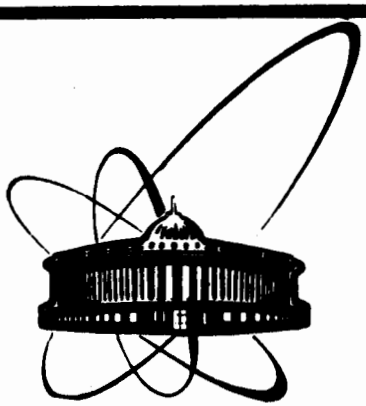

объединенный
институт
ядерных
исследований
дубна

E10-90-410

V.I.Ilyushchenko

AN INTEGRATION WEIGHING METHOD TO EVALUATE EXTREMUM COORDINATES

Submitted to "Nuclear Instruments and Methods"

1990

1. Introduction

Presently, the coordinates of local and global extrema of continuous objective functions are mainly searched by the conventional methods of zero, first and second order [1-5], which are based on the analysis of numerical values of the objective function itself, its first or second derivative, respectively. Besides, all the optimization problems can be subdivided into the nonconstrained or constrained ones depending on the fact whether the constraints have not or have been, respectively, applied to the parameters under study.

The general strategy in searching for global extremum consists in multiple evaluations of the extremal coordinates starting from different initial (seed) parameter vectors. This strategy, however, does not result in the reliable end product due to the principal impossibility of proving the latter to be the true global extremum. Therefore, it is highly desirable to find out some independent methods for a reliable identfication of the extremal coordinates.

On the other hand, discontinuous objective functions have become the subject of the optimization analysis only very recently [6-7]. To out it briefly, the search for the extremal coordinates of the discontinuous objective functions is performed by means of different modifications of Monte Carlo techniques.

The present paper considers an integration method based on a "weighing" of the extremal coordinates by means of the multiple Laplace integrals.

2. The fundamentals of the method

It is well known [8] that "...the mean result of measuring any value is equal to the sum of products of each result by its weight, this sum divided by the sum of all the weights in turn...", i.e. a weighted mean is

$$R_{av} = \frac{\sum_{i=1}^{M} (R_i W_i)}{\sum_{i=i}^{M} (W_i)}, \qquad (1)$$

where W_i - the weight of the i-th measurement (R_i). By passing to an integral presentation and assuming $W_i = W_i(R_i)$ one can write down

$$R_{av} = \frac{\int_{-\infty}^{+\infty} R(x)W(x)dx}{\int_{-\infty}^{+\infty} W(x)dx}, \qquad (2)$$

where x is an independent variable.

On the other hand, the conventional Laplace method [9] for the evaluation of an integral like

$$IL = \int_{-\infty}^{+\infty} (G(x)*\exp(\Lambda*H(x)))dx \qquad (3)$$

$$\Lambda \longrightarrow +\infty$$

is based on a substitution of the IL by an $IL(\Lambda)$ over a small vicinity (+-d) of the point of a maximum, x*, of a function H(x), i.e.

$$IL(\Lambda) = \int_{x^*-d=a}^{x^*+d=a} (G(x)*\exp(\Lambda*H(x)))dx \qquad (4)$$

$$\Lambda \longrightarrow +\infty$$

The asymptotic estimates given in ref.[9] suggest that, for the points within an interval [a,b], one gets

$$\lim \frac{\int_{-\infty}^{+\infty} (G(x)*\exp(\Lambda*H(x)))dx}{\int_{-\infty}^{+\infty} (\exp(\Lambda*H(x)))dx} = G(x^*) \qquad (5)$$

$$\Lambda \longrightarrow +\infty$$

Thus, by taking H(x) as an objective function and using exp(Λ*H(x)) as a weighing function,i.e. W(x) = exp(Λ*H(x)), one obtains a formula to find out the coordinates of a maximum, R = G(x*), which is positioned within a multidimensional closure C, as follows

$$\lim \frac{\int\cdots\int_C (G(v)*\exp(\Lambda*H(v)))dv}{\int\cdots\int_C (\exp(\Lambda*H(v)))dv} = G(v^*), \qquad (6)$$

$\Lambda \longrightarrow +\infty$

where v is a vector of independent variables.

However, ref.[10] states that "...one can enumerate by the fingers of one hand those cases when such integrals (as (3) - VII) are calculated explicitly. Besides, at large values of the parameter (Λ - VII) the calculation of such integrals exceeds the strength of even the most powerful modern computers...".

3. Numerical tests

The problem of accuracy of the asymptotical estimates is a separate item considered, e.g. in ref. [11].

The multiple Laplace integrals have been calculated here by means of the single precision version of the program RGAUSS (D112 from the CERN program library LIBCERN). The calculations have been performed on EC-1055M (IBM compatible) computer of the High Energy Laboratory of JINR (Dubna).

The continuous two-dimensional test functions given in Table 1 have been used as objective ones. These test functions are widely applied to test the standard optimization programs based on the above-mentioned methods of the zero, first and second order [1].

Besides, there have been performed numerical experiments with the discontinuous test functions given in Table 2.

3.1 Continuous test functions

All the numerical experiments have dealt with the dependence of a Euclidean l_2-norm

$$\|x\|_2 = l_2 = ((x1^2 + x2^2))^{1/2} \qquad (7)$$

on the value of parameter Λ --- > 0X0, integration limits [a,b], initial (seed) values (x10,x20), and the number of subdivisions of integration region, NINT. It appears that the l_2-norm is saturated within the range of Λ = 10.0 - 200.0 at NINT = 3-5. The final results are given in Table 3.

Table 1

Continuous two-dimensional test functions (TF)

N	Name	Analytical expression and initial (seed) vector	Minimum
1	Rosenbrock's TF	$100.0(x2 - x1^2)^2 + (1.0 - x1)^2$ $F(-1.2,1.0) = 24.2$	$F(1.0,1.0)=0.0$
2	Beale's TF	$(1.500 - x1(1.0 - x2))^2 +$ $(2.250 - x1(1.0 - x2^2))^2 +$ $(2.625 - x1(1.0 - x2^3))^2$ $F(1.0,-1.2) = 18.95$	$F(3.0,0.5)=0.0$
3	Himmelblau's TF	$(x1^2 + x2 - 11.0)^2 +$ $(x1 + x2^2 - 7.0)^2$ $F(1.0,1.0) = 106.0$	$F(3.6,-1.8)=0.0$ $F(3.0,2.0)=0.0$ *)

*) When starting from the (1.0,1.0) vector all standard programs finish at the second (3.0,2.0) minimum.

Table 2

Discontinuous two-dimensional test functions (TF)

N	Name	Analytical expression and initial (seed) vector	Minimum
1	Wheeling's TF	$-3.0\|x1\| - \|x2\|$ $F(10.0,10.0) = -40.0$	$F(0.0,0.0)=-0.0$
2	Batukhtin's TF	$(\|x1*x2\|)^2 - 1.0$ for $x1.x2 < 0.0$ $(\|x1*x2\|)^2$ for $x1.x2 > 0.0$ $F(-0.75,-1.0) = -0.4375$	$F(0.0,0.0)=-1.0$
3	Joker's TF	$(1.0/(x1^2 - 1.0)) +$ $(1.0/(x2^2 - 1.0))$ $F(0.0,0.0) = -2.0$	$F(1.0,1.0) = 0.0$ $F(-1.0,-1.0)=0.0$

Table 3

The final results of numerical experiments by the integration method (continuous test functions)

N	Name	NINT	N	x1	x2	Integration interval (-a = a)
1	Rosenbrock's TF	1	2.0	2.36	6.55	10.0
2	Rosenbrock's TF	2	7.0	0.33	0.33	10.0
3	Rosenbrock's TF	3	4.0	0.79	0.79	10.0
4	Rosenbrock's TF	4	8.0	0.84	0.84	10.0
5	Rosenbrock's TF	3	100.0	1.06	1.07	1.2
6	Beale's TF	3	60.0	2.94	0.45	5.0
7	Himmelblau's TF	3	200.0	2.94	2.13	5.0

3.2 Discontinuous test functions

The first test function, F(x) = -3.0*(|x1| - |x2|, has been proposed by Wheeling R.F. [12] to evaluate the ability of an optimization algorithm to overcome discontinuities. The function has the form of a three-dimensional pyramid, with the level lines on the (x1,x2)-plane being rhombuses with the breaks along principal axes. It was necessary to detect a maximum of F(x) starting from the seed point (10.0, 10.0), where F(x) = -40.0. The second and third test functions have been investigated for the coordinates of minima. The final results are presented in Table 4.

Table 4

The final results of numerical experiments by the integration method (discontinuous test functions)

N	Name	NINT	N	x1	x2	Integration interval (-a = a)
1	Wheeling's TF	3	40.0	0.9E-4	0.3E-4	20.0
2	Batukhtin's TF	3	80.0	-0.05	-0.05	2.0
3	Joker's TF	3	20.0	0.98	0.98	0.0-1.0

4. Conclusion

The proposed integration method of "weighing" the coordinates of extrema of continuous and discontinuous objective functions provides the extremal coordinates within the accuracy of a few percent with a single precision multiple integration code.

The further development of the integration method is possible due to both an improvement in the weighing function used and a shift to more precise specialized codes for the calculation of multiple integrals.

The cooperation of Dr. Zangieva S.M. and the staff members of the EC-1055M team is gratefully acknowledged.

References

1. D.M.Himmelblau, Applied nonlinear programming (Mc-Graw Hill, N.Y.,1972).
2. Numerical methods for constrained optimization, eds. P.E.Gill and W.Murray (Academic Press, London, 1974).
3. M.Aoki, Introduction to optimization techniques (MacMillan , N.Y.,1974).
4. P.E.Gill, W.Murray and M.H.Wright, Practical optimization (Academic Press, London, 1981).
5. G.V.Reklaitis, A.Ravindran and K.M.Ragsdell, Engineering optimization (Wiley, N.Y.,1983).
6. F.Archetti, A sampling technique for optimization (Editrice Tecnico Scientifica ,Pisa, 1975).
7. V.D.Batukhtin and L.A.Maiboroda, Optimization of discontinuous functions (Nauka, Moscow, 1984) (in Russian).
8. P.S.Laplace, Essai philosophique sur les probabilites (Paris, 1812).
9. N.G.De Bruijn, Asymptotic methods in analysis (North Holland, Amsterdam, 1958).
10. M.V.Fedoryuk, The method of the steepest descent (Nauka, Moscow, 1977) (in Russian).
11. E.Y.Riekstins, The estimated remainders in asymptotic expansions (Zinatne, Riga, 1986) (in Russian).
12. R.F.Wheeling, Comm. ACM, 3, 632 (1960).

Received by Publishing Department
on June 12, 1990.