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GAUSS ELIMINATION METHOD (1849 A.D.) IN THE ANCIENT CHINESE SCRIPT "MATHEMATICS IN NINE CHAPTERS" (152 B.C.)

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1. Introduction

The vast majority of integral, differential, nonlinear algebraic etc equations can be reduced to systems of linear algebraic equations (SLAE) like

$$At = f \qquad (1)$$

where A - the coefficient (apparatus) nxn-matrix, t - a true solution column vector and f - a noisy r.h.s. input data column vector. The two main techniques for solving (1) are known to be Cramer determinant method and Gauss elimination method[1]. The first one can be traced down to 1750, while the second one, to 1849 A.D.

2. JU-ZHANG-SUAN-SHU[2]

However, the first rigorous elimination algorithm for solving SLAE was presented in the ancient Chinese mathematical script JU-ZHANG-SUAN-SHU (these Chinese characters mean NINE-CHAPTER-COUNT-ART) which can be translated as "Mathematics in nine chapters".

It is very essential that the ref.[2] is the only translation of the above script into European languages.

The compiler of that script is ZHANG CANG (252-152 B.C.) also known as a Prime Minister of the then Chinese imperial government.

The script is composed of nine chapters and 246 problems. The eigth chapter is called FANG-CHENG (DIRECTION-RULE), i.e. "Directional ordering (of numbers)". This algorithm describes rules for manipulating matrix columns written down according to the well-known Chinese writing habits - from above to below and from right to left.

The first problem of the eigth chapter runs as follows:

"3 sheaves of good harvest, 2 sheaves of average harvest and 1 sheaf of bad harvest yield 36 dous (of grain, 1 dou = 18.039 liters - VII).

2 sheaves of good harvest, 3 sheaves of average harvest and 1 sheaf of bad harvest yield 34 dous.

1 sheaf of good harvest, 2 sheaves of average harvest and 3 sheaves of bad harvest yield 26 dous.

The guestion is - what amount (of grain) does each sheaf of good, average and bad harvest produce?"

In the modern notation this SLAE can be written down as

$$\begin{aligned} 3t_1 + 2t_2 + 1t_3 &= 39 \\ 2t_1 + 3t_2 + 1t_3 &= 34 \\ 1t_1 + 2t_2 + 3t_3 &= 26 \end{aligned} \qquad (2)$$

However, according to the FANG-CHENG rule and writing habits of the ancient Chinese it is represented in the script by the following input table:

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{vmatrix} \qquad (3)$$

which, after many elementary elimination transformations, is reduced to the final form

$$\begin{vmatrix} 0 & 0 & 3 \\ 0 & 5 & 2 \\ 36 & 1 & 1 \\ 99 & 24 & 39 \end{vmatrix} \qquad (4)$$

or, in the modern notation:

$$\begin{vmatrix} 3 & 2 & 1 & 39 \\ 0 & 5 & 1 & 24 \\ 0 & 0 & 36 & 99 \end{vmatrix} \qquad (5)$$

where one can easily recognize the well-known upper triangular matrix

$$U = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 36 \end{vmatrix} \qquad (6)$$

so characteristic of one of the versions of the GAUSS elimination technique[1].

However, the author of the cited Russian translation[2] erroneously claims the FANG-CHENG rule to be an analogue of the Cramer determinant method. It must be also noted that the text of that translation contains many stylistical and mathematical errors making the precise semantics of the original to be very fuzzy.

Nevertheless, it is notable that the true solutions of the SLAE (2) are

$$t_1 = 9\tfrac{1}{4} \qquad t_2 = 4\tfrac{1}{4} \qquad t_3 = 2\tfrac{3}{4} \qquad (7)$$

i.e. expressed as fractions.

Moreover, the script contains the precise recommendations to implement the FANG-CHENG algorithm by means of a Chinese abacus, i.e. a simple mechanic analogue of modern electronic personal computers.

3. Comments

It will be of interest to compare the main SLAE solution techniques described in refs.[1-2].

Both ancient Chinese and modern computerized users write down the relevant SLAE in a matrix-vector form, then formulate a solution algorithm and realize it by means of some computing tool within a decimal basis.

The main difference between these two groups is due to the problems of the explicit indication of zero, different errors and general stability of the obtained solutions.

On the other hand, the influence and importance of the ancient Chinese mathematical tradition can be exemplified by the modern Chinese mathematical term for equation - FANG-CHENG-SHI[3]. Quite naturally, the same is true for Japanese mathematical texts, where the same Chinese characters are pronounced as HO-TEI-SHIKI[4].

4. Conclusions

During a rather long period the author has been specialized in solving unstable inverse problems (UIP) for n-dimensional SLAE´s with very noisy r.h.s. vectors, f, corresponding to low Poisson statistics samples with relative errors, E = 1 - 100 %/5/. The main difficulty with the UIP consists in identifying multiple sources of information losses due to physical, mathematical and computational errors. These losses are the main cause of different solution instabilities. Under these conditions, each established reliable method of solution must be checked in terms of different arguments including historical ones.

The presented data confirm the validity of the mathematical basis of the elimination techniques to trace them from 1849 A.D. down to 152 B.C. at last.

The author would like to express his profound gratitude to the late Profies Zhang Cang and C.F.Gauss for their deep cognitive insight into the problems under study as well as to Prof. Vi-Ai-Ai, for his recommendations and consultations concerning the linguistic environment of both Chinese and Japanese.

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Метод исключения Гаусса (1849 г.) в древнекитайском трактате "Математика в девяти главах" (152 г. до н.э.)

Основными методами решения систем линейных алгебраических уравнений являются детерминантный метод Крамера (1750 г.) и метод исключения (Гаусс, 1849 г.). Анализ древнекитайского источника ЦЗЮ-ЧЖАН-СУАНЬ-ШУ "Математика в девяти главах", известного со времени не позже 152 года до н.э., показывает, что, кроме метода исключения, древние китайцы широко использовали такие фундаментальные понятия как упорядоченная таблица (матрица), порядок вычисления (алгоритм), счетная доска (компьютер) и дроби. Существенное расширение исторического диапазона указывает на высокую надежность и фундаментальность метода исключения.

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Gauss Elimination Method (1849 A.D.) in the Ancient Chinese Script "Mathematics in Nine Chapters" (152 B.C.)

The basic techniques for solving systems of linear algebraic equations are Cramer determinant method (1750) and Gauss elimination method (1849). The analysis of the ancient Chinese script JU-ZHANG-SUAN-SHU ("Mathematics in nine chapters") known not later then 152 B.C. demonstrates that, in addition to the elimination algorithm, the ancient Chinese widely used such fundamental mathematical notions as ordered table (matrix), the order of computations (algorithm), abacus (computer) and fractions. The substantial increase in the historical range supports fundamental nature, validity and reliability of the elimination method.

The investigation has been performed at the Laboratory of High Energies, JINR.

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