сообщения
объединенного
института
ядерных
исследований
дубна

E10-91-374

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FAST VERTEX SEARCH (IN PLANE)

1991

1 Introduction

The vertex search is a common and important problem in the analysis of experimental data obtained in high energy physics. The difficulty of this problem increases when the region of the vertex is distant from the detector, for example in the target. The fact of the vertex presence may be a criterion for rejecting accidental events. In these cases it is desirable to use a fast algorithm. The vertex search in plane is the simplest initial approximation.

Usually the least-square method (LSQ) is applied for this purpose. But in the case of arc tracks the problem becomes non-linear and for its solution it is necessary to use cumbersome iterational algorithms, like the Gauss-Newton method. This in turn leads to great computer time expenditure and a necessity of a good initial approximation arises especially for a big target. As a result, the search procedure becomes too difficult, which does not correspond to the simplicity of the problem. And very often, the rejection of the event is decided. It makes expenditure completely senseless. This situation appears, for example, during the search for rare or forbidden decays, when events are accidental in most cases.

In this article the fast vertex search method is suggested for plane geometry. Its essence is using the functional different from the LSQ. The search for the vertex reduces to comparison of values of the functional in a few easy-to-find points. This method also provides a simple procedure for 'pulling' the vertex to the target. The method was examined on three-track events, obtained in the experiment on the search for the forbidden decay $\mu \to 3e$ on the ARES facility in JINR, Dubna [1, 2, 3].

2 Problem formulation

The problem of the vertex search consists in minimizing some functional. The LSQ functional is applied for the detectors with Gaussian distributed errors of measurements, like bubble or streamer chambers. It has a sense of a mean-square distance from a point to the tracks. For arc tracks the

LSQ functional has a form :

$$S_2(x,y) = \sum_{i=1}^{3}(\sqrt{(x-a_i)^2+(y-b_i)^2} - R_i)^2 \to \min, \tag{1}$$

here (a_i, b_i) are coordinates of centers, R_i — radii of the tracks.

But many detectors, like multiwire proportional chambers, have a discrete structure and here the errors of measurements are not Gaussian distributed. In this case there is no reason for using the LSQ functional [4]. The more suitable functional is the one equal to the longest distance from a point to the tracks:

$$\mathcal{L}(x,y) = \max_i \left|\sqrt{(x-a_i)^2+(y-b_i)^2} - R_i\right|. \tag{2}$$

Functional $\mathcal{L}$ is better to express the discrete structure of the detector and to take into account the effect of multiple scattering.

Thus, the vertex search problem is formulated as follows:

$$\mathcal{L}(x,y) = \max_i \left|\sqrt{(x-a_i)^2+(y-b_i)^2} - R_i\right| \to \min. \tag{3}$$

3 Solution of the problem

As a consequence of this choice of the functional the vertex search becomes a simple and clear geometric problem. Let's take three random circles on a plane and any point with coordinates (x, y) (fig. 1). Denote: O_1, O_2, O_3 — centers of these circles, l_1, l_2, l_3 — distances from point A to the corresponding circle.

Later on, according to the form of the functional, we will look for the longest distance. Let's $l_3 > l_2$ and $l_3 > l_1$. Then moving point A along the straight line (AO_3) to point O_3, one can decrease l_3 until it becomes equal to the distance to any other circle, for example, l_2.

If $l_3 = l_2 > l_1$, then moving along the curve, given by equation:

$$l_3 = l_2, \tag{4}$$

one can further decrease $\mathcal{L}$. The minimum is achieved if:

$$\begin{array}{llll} \text{either} & l_3 = l_2 = \min, & l_3, l_2 > l_1, & (5) \\ \text{or} & l_1 = l_2 = l_3. & & (6) \end{array}$$

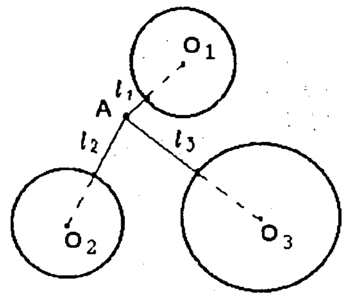

Figure 1: Basic notation.

Equation (4) defines a set of points that have the same distances from two circles. Its solutions are:[1]

1. A branch of a hyperbola for separate (fig. 2a) or crossing (fig. 2b) circles.
2. An ellipsis for crossing (fig. 2b) or enclosed (fig. 2c) circles.
3. A segment and a ray for touching circles.
4. A straight line for equal, but not coinciding circles.
5. Any point for coinciding circles.

Consider the first condition of the minimum of the functional $\mathcal{L}$ — equation (5). If the circles cross (fig. 2b), then the minimum is achieved at the cross points. In this case $l_2 = l_3 = 0$ but $l_1 > l_2, l_3$, i.e. condition (5) is not fulfilled. Therefore, it is sufficient to examine the pairs of non-crossing circles (fig. 2a, 2c). For them the minimum lies in the center of the segment $[MN]$.

Let's proceed to equation (6). This case of the minimum of the functional $\mathcal{L}$ is a part of Apollonius problem [5, 6], which is formulated as follows: given three circles, find all circles touching them. To solve our problem it is necessary to find the roots of the following system of the equations:

$$\begin{cases} l_1 = l_2, \\ l_1 = l_3. \end{cases} \tag{7}$$

[1]Cases 3-5 are not interesting for practice.

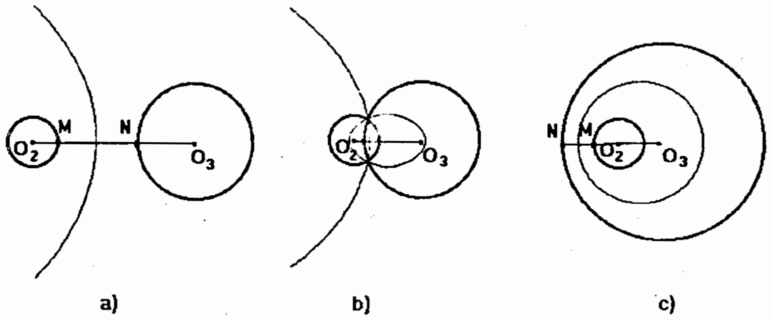

Figure 2: Curves lying at the same distances from two circles.

i.e. to find points, that have the same distance from all the circles. But in general this problem reduces to an equation of forth power. Following reasoning allows one to simplify it significantly.

Let's the tracks have a common vertex. Then it lies near each circle and it is possible to write:

$$\begin{aligned}\sqrt{(x-a_i)^2+(y-b_i)^2}-R_i &= \\ = \frac{(x-a_i)^2+(y-b_i)^2-R_i^2}{\sqrt{(x-a_i)^2+(y-b_i)^2}+R_i} &\approx \frac{(x-a_i)^2+(y-b_i)^2-R_i^2}{2R_i}.\end{aligned}$$

Using this approximation, we can write the equation (4) as:

$$\left|\frac{(x-a_2)^2+(y-b_2)^2-{R_2}^2}{2R_2}\right| = \left|\frac{(x-a_3)^2+(y-b_3)^2-{R_3}^2}{2R_3}\right| \tag{8}$$

After some transformation and reduction to the canonical form one can obtain:

$$\begin{aligned}\left(x-\frac{R_2a_3\pm R_3a_2}{R_2\pm R_3}\right)^2+\left(y-\frac{R_2b_3\pm R_3b_2}{R_2\pm R_3}\right)^2 &= \\ = \pm\frac{R_2R_3}{(R_2\pm R_3)^2}[(R_2\pm R_3)^2-(a_2-a_3)^2-(b_2-b_3)^2]\end{aligned} \tag{9}$$

Sign '−' corresponds to the inside and outside regions for both circles of the tracks, '+' — to the inside region for one and the outside region for

the other circle. Thus, the solution of the system (7) consists in finding the cross points of the circles (9).

An example of possible points of minimum of the functional $\mathcal{L}$ is shown in fig. 3. The points X_1, X_2, X_3 are centers of the corresponding segments $[M, N]$, and point X_4 is the solution of the Apollonius problem.

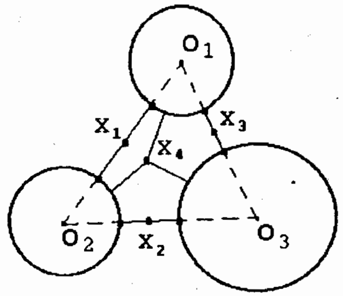

Figure 3: Example of the possible points of the minimum of the functional $\mathcal{L}$.

4 Effect of the target

Previous reasonings allow us to find the point minimizing $\mathcal{L}$ in the whole plane. But this point may lie out of the target, which is impossible because of physics restrictions. So the minimization problem (3) should have an additional condition:

$$\sqrt{x^2+y^2} \leq R_t, \tag{10}$$

here R_t — radius of the target .

Obviously, new possible points of the minimum of the functional $\mathcal{L}$ lie on the target surface. Repeat reasonings of chapter 3, but now point A will be fixed on the target border. Let's have $l_3 > l_2$, $l_3 > l_1$, as before. Consider two cases:

1. Some tracks cross the target (fig. 4a). Then moving point A to the nearest cross point of the circle with the target surface one can decrease l_3 until it becomes equal to another distance, for example l_2.

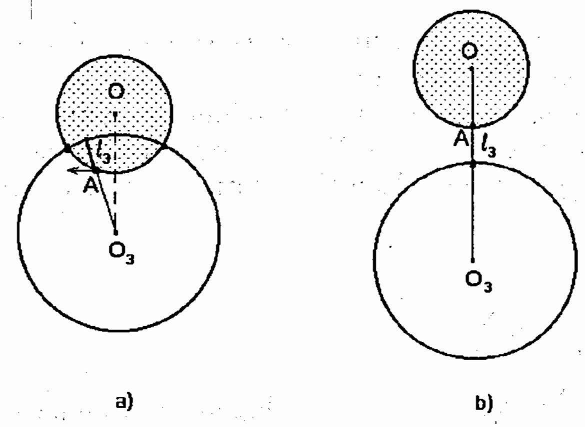

Figure 4: Additional points of the possible minimum of the functional $\mathcal{L}$.

Thus the new points are given by the following system:

$$\begin{cases} l_3 = l_2, \\ x^2 + y^2 = R_t^2, \\ l_3, l_2 > l_1, \end{cases} \tag{11}$$

2. Some of the tracks do not cross the target (fig. 4b). Suppose now that point A lies on the segment $[O, O_3]$, which connects the target center with the center of the circle. Then any shift of point A along the target surface will increase l_3. Therefore, besides the previously found points of the possible minimum of the functional $\mathcal{L}$ new points will appear. They satisfy the following conditions:

$$\begin{cases} l_3 > l_2, l_1, \\ x^2 + y^2 = R_t^2, \\ (x, y) \in [O, O_3], \end{cases} \tag{12}$$

5 Algorithm

The algorithm of the fast vertex search consists of the following steps:

1. For non-crossing circles, calculate the values of the functional $\mathcal{L}$ at the points, given by equation (5).

2. Using approximate equation (9), find the solutions of Apollonius problem.

3. Among these points chose the one with the minimal value $\mathcal{L}_{min}$. If $\mathcal{L}_{min} > \mathcal{L}_{cut}$, the event is interpreted as accidental. Otherwise check that this point is inside the target.

4. If the event is not accidental but the point of the minimum of the functional $\mathcal{L}$ lies outside the target, then calculate the values of $\mathcal{L}$ at the points given by equations (11) and (12). Among *all* points obtained inside the target, chose the one at which the functional has the minimal value $\mathcal{L}_{min}^t$. If $\mathcal{L}_{min}^t > \mathcal{L}_{cut}$, then the event is accidental. Otherwise the point found is the vertex, and the error of the vertex is $\sigma = \mathcal{L}_{min}^t$.

6 Numerical results

The program of the fast vertex search was realized in *Turbo Pascal*. The data obtained in the experiment on the search for the decay $\mu \to 3e$ on the ARES facility [1, 2, 3] were used to examine the method. In this experiment $R_t = 5$ cm, radii of tracks lie in the region from 15 cm to 70 cm, the mean radius is ≈ 20 cm.

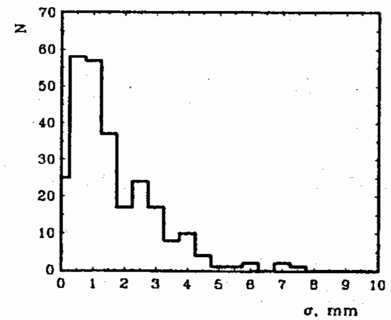

Figure 5: The error of the vertex for simulated events.

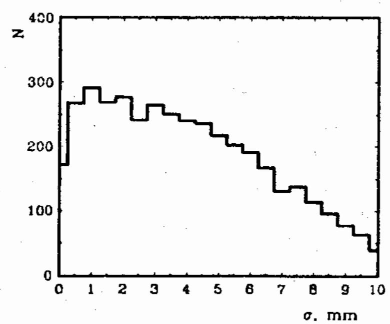

Figure 6: The error of the vertex for real events.

The distribution of the errors of the reconstructed vertex for simulated $\mu \rightarrow 3e$ events is shown in fig. 5. It is clear that the restriction 5 mm preserve practically all true events.

In fig. 6 the vertex error distribution is shown for real events obtained in the experiment on the search for the forbidden $\mu \rightarrow 3e$ decay. These events have already undergone some selection. Most of them are accidental. At this stage of data processing we can reject about a third of them with error $\sigma > 5$ mm.

7 Conclusion

Defining the vertex as a point minimizing the maximal of the distances from it to the tracks, makes the search procedure simple, handy and fast. These properties are extremely important for processing the data, obtained in experiments on the search for rare and forbidden decays, when registered events are accidental in most cases. If necessary, the vertex found by this method may be used as an initial approximation for the LSQ, which is important for experiments with a big target.

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Received by Publishing Department
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Быстрый поиск вершины (в плоскости)

В работе предлагается быстрый метод поиска вершины в плоскости. Поиск вершины сводится к сравнению значений функционала в нескольких легко находимых точках. Этот метод позволяет также легко реализовать процедуру "притягивания" вершины к мишени. Эти свойства являются чрезвычайно важными при обработке данных, полученных в экспериментах по поиску редких и запрещенных распадов, когда зарегистрированные события в основном являются случайными. При необходимости вершина, найденная этим методом, может быть использована как начальное приближение в методе наименьших квадратов, что важно в экспериментах с большой мишенью. Данный метод был апробирован на трехтрековых событиях, полученных в эксперименте по поиску запрещенного распада $\mu \rightarrow 3e$ на установке АРЕС.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

Glazov A.A. et al. E10-91-374
Fast Vertex Search (in Plane)

In this article the fast vertex search method is suggested for plane geometry. The search for the vertex reduces to comparison of values of the functional in a few easy-to-find points. This method also provides a simple procedure for "pulling" the vertex to the target. These properties are extremely important for processing the data, obtained in experiments on the search for rare and forbidden decays, when registered events are accidental in most cases. If necessary, the vertex found by this method may be used as an initial approximation for the least square method, which is important for experiments with a big target. The method was examined on three-track events, obtained in the experiment on the search for the forbidden decay $\mu \rightarrow 3e$ on the ARES facility.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1991