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INVERSE COMBINATORIAL VECTOR-MATRIX TRANSFORMS OVER FINITE AND INFINITE ALGEBRAIC FIELDS

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1 Introduction

The matrix determinant, DetA, can be considered as a polynomial with a specific sign alternance [1]:

$$\mathrm{DetA} = \sum_{i=1}^{n} (-1)^{i+j} a_{ij} \mathrm{DetA}_{ij} \tag{1}$$

on the other hand, any polynomial can be treated as a vector or a matrix determinant so that, e.g.

$$\mathrm{DetA} \begin{vmatrix} a & -b & c \\ 1 & x & 0 \\ 0 & 1 & x \end{vmatrix} = ax^2 + bx + c \tag{2}$$

The recent studies of the solutions of unstable inverse problems for systems of linear algebraic equations (SLAE) demonstrated that a vector-matrix transform of the SLAE's r.h.s. results in a significant improvement of solution techniques as compared with standard ones [2]. From the standpoint of reversible mathematics [3] there the main qualitative gain has been made due to a dramatic increase in mathematical reversibility of the problems under study. To state it in other words, the degree of reversibility defines the level of irreducible information losses.

2 Irreducible polynomials over the Galois field GF(2) [4]

These polynomials are used to be treated as basic vectors for constructing different error-correcting codes. For example, all the irreducible polynomials for n = 1-4 are shown below in matrix patterns (3-10).

2.1 Matrix patterns for n=1 (I_1=2)

These can be reconstructed, with +1 = -1 in GF(2), as

$$\mathrm{Det} \begin{vmatrix} x & 0 \\ 0 & 1 \end{vmatrix} = x \tag{3}$$

and

$$\mathrm{Det}\begin{vmatrix} x & 1 \\ 1 & 1 \end{vmatrix} = x+1 \tag{4}$$

2.2 Matrix patterns for n = 2 (I_2=1)

Here the only irreducible item is

$$\mathrm{Det}\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 0 \\ 0 & 1 & x \end{vmatrix} = x^2+x+1 \tag{5}$$

2.3 Matrix patterns for n=3 ($I_3 = 2$)

Here again we deal with two items as follows:

$$\mathrm{Det}\begin{vmatrix} x & 1 & 1 \\ 1 & x & 0 \\ 0 & 1 & x \end{vmatrix} = x^3+x+1 \tag{6}$$

$$\mathrm{Det}\begin{vmatrix} x & x & 1 \\ 1 & x & 0 \\ 0 & 1 & x \end{vmatrix} = x^3+x^2+1 \tag{7}$$

Finally, for n = 4 we have three matrix patterns shown below in section 2.4.

2.4 Matrix patterns for n = 4 ($I_4 = 3$)

These three polynomials can be reconstructed as follows:

$$\mathrm{Det}\begin{vmatrix} x & x & x & 1 \\ 1 & x & 0 & 0 \\ 0 & 1 & x & 0 \\ 0 & 0 & 1 & x \end{vmatrix} = x^4+x^3+x^2+1 \tag{8}$$

$$\mathrm{Det}\begin{vmatrix} x & x & 0 & 1 \\ 1 & x & 0 & 0 \\ 0 & 1 & x & 0 \\ 0 & 0 & 1 & x \end{vmatrix} = x^4+x^3+1 \tag{9}$$

$$\mathrm{Det}\begin{vmatrix} x & 0 & 1 & 1 \\ 1 & x & 0 & 0 \\ 0 & 1 & x & 0 \\ 0 & 0 & 1 & x \end{vmatrix} = x^4+x+1 \tag{10}$$

3 A qualitative analysis of matrix pattern structures

The accepted algorithm of computing the above matrix determinants defines the main structural features of the reconstructed matrices: the variable first row used to select the relevant polynomial version and the invariant remaining matrix part. All the reconstructed matrices are specified by the main diagonal filled with x's and the lower diagonal filled with ones. The increased computational complexity of matrices as compared with the relevant vectors is imaginery since the only variable element of these matrices is the first row.

4 Multivariate vector-matrix transforms over infinite algebraic fields

The situation drastically changes when one tries to do the same vector-matrix transform in a multivariate case. Let us take a standard matrix-vector SLAE like

$$At = f + n \tag{11}$$

where A - a coefficient matrix, t, f and n - column vectors of a true solution, an input data set and an additive error (noise), respectively.

For two variables, x_1 and x_2, the solution is quite simple:

$$\begin{vmatrix} a_1 & -a_2 \\ x_1 & x_2 \end{vmatrix} = a_1x_1 + a_2x_2 \tag{12}$$

However, there is no simple combinatorial solution for three or more variables. For example, in trying to solve this problem for four variables, x_1,x_2,x_3 and x_4, by simple complementary techniques one arrives at a 6x6-matrix like

$$\begin{vmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & a_1 & -a_2 & 0 & a_3 & -a_4 \\ 0 & x_2 & x_1 & 0 & x_4 & x_3 \end{vmatrix} \tag{13}$$

where three upper rows must be filled with zeros and ones in a still unknown pattern.

5 Conclusions

Nonstandard vector-matrix transform techniques have been applied to obtain a novel algebraic representation of different polynomials over finite and infinite algebraic fields. The irreducible basic vector polynomials with n = 1-4 allow a simple matrix form to be obtained over the galois field GF(2). The most difficult case corresponds to multivariate polynomials over infinite algebraic fields.

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Обратные комбинаторные вектор-матричные преобразования над конечными и бесконечными алгебраическими полями

Показано, что степенные полиномы можно рассматривать либо как векторы, либо как детерминанты квадратных матриц. Найдены комбинаторные вектор-матричные преобразования для всех полиномов степени n = 1-4, неприводимых над конечным полем Галуа GF(2). Показано, что увеличение размерности переменной в полиномиальных выражениях приводит к существенному увеличению сложности решения соответствующей обратной комбинаторной задачи реконструкции искомой матрицы.

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Inverse Combinatorial Vector-Matrix Transforms over Finite and Infinite Algebraic Fields

It has been shown that power polynomials can be considered either as vectors or determinants of square matrices. Combinatorial vector-matrix transforms have been found for all the polynomials of degree n = 1-4 irreducible over the finite Galois field GF(2). An increase in the dimension of polynomial variables is shown to result in a substantial increase in the complexity of solving the corresponding inverse combinatorial problem of the reconstruction of the searched matrix.

The investigation has been performed at the Laboratory of High Energies, JINR.

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