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DISTRIBUTION RESTORATION METHOD FROM DATA WITH DISTORTION DUE TO DETECTOR RESOLUTION

(Unfolding Problem — Analysis and Constructive Solving Method)

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1. Introduction

Besides statistical errors, the experimental measured distribution of events has got certain distortions due to the finite resolution of the detector. The detector resolution is defined by its technical characteristics and calculation method of event parameters.

To interpret the experimental distribution with theoretical models, the distortion should be corrected. Certainly, the information about the resolution of the detector, should be used. This information could be obtained from calibration measurements and simulation of the measuring process.

The heuristic method of correction with acceptance was used in many experiments [1,2]. Acceptance is a ratio of true distribution to the measured one. The acceptance is calculated with simulated data.

At first, for the simulation a physicist uses a more probable, in his opinion, theoretical distribution. Further, the iteration goes with respect to the corrected distribution or its parameters. The parameters are obtained by fitting the corrected distribution with the theoretical model.

There are more rigorous approaches to this problem, and the most famous of them are reduced to the solution of Fredholm integral equation of the first kind [3,4]. This problem is an ill-posed problem and the solution is obtained using a priori information about the solution (smoothness, correlation relations and so on).

The main demerits of the famous methods are:

a)difficulties in bias analysis, given by the method itself;

b)difficulties in interpretation of statistical errors for the corrected distribution. For example, discrepancy of error values and smoothness of the corrected distribution.

In the presented work the problem of data processing will be discussed in connection with the problem of information parametrization about the detector and correction procedure of the measured distribution.

To obtain a stable solution of the problem (or stable variant of the problem) with minimal bias and statistical errors, we propose a constructive method,based on:

a) using a priori information, agreeable to theoretical models;

b) choice of points net for the corrected distribution;

c) reducing the experimental distribution to the distribution with a better resolution function.

A new method of data processing will be illustrated by an example from [3] where:

$$\Phi(x) = A_1 \frac{C_1^2}{(x-B_1)^2+C_1^2} + A_2 \frac{C_2^2}{(x-B_2)^2+C_2^2}$$ is

true distribution, x is defined on interval [4, 16] ,

$$F(x) = \int_4^{16} \Phi(x')E(x')K(x,x')dx'$$ is

measured distorted distribution,

$$E(x) = 1 - \frac{(x-10)^2}{6}$$ is

efficiency function,

$$K(x,x') = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-x'}{\sigma}\right)^2\right]$$ is

the detector resolution function.

2. The Problem

The distribution that needs to be measured, further is called "true", and it is measured with a detector consisting of an apparatus and calculation programs of event parameters. The measured distribution has statistical errors and distortion due to the resolution of the detector.

Further without limiting the general nature of the discussion, we represent the true distribution with the values on the discrete points net and the measured distribution as a histogram.

Let us denote true distribution with an n-dimensional vector $\vec{\Phi}$, the measured distribution m-dimensional vector $\vec{f}$, components of vector $\vec{\phi}$ are distribution values on the points net, components of vector $\vec{f}$ are channel contents of histogram.

The true distribution is connected with the measured distribution by P-transformation

$$\vec{f} = P\,\vec{\phi} + \vec{\varepsilon}\,, \qquad (1)$$

where $\vec{\varepsilon}$ is m -dimensional random vector (noise) with an average value $E\,\vec{\varepsilon} = 0$ and diagonal variance matrix V

$$V = \mathrm{Var}\,\vec{\varepsilon} = \mathrm{diag}\,(\sigma_1^2,\ \ldots\ \sigma_m^2),$$

σ_i are the statistical errors of the measured distribution.

This relation between the distributions can be represented in the scheme (Fig. 1).

Fig.2 shows true distribution $\vec{\phi}$ that is $\phi(x)$ on the net of points for our example here, $A_1=2$, $A_2=1$, $B_1=10$, $B_2=14$, $C_1=C_2=1$, $\sigma=1.5$.

Fig.3 is a histogram of the measured distribution $\vec{f}$ represented for the example mentioned above, number of true events equals 10000.

The events for the histogram of the measured distribution are simulated according to the scheme (Fig.4).

To solve a restoration problem of the true distribution from the measured distribution, it is necessary:

a) to study the relationship between the measured and true distributions and obtain P-transformation;

b) to solve equation (1) to obtain true distribution $\vec{\phi}$.

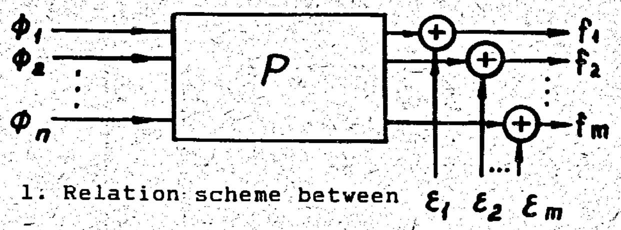

Fig. 1. Relation scheme between true and measured distributions

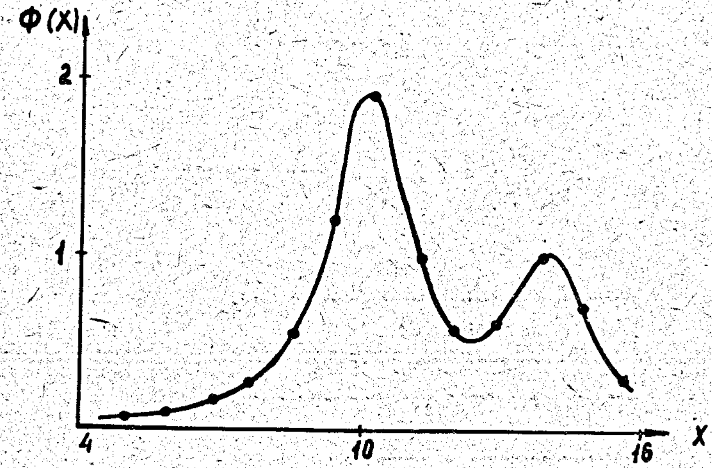

Fig. 2. True distribution $\vec{\phi}$

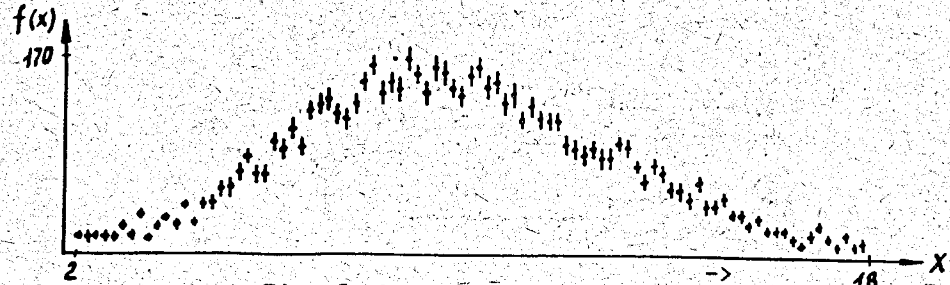

Fig. 3. Measured distribution $\vec{f}$

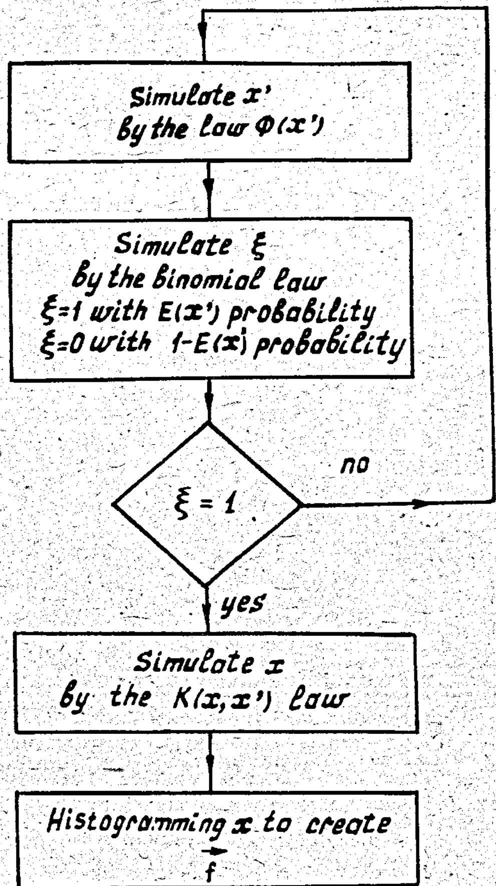

Fig. 4. Histogram simulation scheme of measured distribution

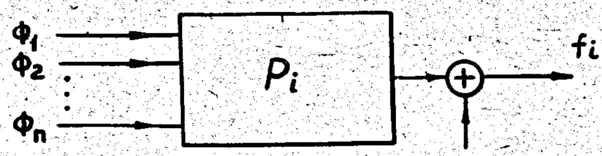

Fig. 5. Relation scheme between true distribution and i-th channel measured distribution

3. Detector Identification

In this chapter we consider the first part of the problem. It is classified as a problem of system identification [5,6].

To get P-transformation it is necessary to reduce the problem to obtain components of transformation P_i, $i=1,m$.

P_i component transforms $\vec{\Phi}$ into f_i that is the i-th component of $\vec{f}$. Schematically it is represented in Fig. 5.

P_i transformation can be linear or nonlinear. As an example of linear transformation we can consider transformation:

$$f_i = p_{i1}\,\Phi_1 + p_{i3}\,\Phi_3 .$$

The nonlinear transformation is a more extensive class of transformations and as an example we can consider the following expression:

$$f_i = p_{i1}\,\Phi_1 + p_{i2}\,\Phi_2^2 .$$

In this publication we discuss only linear case because:

a) linear case is more studied;

b) linear case is approximation for nonlinear case;

c) linear case is a good way to describe distribution distortion connected with particle transport through the matter that most detectors consist of [7].

The identification problem for linear case is to find p_{ij} $j=1,n$ for each P_i transformation.

To calculate P_i we use a system of equations obtained with the simulated data:

$$f_{i,1} = p_{i1}\phi_{1,1} + p_{i2}\phi_{2,1} \dots + p_{in}\phi_{n,1} + \varepsilon_{i,1}$$

$$f_{i,2} = p_{i1}\phi_{1,2} + p_{i2}\phi_{2,2} \dots + p_{in}\phi_{n,2} + \varepsilon_{i,2} \quad (2)$$

$$\vdots$$

$$f_{i,l} = p_{i1}\phi_{1,l} + p_{i2}\phi_{2,l} \dots + p_{in}\phi_{n,l} + \varepsilon_{i,l},$$

where $\phi_{i,l}$ is the variant of generated distribution (true distribution analogue), and $f_{i,l}$ is a reconstructed distribution (measured distribution analogue) for i-th channel, $\varepsilon_{i,l}$ is the calculation noise of $f_{i,l}$ with the simulated data. Solving the system of equations (2) with the least square method for all i, we obtain P transformation - matrix n*m with p_{ij} matrix elements.

To solve identification problem according to this scheme we should answer the following questions:

a) how to choose variants of distributions and how many variants of distribution are necessary;

b) how to choose the net of points for true distribution;

c) how to choose channels for measured distribution.

We choose variants of equations so that P - transformation could in a good way describe the relation between the generated and reconstructed distributions on the distribution set that we are going to measure. That set of distributions could be known from the theory, kinematics and the physical sense, etc.

In our example we shall obtain variants, simulating parameters with random law on intervals:

[1,3]	for A_1;	[10,18]	for B_2;
[0.5,1.5]	for A_2;	[0.5,1.5]	for C_1;
[8,12]	for B_1;	[0.5,1.5]	for C_2.

To calculate reconstructed distribution we simulate events with more probable, in our opinion, parameters of generated distribution $\phi(x)$, for the given example with A_1=2, A_2=1, B_1=10, B_2=14 ,C_1=C_2=1.

After that all the other reconstructed distributions will be obtained histograming the events with weigths $\phi_k(x)/\phi(x)$, where $\phi_k(x)$ - generated distribution with simulated parameters A_1, A_2, B_1, B_2, C_1, C_2.

For reconstructed distribution we choose as many channels as possible. But for the second part of the problem it is important that the error of the channel content should have Gauss distribution ,that is why there should be enough events in the channels. We have taken more than 20 events per one channel for our example.

We choose the points net for true distribution so that the density of points will be proportional to the channel density of measured distribution and the number of points for true distribution should be less than the number of channels of measured distribution.

In practice for many detectors the simulation is very laborious and statistics of the simulated and measured events is of the same order.

This case important for practice, is studied in our work. We take 10000 generated events for our example. The number of points is n=13 for true distribution and number of channels of measured distribution - m=90.

We choose P_i transformation with minimal number of non-zero p_{ij} elements that sufficiently describes the relation of true distribution with i-th channel contents of the reconstructed data.

We calculate p_i by iteration procedure with respect to number of equtions l. It can show that the procedure in most cases converges [5,6].

The number of iteration is limited so that $\max_j |p_{ij}(l)-p_{ij}(l+1)| < \delta$, where δ is a small number defined before.

4. True Distribution Restoration

P transformation has been obtained in the previous chapter and it describes the relation between true and measured distributions.

Equation (1) can be solved in the least squares sense with minimizing

$$\chi^2 = (\vec{f} - P^*\vec{\Phi})^* V^{-1}(\vec{f} - P^*\vec{\Phi}).$$

The $\hat{\vec{\Phi}}$ that yields minimal χ^2, is taken as the estimator of $\vec{\Phi}$ [8]:

$$\hat{\vec{\Phi}} = (P^* V^{-1}P)^{-1}P^* V^{-1}\vec{f} ,$$

where * indicates matrix transposing.

The estimator $\hat{\vec{\Phi}}$ is unbiased and the complete matrix of statistical errors for $\hat{\vec{\Phi}}$ is [8]

$$\mathrm{Var}\ \hat{\vec{\Phi}} = (P^* V^{-1}P)^{-1} .$$

Fig.6 shows the results of $\hat{\vec{\Phi}}$ calculation for our example.

Solving eqution (1) we may obtain $\hat{\vec{\Phi}}$ with very large errors or we may not be able to solve the equation at all if there is no inverse matrix for $P^* V^{-1}P$.

For the χ^2 functional it means the existence of the direction along which χ^2 varies too little and therefore its minimum is very poorly determined.

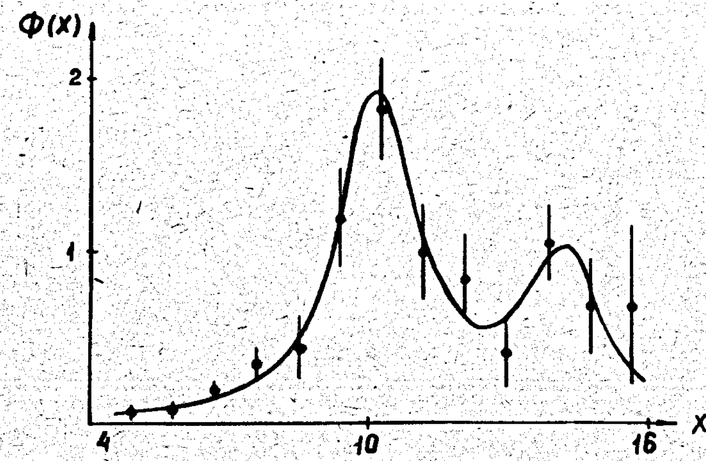

Fig. 6. Restored distribution $\hat{\vec{\Phi}}$

In this paper we propose three ways to overcome these difficulties. They are:

a) to modify the net of points for true distribution;

b) to reduce the class of distributions used for identification. In our example we can reduce simulation intervals for parameters A_1, A_2, B_1, B_2, C_1, C_2;

c) to solve the problem by reducing the experimental distribution to the distribution with a better resolution function. It means to find the experimental distribution that could be obtained at the detector with a narrower resolution function.

To make reduction, we should identify the system that could transform the reduced distribution to a reconstructed one.

Reduced distribution ϕ_R could be calculated on the net numerically. In our example

$$\phi_R(x) = \int \phi(x')\, K(x,x')dx' ,$$

where K(x,x') we can take with $\sigma < 1,5$.

Each of the three ways, or in combination, leads us to the equation that has a solution.

Solving the problem with the method described above, we could specify it from the point of accuracy.

Specification of the solution can be done taking into account statistical errors of P matrix elements.

Full matrix of errors d_i for P_i could be calculated in the framework of the least square method that we use in identification procedure.

Due to the noise of matrix elements and experimental distribution having Gaussian distribution, we can solve the problem in the framework of maximum likelihood method. Logarithm of likelihood function of the problem is as follows:

$$\ln L = -\frac{1}{2}\sum_i (f_i - P_i\vec{\phi})^2/\sigma_i + (P_i - K_i)\, d_i^{-1}\, (P_i - K_i)^* + \text{const} ,$$

where K_i is the true value of P_i.

Maximizing functional only for K_i, we obtain functional

$$\ln L = -\frac{1}{2}\sum_i (f_i - P_i\vec{\phi})^2/(\sigma_i + \vec{\phi}^*\, d_i\, \vec{\phi}) \qquad (3)$$

only for $\vec{\phi}$.

Maximization (3) gives us $\hat{\vec{\phi}}$ as an estimator of $\vec{\phi}$.

Full matrix of errors could be calculated as the inverse matrix to matrix of second derivatives of functional (3) in the maximum with inverse signs [8].

5. Conclusion

We may summarize the main results of this paper as follows.

Data processing method with distortion due to the finite resolution of the detector, is proposed.

The bias of the method is equal to zero and statistical error is minimal for distributions used for identification and its linear combinations.

Stability of restoration problem being solved in the framework of the least square method, is obtained by:

a) characteristic identification of detector for distribution class being supposed for measurements;

b) choosing the points net for restored distribution;

c) reducing the restoration problem of the true distribution to the restoration problem of the distribution with finite resolution better than the measured one.

Identifying the detector characteristics with finite statistics of simulated events, the method generalization in the framework of maximum likelihood method. is proposed.

The data processing method proposed in this paper, could be generalized for the detector with nonlinear distortion.

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