given in the description of the program) one can find additional conditions of coupling between transitions inside each element, i.e. reveal their internal CR's. Then transitions are successfully ordered in each GCR. To this end, the above-described mechanism can be used.

This approach not only enriches the useful information for the analysis. In some cases CR's can contain all reliable information, and thus the generalized approach to CM's and CR's allows one to use the program without any changes.

With these additions, the method shows highly stable work with coincidence data gained under non-optimum experimental conditions, and the new version of the program COIN allows working with real experimental data.

4. Program COIN

The program COIN is written in C++ (Borland C++ 3 medium). To provide a developed and standardized interface, a library of classes and functions Turbo Vision 2.0 is used. The program widely uses an object approach to representation of the data, which makes the text of the program clear and devoid of nonobvious solutions.

The program is divided into two quite independent parts:

- a module for reading data on transitions and coincidences between them and constructing an array of generalized continuity regions (GCR) from these data;
- a module responsible for "assembly" of the decay scheme on the basis of the GCR array.

As most programs involving the Turbo Vision library, this program is an analyser of events generated by the user. Events are selection of a menu item, "keystrokes" in the window, input of a text in editor lines, etc.

4.1. Structure of module for construction of array of generalized continuity regions (GCR).

Two modes of operation are provided:

- the automatic mode, in which the program uses the data from the current configuration file or those set by the user before running the program;
- the interactive mode, in which the program calls the user each time it needs any data.

The program run is similar in both modes and consists of the following stages:

- (a) read-out of the data on transitions found by analysing a one-dimensional γ -spectrum of a nucleus (energies, intensities);
- (b) read-out of the data on coincidences (coincidence

tables with energies and intensities of coinciding transitions) and search for possible multiplets by comparing energies of lines in coincidence with lines of the single spectrum of transitions (the cases where it is impossible to determine which line of the group coincides with another line);

- (c) construction of a coincidence matrix from the data on coincidences of transitions with allowance made for the results of searching multiplets and choosing one of the multiplet lines;

- (d) formation of an array of complete modes (CM) on the basis of the coincidence matrix;

- (e) formation of an array of continuity regions (CR) found inside the complete modes;

- (f) formation of an array of generalized continuity regions (GCR) on the basis of energy criteria defined by the user;

- (g) internal ordering of GCR's and formation of an array of ordered GCR's.

The program successively fulfills the above stages and turns over the control to the user, who can look through results and decide whether to construct another array of internally ordered GCR's or to proceed to "assemble" the decay scheme.

In the interactive mode the program allows the user to choose the stage at which he interrupts the run to look through results and to change, if necessary, the parameters of the search for internally ordered GCR's. At stage (b) an energy window dE is set, within which the program compares the line energy in the coincidence table with line energies in the table of the single transition spectrum. The lines falling into the window (if there are more than one line) make up a multiplet, and the program allows the user to choose one of them (by energy or intensity) to include in the coincidence matrix.

At stage (e) it is possible to influence the search for CR's by intensity cut-off of coincidence matrix transitions to which parallel transitions are found. Transitions of weak intensity may fail to show up in some coincidences, which results in "extra" parallel transitions and further in false CR's or false transitions in them. This cut-off is introduced only in the procedure of searching for CR's and is never used at other stages. The cut-off is quite effective because it substantially reduces the number of false CR's. After the intensities of transitions in the matrix of γ - γ coincidences from the $^{172}\text{Lu} \rightarrow ^{172}\text{Yb}$ decay were cut off by about 3% (of the maximum intensity), the number of modes (of rank 3 and higher) with matching CR's increased from 15 to 90 per cent.

Formation of the GCR array at stage (f) includes procedures which make it possible to modify the formal logical method of decay scheme construction and successively use with real experimental data. CM's and CR's found in them are selected by the energy balance (expression (4)) between them and with single lines of the given nucleus. The necessary conditions for including CM's and CR's in the GCR array may vary depending on the character of the input data. Therefore the program allows quite flexible and wide variation of the selection criteria by offering various criteria and parameter values in appropriate windows.

The first to set is the value of the parameter "r" which determines the tolerable energy measurement error. The possible options are (i) to use or not to use energy coincidences of CM's and CR's with single transitions (this may be done separately for CM's and CR's), (ii) to set the lowest multiplicity of energy coincidences separately for CM's and CR's allowing them to be included in the GCR array, (iii) to use or not to use energy coincidences of

CM's and CR's as a criterion for their inclusion in the GCR array. This ensures quite flexible and differential selection of CM's and CR's for the GCR array.

The criteria vary so greatly to allow for difference in degree of reliability between CM's and CR's (CM's are normally more reliable and their coincidence with single transitions is quite enough), to allow for possible substantial difference in number between CM's and CR's selected for the GCR array from different experimental data, and to make the most use of the experimental information for reconstruction of the decay scheme. Varying the selection criteria one can get several different GCR arrays which, after ordering of transitions in them, are used to construct a decay scheme. The ordering of the GCR's themselves will also depend on their concrete set.

With the energy selection criteria, we appreciably reduce the number of CM's and CR's. The remaining CR's can be too few to provide internal ordering of GCR's. In this connection it is reasonable to extend the list of CR's, which can be done by comparison of CM's and CR's from the GCR array to reveal identical transitions. This may yield additional coupling conditions, namely new internal CR's for CM's and CR's.

Absence of identical transitions in comparison does not yield additional information. If there are identical transitions, one can check whether they are continuity regions as well as whether the nonidentical transitions in both GCR's under comparison are continuity regions too. Two cases are distinguished: (i) comparison of energy-identical (criterion (4)) GCR's to reveal coupling conditions among NONIDENTICAL TRANSITIONS and (ii) comparison of energy-nonidentical GCR's to reveal coupling in IDENTICAL TRANSITIONS.

In the case (i), we compare two GCR's that have P and Q transitions with the equal energy sums and we suppose

they have C identical transitions. Then the next equation takes place:

$$\sum_{i=1}^C X_i + \sum_{i=1}^{P-C} Y_i = \sum_{i=1}^C X_i + \sum_{i=1}^{Q-C} Z_i ; Y_i \neq Z_j \text{ for each } i, j \quad (5)$$

If it is impossible to find at least two particular sums of transitions Y_i and Z_j when the system of the next equations takes place:

$$\sum_{i=1}^m Y_i + \sum_{i=m+1}^{P-C} Y_i = \sum_{i=1}^n Z_i + \sum_{i=n+1}^{Q-C} Z_i ; \sum_{i=1}^m Y_i = \sum_{i=1}^n Z_i \quad (6)$$

then it means that transitions Y_i and Z_j don't create internal CR's which could be separated by C identical transitions and consequently they all are CR. In the particular case, the conditions (6) cannot be fulfilled when:

$$\begin{aligned} & ((P - C = 1) \text{ and } (Q - C > 1)) \text{ or} \\ & ((P - C = 2) \text{ and } (Q - C \leq 3)) \end{aligned} \quad (7)$$

For the case (ii), the condition (5) is not true, it is possible to see whether C identical transitions are CR? These C transitions will be CR if they could not be separated by the groups with the identical sums of transitions Y_i and Z_j . It means that the following equations are not true:

$$\sum_{i=1}^{m \leq P-C} Y_i = \sum_{i=1}^{n < Q-C} Z_i \text{ or } \sum_{i=1}^{m < P-C} Y_i = \sum_{i=1}^{n \leq Q-C} Z_i \quad (8)$$

If they really are not true, the transitions X_i create CR. For the particular case, equations (8) are not true,

when:

$$\begin{aligned} & P - C = 0 \text{ and } Q > P \\ & P - C = 1 \text{ and } Q - C \leq 2 \\ & P - C = 2 \text{ and } Q - C \leq 2 \end{aligned} \quad (9)$$

So far the program involves only this simple algorithm for analysis. If conditions (7) and (9) are not satisfied, we cannot say anything and get additional coupling conditions. A more detailed analysis with much more complicated algorithm is required in this case. But a full analysis like this, involving energy relations, is rather cumbersome in a general case and cannot be also absolutely reliable as in the case of searching for and selecting CM's and CR's by energy coincidences.

The result of this phase is an array of GCR's with CR's found for each of them.

Internal ordering of each GCR (its principle is described in subsection 2.2) is carried out by a simple method given in Appendix 2. The result is an array of internally ordered GCR's. The first phase of the program run is over.

4.2. Program of "assembling" decay scheme based on array of generalized continuity regions (GCR)

Unlike the GCR formation phase, the decay scheme construction is not algorithmized rigorously enough to be fully performed by a program. Experience shows that correct and flexible construction of the decay scheme requires active participation of a man. Therefore the program block responsible for this part of work provides much better possibilities of controlling the work and allows better flexibility of the control than in the previous phase.

Since the coincidence matrix may correspond to fragments of one decay scheme not connected with one another or to different decay schemes, it is reasonable to

divide the GCR array into groups connected to one another by common transitions (or through common transitions). For an isolated GCR group a "prime" GCR is chosen with which the construction of the decay scheme begins. Then GCR's of this group are analysed by the exhaustive method for (i) presence of a transition which populates the ground state in the decay scheme built so far (it may be the prime GCR), (ii) presence of two and more transitions already included in the decay scheme, and (iii) being in the same energy group with a GCR already included in the decay scheme. If the GCR under examination has one of the above features it is put into the existing decay scheme after "orientation" on the energy scale.

The choice of the prime GCR is an important step in the construction of the decay scheme. Sometimes several versions must be tested to make the proper choice suitable for further work. Several interactive windows provide wide possibilities of making the choice. The user can choose one of the formed groups for common transitions, and inside the chosen group he can select a prime GCR for different properties, including the maximum energy in the group, the maximum energy among the GCR's with the most frequently occurring extreme transition, or he can pick a GCR from an equal-energy group of GCR's (e.g. a group with maximum energy, a group with the largest number of members), a group with energy equal to the single transition energy. Inside these groups (based on equal energy) the user may choose a prime GCR for the maximum number of transitions, for the maximum intensity transition, or simply indicate a certain GCR. Following the user's instruction, the program finds the GCR by the preset criteria and displays it on the screen. Then the chosen region is oriented on the energy scale. The interactive window offers to choose between the most frequently occurring extreme transition and the most intense extreme transition as the lower transition. The

program displays the initial scheme and stores it in a file with a user-defined name.

Then the decay scheme extension stage begins. GCR's of a given group are examined one by one for satisfying the above-mentioned three criteria. If the region satisfies them, the user is allowed viewing of the decay scheme with and without the given GCR in a graphic mode. It is also possible to invert the GCR, display the resulting scheme and decide whether or not it should be included in the decay scheme. All operations are performed by appropriate "keystrokes" in a relevant interactive window. On deciding to omit the GCR, one can mark it as "false" and reject from further consideration or keep it and return to it later as the decay scheme becomes fuller.

If the GCR is included in the decay scheme, the program again displays the scheme in the graphic mode for assessment. In this case the program recognizes the following estimations: (j) the GCR is beyond the energy limits of the current decay scheme. It can be either correct or wrong. The program leaves it to the user to make the ultimate decision and displays a window which allows the region to be inverted, kept, or rejected; (jj) putting the GCR into the decay scheme results in a "hole", or break, in the GCR. The transitions that are neighbouring in the GCR de-excite/populate levels of different numbers in the scheme. This indicates that the GCR in question is defective or that a false GCR was previously included in the decay scheme. The user chooses the option by a "keystroke" in the displayed window. Then the program analyses the next GCR and so on until the group is exhausted. After that the main menu takes over the control and the user can view the results, construct a new version of the decay scheme (under different conditions), or view again the group of GCR's to extend the decay scheme based on it.

4.3. Example of COIN operation

Operation of the program can be illustrated by construction of the $^{166}\text{Tm} \rightarrow ^{166}\text{Er}$ beta-decay scheme. This decay was studied previously [12] and a scheme of excited levels in ^{166}Er was constructed but without the data on γ - γ coincidences as they were not the subject of that investigation. We used the γ - γ coincidence data gained with a multidetector set-up at the YASNAPP complex in Dubna [13]. A coincidence matrix of 68x68 γ -transitions was constructed on the basis of the γ - γ coincidences. Comparison of coincidence transitions with single spectrum transitions yielded multiplets from which γ -rays of maximum intensity were selected. Complete modes (155) and incomplete modes (720) were found. As many as 423 continuity regions were derived (without intensity cut-off for matrix γ -quanta). Complete modes and continuity regions were grouped into an array of GCR's subject to the requirement that - selected CM's are equal in energy to any coincident transition or to a CR, or to other CM's with coincidence folding of 2 (a total of 67 CM's are selected); - selected CR's are simultaneously equal in energy to any transition and a CM or CR (50 CR's are selected); - the total energy error in the comparison is below 0.2 keV. The number of the GCR array members is 117. By finding internal CR's for the GCR array members and ordering transitions we ordered 74 GCR's. This ends the first stage of the program operation.

At the second stage, which is reconstruction of the decay scheme by the data gained at the first stage, the following procedures were carried out. A GCR not obligatory equal in energy to any transition and with the maximum intensity transition as an extreme one was picked as a prime one from the maximum energy group by means of the

interactive window. Meeting these requirements was the GCR:

Mode : Energy: 2160.124; dE : 0.027; NumMembers: 3;
Transition numbers in coincidece matrices :
49 --> 1300.725(16) ; 29 --> 757.798(15) ; 1 --> 80.585(15)

The chosen prime GCR was oriented by means of the interactive window in such a way that the maximum intensity transition "1" is at the bottom. In Fig.2 this prime GCR is shown by heavy arrows on the energy scale. For further reconstruction of the decay scheme GCR's were selected for the following properties:

- they have an extreme transition in common with the prime GCR, which is taken to be the lower one;
 - they are members of an energy group with GCR's like this.
- After placement of these GCR's in a prime scheme we got a fragment of the decay scheme shown in Fig.2 by medium-bold lines (only additionally included transitions from the GCR's used are given, repetitive transitions are shown only once).

In further construction of the decay scheme we used GCR's with two or more transitions already included in the scheme and others from their energy groups. Some GCR's had to be re-orientated, some were marked as "wrong" because they contradicted the current decay scheme. This selection of new GCR's was repeated twice. At this stage another 35 GCR's were added to the prime one and those 11 GCR's included in the scheme at the previous stage. The last stage's addition to the scheme is shown in Fig.2 by thin lines. This decay scheme fragment contains 47 of 74 internally ordered GCR's. We give only one possibility of using the GCR array to construct a decay scheme.

Varying the criteria for selection of CM's and CR's to be included in the GCR array one build several versions of the decay scheme, compare them, select the most stable

levels and transition positions. Besides, on obtaining the framework of the scheme, one can continue reconstructing it by hand, using the techniques that we have not algorithmized at this stage yet.

The given fragment of the $^{166}\text{Tm} \rightarrow ^{166}\text{Er}$ decay scheme in the range up to 1572.2 keV reproduces all levels, except the 545.434 keV one, obtained in [12]. In the range up to 2160.110 keV about 50% of the levels are reproduced and there are no disagreement with the above paper. It should be mentioned that we used coincidence data for only 68 of 300 transitions recorded in the one-dimensional spectrum and avoided any other approaches to reconstruction of the decay scheme.

With the γ - γ coincidence data for the decays $^{169}\text{Lu} \rightarrow ^{169}\text{Yb}$ [14] and $^{172}\text{Lu} \rightarrow ^{172}\text{Yb}$ [15], the program fully reconstructed the decay schemes described in those papers.

5. Technical characteristics of program

The program is a 370kb .exe file executable in computers with an INTEL 80x86 processor. The required RAM capacity is not less than 640kb and free disc space about 1Mb. When operating, the program creates several temporary files (deleted on exit) and a protocol file. The run time greatly depends on the data amount and the rate of exchange with the disc. The program was tested at a PC IBM AT 386 DX-40 with a Smartdrive MS-DOS 6.20 read-write hard-disc cache of 1 Mb. In the case of processing data with 320 transitions in a one-dimensional γ -spectrum and a 70x70 coincidence matrix it took below 5 minutes to construct an array of internally ordered GCR's, and the program almost promptly responded to the user's instructions during the construction of the decay scheme. The program was successfully used to process the data with a 198x198

coincidence matrix. The array of internally ordered GCR's for these data was built for 15 minutes in the automatic mode. Working without hard-disc cache or starting the program from a floppy disc may substantially slow down the run.

6. Conclusion

The proposed method and program for analysis of coincidences and reconstruction of decay schemes allow quite full and diverse use of the experimental data and facilitate the work of a researcher. With the program, it is possible not only to select reliable γ - γ coincidence data and reconstruct a decay scheme from them but also to use "wastes" of this process to extend the scheme, to assess the operation of the equipment, and to analyse experimental data.

CM's and CR's from the GCR array which did not fit into the decay scheme as contradictory may have defects of different types. First of all, they can be wrongly ordered inside and yet have a "correct" set of transitions which fits into scheme if one ignores the arrangement of the transitions in the GCR. These GCR's can be put into the scheme and usefully add new transitions to it. There can be GCR's with omitted transitions or "extra" transitions parallel to others. The defects often occur together. Analysis of these GCR's can also yield useful information. Omission of transitions, which shows up as breaks arising in the GCR when it is placed in the decay scheme, indicates that there are weak (strongly converted) transitions which do not manifest themselves in coincidences.

Parallel transitions in GCR's indicate defects of data processing or coincidence analysis and also allow useful information for evaluating the quality of measurements and of their processing. We think that the given approach and

program COIN together with other methods and programs will speed up and facilitate analysis of coincidences and construction of decay schemes.

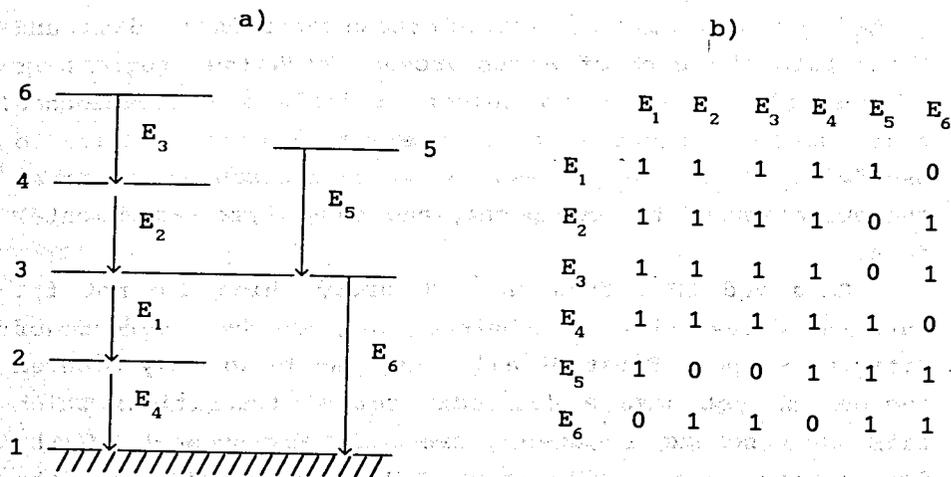


Fig.1. a - "the decay scheme"; b- the matrix of γ - γ coincidences of the decay scheme (a), but with the "units" instead "zeros" on the main diagonal.

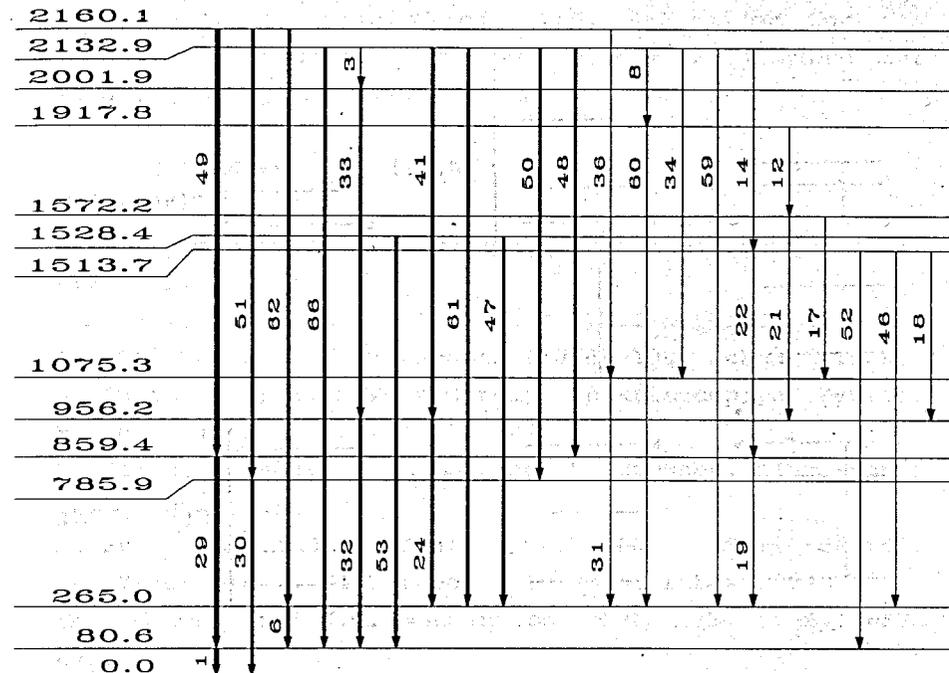


Fig.2. Illustration to the decay scheme construction of $^{166}\text{Tm} \rightarrow ^{166}\text{Er}$ on the base of γ - γ coincidence data. On the left, there is the mode with the sum energy of its three transitions 2160.124(27) keV, which is taken as starting. This mode is shown by heavy arrows on the energy scale. The transitions placed to the scheme on the second stage are shown by the half-heavy arrows. On this stage the GCR's are used which have common extreme transition or belong to the same energy group with the starting mode. The transitions which are built to the scheme on the third stage are shown by thin arrows. The transitions common for different modes are shown only one time. At the left side of transitions the vertical numbers show their places in coincidence matrix.

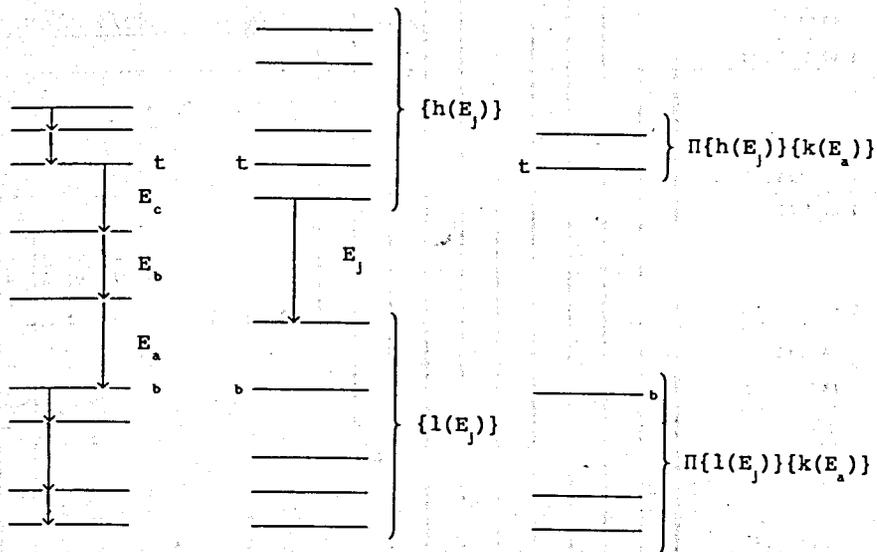


Fig.3. Illustration to the third theorem proof. At the centre the position of the transition E_j in the system of the decay scheme levels is shown. At the top, there is the system of the levels $\{h(E_j)\}$ being on which the nucleus suffers transition E_j . At the bottom, there is the system of the levels $\{l(E_j)\}$ which the nucleus occupies after transition E_j . At the right top side, there is $\Pi\{h(E_j)\}\{k(E_a)\}$ - the intersection of the levels $\{h(E_j)\}$ and the levels of the mode $\{E_a\}_b$. Here "t" is the lower level among this group. At the right bottom corner, there is $\Pi\{l(E_j)\}\{k(E_a)\}$ - the intersection of the levels $\{l(E_j)\}$ and the levels of the mode $\{E_a\}_b$, and "b" is upper of them. At the left, there is the mode $\{E_a\}_b$ in the system of its levels $\{k(E_a)\}$ and the continuous region (E_a, E_b, E_c) in the system of decay scheme levels.

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Appendix 1.

Theorems and their proofs.

Theorem 1.

If there exists a set of coincidence matrix rows $\{\text{line}E_i\}_k$ with a set of indices of its component rows $\{i\}_k$ such that logical multiplication of its component rows yields a row with unities over the entire set $\{i\}_k$ of indices, the transitions with labels from this set $\{E_i\}_k$ make up at least an incomplete mode. The opposite is valid too.

The direct theorem is proved by contradiction. Let the set $\{\text{line}E_i\}_k$ with the above properties define the set $\{E_i\}_k$ which is not an incomplete mode (or a complete mode). Then there exists at least two transitions E_b and E_c from $\{E_i\}_k$ which are parallel in the decay scheme. But in this case E_b will not coincide with E_c (and vice versa). This means that in row "b" there will be "0" in position "c", and in row "c" there will also be "0" in position "b". Their logical multiplication will not make "1" in positions

"b" and "c" either, which contradicts the condition of the theorem.

The inverse theorem (proof). If $\{E_i\}_k$ is an incomplete (complete) mode, then all transitions in it coincide by definition. For each E_a from $\{E_i\}_k$ the row $\text{line}E_a$ has unities in all positions of the set $\{i\}_k$ (including position "a" because of the diagonal of unities) but it is not obligatory only them. Since it is valid for all rows, their logical product will also have unities over the $\{i\}_k$ set of indices. Theorem 2.

If there is a set of coincidence matrix rows $\{\text{line}E_i\}_k$ with indices of its component rows $\{i\}_k$ such that logical multiplication of all its component rows yields a row with unities and only unities over the entire set of indices and no unities in another places, the transitions of this set $\{E_i\}_k$ make a complete mode (CM). The opposite is valid too.

Let $\{E_i\}_k$ not be a complete mode. By theorem 1, the set $\{E_i\}_k$ must then be only an incomplete mode and not an arbitrary set. Then there exists at least one transition E_j not from $\{E_i\}_k$ for which $\{\{E_i\}_k + E_j\}$ is a complete or incomplete mode. By the theorem inverse to theorem 1, in all $\{\text{line}E_i\}_k$ there must be unities in the j-th position. This means that logical multiplication makes a unity in the j-th place too. It contradicts the condition of the theorem, under which unities must be only in positions from the set $\{i\}_k$.

The inverse theorem (proof). Indeed, if we considered $\{E_i\}_k$ and transitions formed by "extra" unities in accordance with theorem 1, we would find that $\{E_i\}_k$ and those transitions are at least an incomplete mode. This contradicts the mode completeness, which proves the theorem.

Theorem 3.

Let E_j be an arbitrary transition, $\{PE_j\}$ a set of

transitions parallel to it. If intersection of this set $\{PE_j\}$ with an arbitrary complete mode $\{E_a\}_k$ consists of more than one element, the part of this complete mode that fully belongs to the intersection

$$\Pi\{PE_j\}\{E_a\}_k$$

is a continuity region (CR), i.e. consists of a continuous succession of transitions of this complete mode.

Proof. Let $\{l(E_j)\}$ be a set of all levels (and only of them) in the decay scheme which are possible for a nucleus after the transition E_j and later, as a result of other transitions as well (see fig. 3). Let $\Pi\{l(E_j)\}\{k(E_a)\}$ be intersection of the set $\{l(E_j)\}$ and the levels underlying the mode $\{E_a\}_k$. Let $\{h(E_j)\}$ be a set of all levels (and only of them) being at which a nucleus can undergo the transition E_j . $\Pi\{h(E_j)\}\{k(E_a)\}$ is the intersection of this set with the set of all levels underlying the mode $\{E_a\}_k$. We designate the lowest level from the set $\Pi\{h(E_j)\}\{k(E_a)\}$ in the decay scheme by t and the highest level from $\Pi\{l(E_j)\}\{k(E_a)\}$ by b . If $\Pi\{h(E_j)\}\{k(E_a)\}$ is empty, t will be the highest level from the mode $\{E_a\}_k$ in the decay scheme. If $\Pi\{l(E_j)\}\{k(E_a)\}$ is empty, b will be the lowest level from the mode $\{E_a\}_k$ in the decay scheme.

Let us take an arbitrary transition E_b from those belonging to the mode $\{E_a\}_k$ and parallel to E_j , i.e. those not belonging to the same mode as E_j . Then t is not lower in the decay scheme than the level at which the transition E_b begins. If it were lower, E_b would proceed to it or ended above it. Thus, it would be in the same mode as E_j . On the strength of similar considerations level b is not higher than the level at which the transition E_b ends. As E_b is an arbitrary transition from the above-mentioned intersection, all transitions from it are between t and b . On the other hand, they all belong to the same mode and are parallel to E_j , which perfectly complies with the condition of the theorem.

Appendix 2.

The position of each CR in a GCR is represented by binary numbers, unities standing for positions that can be occupied by transitions of the given CR and zeros for impossible ones. Then different CR arrangement variants are logically multiplied bit by bit, which reveals permissible transition positions and allows their matching.

Let us consider a GCR $\{E_a, E_b, E_c, E_d\}$ for which CR's (E_a, E_b, E_c) , (E_a, E_b, E_d) , and (E_a, E_b) are found. The table lists possible arrangements of transitions from the first CR in the GCR:

	E_a	E_b	E_c	E_d		E_a	E_b	E_c	E_d
(I)	0111	0111	0111	1000	and in short form:	7	7	7	8
(II)	1110	1110	1110	0001	and in short form:	E	E	E	1

where E is equal to the binary number 1110=14 in the hexadecimal system. For the second and third CR we can write down the following arrangement tables

	E_a	E_b	E_c	E_d		E_a	E_b	E_c	E_d
(III)	7	7	8	7	(V)	3	3	C	C
(IV)	E	E	1	E	(VI)	6	6	9	9
					(VII)	C	C	3	3

where C is the hexadecimal number 1100. The true arrangement of transitions in the GCR must be consistent with one of the combinations of possible arrangements of transitions for all three CR's. This arrangement can be found by logical bit-by-bit multiplication of possible variants from a set of rows for different CR's. By exhaustion of possible triple variants, we find that only two of them are acceptable and others yield zero products

for at least one transition, which shows impossibility of placing it:

	E_a	E_b	E_c	E_d
(I) & (IV) & (VI)	6	6	1	8
(II) & (III) & (VI)	6	6	8	1

The resulting two mirror sets reveal an objective difficulty of establishing the "top-bottom" of the decay scheme from coincidences alone, and the method allows two options. In the example the positions of the transitions E_a and E_b are not unambiguously determined inside each set because the coupling conditions chosen by us are insufficient for this determination. Unambiguous ordering requires one more GCR, e.g. $\{E_a, E_c\}$, for which we get the following transition arrangement variants in the given GCR:

	E_a	E_b	E_c	E_d
(VIII)	1100	0011	1100	0011
(IX)	0110	1001	0110	1001
(X)	0011	1100	0011	1100

Examining combinations of transition arrangement variants in the GCR, we find that only the combinations (logical products) of variants [I, IV, VI, VIII] and [II, III, VI, X] have not zeros for all transitions. Thus, we arrive at two symmetrical solutions

$$\{E_c, E_a, E_b, E_d\} \text{ and } \{E_d, E_b, E_a, E_c\}.$$

This is virtually an unambiguous solution up to inversion of the mode.

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