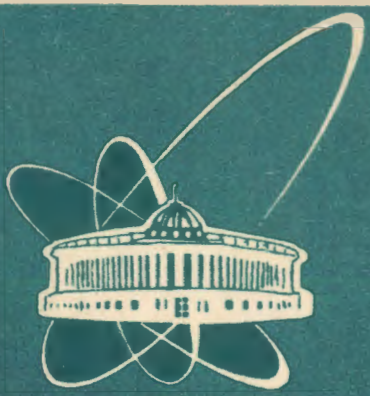

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COMPARISON OF MULTICHANNEL SPECTRAL MEASUREMENTS

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1 Introduction

Recognition of objects on the basis of their spectral reflective characteristics is one of the central problems in remote sensing of the Earth surface. (Note that similar problems involving emission or absorption spectral characteristics instead of the reflective ones arise in many other fields of applied spectroscopy as well.) In most of the sensors, used in remote sensing, the spectral range is split into separate non-overlapping bands (or "channels") and the output is vectorized. The number, width and location of the channels are important characteristics of the sensors and may vary in very large limits. Here are a few examples: the Nimbus-7 system has 4 channels (20 *nm* wide each) in the visible region (VIS) and 1 of 100 *nm* – in the near infrared region (NIR); Landsat MSS has 4 channels, 100 *nm* wide each; the width of the 4 channels of Landsat TM varies from 60 to 140 *nm*; Skylab MSS has 12 cannels in VIS and NIR, about 50 *nm* wide each [1]. The russian earth resource satelites "Meteor" are supplied with scanners MSU–M – with 4 – and MSU–S – with 2 channels; "Fragment" has 8 cannels in VIS and NIR [2]. The trace spectrometers developed in the Bulgarian Academy of Sciences can operate upto 256 narrow spectral bands in VIS and NIR range [3, 4]. The data volume from aircrafts can be still higher [1].

Multichannel experimental data may be represented by points in the n-dimensional linear space R^n, referred to as *characteristic space* (n standing for the number of channels of the spectrometer). Points that correspond to different measurements of the same object normally lie close to each other, thus forming *clusters*; points from different objects are expected to belong to separate clusters [1, p.76]. Most of the methods for classification and recognition of objects acquire quite a simple geometric meaning in terms of characterisic spaces; these methods can only be applied, however, if all the experimental data under consideration belong to the same characteristic space, i.e. if they are obtained with spectrometers with *the same number, location and width* of the channels. If this is not the case, the spectral data from different spectrometers cannot be compared directly. The commonly used procedure here consists in constructing models for the spectral densities that are being compared, with some parameters which are later determined from the experimental data. If the different sets of parameters (as extracted from the different sets of data) are consistent within the uncertainty related to their specific statistical properties, the corresponding objects cannot be

distinguished, and *vice versa* [5, p.25]. In the present paper we suggest an alternative method for comparison of experimental data obtained with different spectrometers. We derive a model-independent linear transformation of the experimetal data that maps the characteristic space associated with one of the spectrometers onto the characteristic space of the other one. The transformed data - now belonging to the same characteristic space - are shown to be distributed by the multivariate normal distribution that allows for standard statistical methods to be applied in the classification and object recognition tasks [5].

2 The interpolation formula

Let $\rho(\lambda)$ be a smooth enough function of λ: $\rho(\lambda) \in \mathcal{C}^n[\lambda_{min}, \lambda_{max}]$, and denote

$$Y_k = \int_{a_k}^{b_k} \rho(\lambda)\, d\lambda, \quad \lambda_{min} < a_k < b_k < \lambda_{max}, \quad k = 1, \ldots, n \quad . \tag{1}$$

We shall now derive an interpolation formula for the integral

$$Y = \int_a^b \rho(\lambda)\, d\lambda, \quad \lambda_{min} < a < b < \lambda_{max}, \tag{2}$$

in the form of a linear combination of Y_k with coefficients u_k depending on a_k, b_k, a, b only:

$$Y = \sum_{k=1}^{n} u_k Y_k + R_n(a,b), \tag{3}$$

such that Eq.3 is *exact* (i.e. the remainder $R_n(a,b)$ vanishes) for $(a,b) = (a_k, b_k)$, $k = 1, \ldots, n$.

Consider the polynomial $P(\lambda) = \sum_{i=0}^{n-1} c_i \lambda^{i-1}$ with coefficients c_i determined from the requirement that

$$\int_{a_k}^{b_k} P(\lambda)\, d\lambda = \sum_{i=1}^{n} c_i (b_k^i - a_k^i)/i = Y_k, \; k = 1, \ldots, n.$$

Since $\int_{a_k}^{b_k} (P(\lambda) - \rho(\lambda))\, d\lambda = 0$, there exist n points $\xi_k, a_k < \xi_k < b_k, k = 1, \ldots, n$, such that $P(\xi_k) = \rho(\xi_k)$; therefore $P(\lambda)$ is a Lagrange's interpolation

polynomial of $\rho(\lambda)$ through the points $\xi_k, k = 1, \ldots, n$:

$$\rho(\lambda) = P(\lambda) + r(\lambda), \tag{4}$$

$$r(\lambda) = \prod_{i=1}^{n} (\lambda - \xi_i) \cdot \frac{\rho^{(n)}(\xi)}{n!}, \; \min_i a_i < \xi < \max_i b_i$$

The coefficients c_i satisfy the system of linear equations

$$\sum_{i=1}^{n} M_{k,i} c_i = Y_k, \; M_{i,k} = (b_k^i - a_k^i)/i \tag{5}$$

For noncoinciding (a_k, b_k), $k = 1, \ldots, n$ (with the exception of the few peculiar cases when occasionally $det(M) = 0$) Eq.5 has a unique solution $c_i = \sum_{k=1}^{n} M_{i,k}^{-1} Y_k$. After substituting these c_i's back in $P(\lambda)$ we obtain for the integral of $P(\lambda)$ over an **arbitrary** interval $[a,b]$ the expression:

$$\int_a^b P(\lambda)\, d\lambda = \sum_{i=1}^{n} c_i (b^i - a^i)/i = \sum_{k=1}^{n} u_k Y_k \tag{6}$$

$$u_k = \sum_{i=1}^{n} \frac{b^i - a^i}{i} M_{i,k}^{-1} \tag{7}$$

Here are the explicit expression of u_k for the few lowest values of n:

$$n = 2: \; u_i = \frac{(b-a)(b_j + a_j - b - a)}{(b_i - a_i)(b_j + a_j - b_i - a_i)}, \; i = 1,2; \; j = 3 - i \tag{8}$$

$$\begin{aligned} n = 3: \; u_i = \; & d^{-1} \sum_{j,k=1}^{3} \epsilon_{ijk} \; [(b-a)\left(b_j^2 - a_j^2\right)\left(b_k^3 - a_k^3\right) + \\ & \left(b^2 - a^2\right)\left(b_j^3 - a_j^3\right)(b_k - a_k) + \\ & \left(b^3 - a^3\right)(b_j - a_j)\left(b_k^2 - a_k^2\right)] \end{aligned} \tag{9}$$

$$d = \sum_{i,j,k=1}^{3} \epsilon_{ijk} \; (b_i - a_i)\left(b_j^2 - a_j^2\right)\left(b_k^3 - a_k^3\right)$$

ϵ_{ijk} being the 3-dimensional antisymetric tensor, $\epsilon_{123} = 1$. For higher n the explicit expressions of u_k become far too complicated; solving numerically Eq.5 is much simpler in this case.

Integrating Eq.4 with respect to λ over $[a,b]$ now leads to the interpolation formula of Eq.3 with the following explicit expression for the remainder $R_n(a,b)$:

$$R(\lambda) = \int_a^b r(\lambda)\, d\lambda, \quad |R(\lambda)| \le (b-a) \max_\lambda \left|\rho^{(n)}(\lambda)\right| (b_n - a_1)^n / n! \tag{10}$$

3 Transformation of the characteristic space

Let $\rho(\lambda)$ be the spectral characteristic of an object as a function of the wavelength λ, and let the spectrometer we are using have n channels with constant sensitivity within the bands $a_k < \lambda < b_k$, $k = 1, \ldots, n$, and 0 outside them. What we shall obtain as an output from our spectrometer are the quantities Y_k (see Eq.1), and Y (see Eq.2) should be measured in the spectral band $a < \lambda < b$. The interpolation formula of Eq.3 allows for expressing Y in terms of Y_k in a *model-independent way*, provided that $\rho(\lambda)$ is smooth enough; $R_n(a,b)$ (see Eq.10) gives an estimate of the possible inaccuracy of our guess.

In case the Y_k's are experimental values, they should be regarded as random variables. Under quite general assumptions [1, p.209] they can be shown to be independent and the individual distribution of any of the Y_k's to be normal with mean value y_k and dispersion σ_k^2 (also denoted as $N(y_k, \sigma_k^2)$). Leaving out the remainder R_n we now may interprete the interpolation formula

$$Y^* = \sum_{k=1}^{n} u_k Y_k \tag{11}$$

as a linear transformation of the random variables Y_k. Y^* will then also be a random variable with $N(y, \sigma^2)$-distribution, where (see [6]):

$$y = \sum_{k=1}^{n} u_k y_k; \;\; \sigma^2 = \sum_{k=1}^{n} u_k^2 \sigma_k^2 \tag{12}$$

Eqs.11,12 may be used in solving the following problem, closely related to object recognition: *Let y_k, $k = 1, \ldots, n$ be the experimental data from the spectral measurement of a sample object in n spectral bands, and Y' - be the result from the measurement of some unknown object in a spectral band that may not coincide with any of the former n. What is the probablity p that the unknown object be distinguished from the sample one on the basis of these spectral measurements? Or, vice versa, what is the probability $\bar{p} = 1 - p$ that Y^* be a measurement of the same sample spectrum as the Y_k's?* Here is the algorithm we suggest:

1. Calculate the coefficients u_k in the transformation formula (3) (by either using Eqs.8-9 or solving Eq.5 numerically and then using Eq.7);

2. Calculate from Eq.12 the parameters y and σ^2 of the normal distribution of the transformed variable Y^* (using the sample values for y_k, σ_k^2 if no other estimates are available);

3. Apply the appropriate statistical criteria (see [6], [7] or any other handbook in mathematical statistics) to check the consistency of the experimental value Y' with the $N(y, \sigma^2)$-distribution obtained above. (For example, in the simplest case when y_k and σ_k^2 are the *exact* (rather than sample) values of the parameters of the distribution of Y_k, and Y' is a single measurement, $\bar{p} = \mathrm{erf}(\frac{Y'-y}{\sigma\sqrt{2}})$.)

The generalization to multichannel data is quite straightforward. *Let Y_k, $k = 1, \ldots, n$ be the experimental data from the spectral measurement of a sample object in n spectral bands, and $Y'_l, l = 1, \ldots, m$, $m \leq n$ be the spectral measurements of some unknown object in m spectral bands that may not coincide with any of the former n. What is the probablity p that the unknown object be distinguished from the sample one on the basis of these spectral measurements?* Here is the "multidimensional" form of the algorithm described above:

1. Similarly to Eq.7 define

$$u_{l,k} = \sum_{i=1}^{n} \frac{b_l^i - a_l^i}{i} M_{i,k}^{-1}, \; l = 1, \ldots, m$$

where (a_l, b_l) denotes the spectral band of the l-th channel of the second spectrometer, and consider the linear transformation:

$$Y_l^* = \sum_{k=1}^{n} u_{l,k} Y_k. \tag{13}$$

$u = \{u_{l,k}\}$ is a linear mapping of R^n onto R^m: $R^n \overset{u}{\mapsto} R^m$. If $m \leq n$ and $\mathrm{rank}(u) = m$, Y_l^* will have multivariate normal distribution [6] with density

$$\phi_{Y_1^*,\ldots,Y_m^*}(Y_1, \ldots, Y_m) = ((2\pi)^m \det(\Sigma))^{-1/2} \exp\left(-\frac{1}{2}(Y-y)^T \Sigma^{-1} (Y-y)\right)$$

Note that the linear transformation (13) leaves unchanged the values of those of the Y_k's for which the spectral band (a_k, b_k) is not altered.

2. The parameters of the distribution of Y_l^* are now given by:

$$y_l = \sum_{k=1}^{n} u_{l,k} y_k; \ \Sigma_{i,j} = \sum_{k=1}^{n} u_{i,k} u_{j,k} \sigma_k^2 \qquad (14)$$

where y_i and σ_i^2 stand for the sample mean value and dispersion of Y_i, $i = 1, \ldots, n$ (or for some other estimate of the latter).

3. Both Y_l^* and Y_l', $l = 1, \ldots, m$ will now belong to the characteristic space R^m associated with the m-channel spectrometer, and will have multivariate normal distribution. The statistical criteria for studying the hypothesis that the random vectors Y_l^* and Y_l', $l = 1, \ldots, m$ have (or have not) the same distribution, and therefore, the corresponding objects may not (or may) be distinguished on the basis of the available experimental data are described in details, among other, in [1, 5] and will not be discussed here. Just note that the general criteria can be substantially optimized when applied to specific classes of experimental data [5, p.89].

4 Concluding remarks

The present work was based on two assumptions that need to be discussed in more details.

The requirement that the sensitivity of the sensors be constant within the spectral bands and 0 outside is of great importance for the derivation of the interpolation formula of Eq.3. We believe we shall be able to derive in the future similar formulae for other specific shapes of the sensitivity too (e.g. gaussian); for the moment being we suggest to use in Eqs.5,7 the *effective width* of the channels.

The requirement that the dimension of the characteristic space associated with the sample spectrometer, n, be greater or equal to the dimension m of the space with the data about the unknown object, $n \geq m$, is not such a severe restriction: you always can map R^m onto R^n and compare the spectra in R^n instead. The symmetry between R^m and R^n is broken, because the mapping $R^n \stackrel{u}{\mapsto} R^m$, $n \geq m$ preserves the multivariate normal distribution of the transformed variables Y_l^*, $l = 1, \ldots, m$, thus allowing for standard

methods for the comparison of the spectra to be applied, while the inverse mapping $R^m \stackrel{u^{-1}}{\mapsto} R^n$ leads to linearly dependent Y_k^*, $k = 1, \ldots, n$ [7].

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References

[1] J.A.Richards. Remote Sensing Digital Image Analysis. Springer Verlag, 1986.

[2] G.A.Avanesov et al. Television observation of Phobos. Nature, v.341, p.585.

[3] D.Mishev et al. New generation of spectrometers for measurement of spectral reflective characteristics. Acta Astronautica, v.12, No 11, p.973.

[4] S.Kovachev, D.Krezhova, D.Petkov. High-resolution multichannel spectrometric system "Spectrum 256" for manned space objects. Analytical Space Laboratory, v.1, No 3, 1993.

[5] Gol'zman. Fizitsheskij eksperiment i statistitsheskie vyvody. Leningrad, 1982 (in russian).

[6] H.G.Tucker. An Introduction to Probability and Mathematical Statistics. Academic Press, NY, 1962.

[7] . G.Korn, T.Korn. Mathematical Handbook. McGraw Hill, 1961.

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