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USE OF THE ALGEBRAIC CODING THEORY IN NUCLEAR ELECTRONICS

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INTRODUCTION

The method of coding previously suggested in paper [1] is used for the creation of fast tracking processors[2,3], parallel encoders with wide functional possibilities and economic majority coincidence units for a large number of inputs $n \gtrsim 30$ [3-5]. These devices are algebraic in structure and have a number of advantages in comparison with the usual method of constructing special-purpose processors (SP) for fast event selection registered in multichannel charged particle detectors (MCPD). These advantages are due to the use of finite field algebra and, in particular. Galois field algebra [3-5], where multiplication, division and power raising are executed simpler than in the infinite field with a position number system. New results of studies of the development and use of the syndrome coding method in nuclear electronics are described.

1. NEW TYPE OF TIME-DIGITAL CONVERTERS

Along with parallel methods of data registration and processing in MCPD, sequential methods are widely used which differ in economic and simple registration electronics. Among them are counters, shift registers and circular counters. Shift registers are used in systems of data registration from drift chambers [6]. However, shift registers containing some hundred bits are required to register several time intervals and such systems are not effective for a large number of registration channels. There are systems where RAMs are used as shift registers [7], but the time resolution T_R for such devices is large. As shown below, coders and decoders used in technical error-correcting codes [8] and in signature analysers [9] can be applied for the creation of effective time-to-digital converters of the "start-stop" type. It should be noted that timing measurements have been always important in particle physics experiments. These measurements allow multitrack events to be registered. As an example, the development of signals in a drift chamber in time is shown in fig.1. The tracks of charged particles can be restored if we know three coordinates of

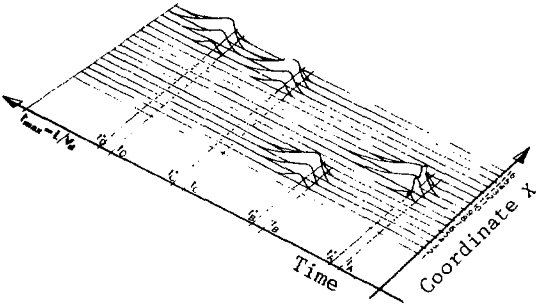

Fig.1. Development of signals in a drift chamber when multitrack events are registered.

events: X, Y and t_s, i.e. the propagation time of signals relative to a start pulse at the ends of the detector [10].

1. Circular counters in the residual system. If a shift register having n bits is divided into L unequal parts such that p_1, p_2, $p_3 \ldots p_i$ are respectively the numbers of bits in each part, then an error-correcting counter can be constructed in the residual system. The numbers p_i are mutually simple ones. The compression coefficient of such a counter equals the product of p_i divided by their sum. Figure 2 gives a scheme of such a counter in the residual system with $p_1 = 2$, $p_2 = 3$ and $p_3 = 5$ ($M = 30$ and $k_c = 30/10$). If one more module $p_4 = 7$ is added, $k_c = 210/17 = 12$ [11]. The operation of the counter is explained in table 1, where the states of triggers over one period of $M = 30$ shift pulses are presented. The event signal is

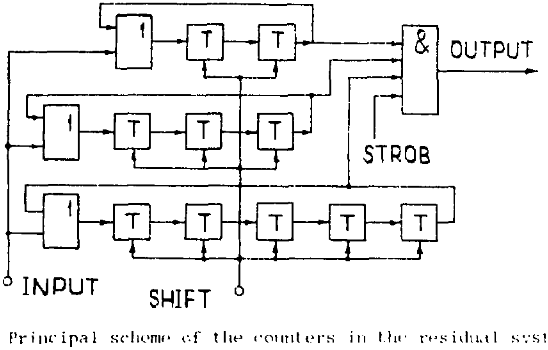

Fig.2. Principal scheme of the counters in the residual systems. T - triggers, 1 - OR, & - AND.

Table 1

State of the schift registers

Triggers Time	1	2	3	4	5	6	7	8	9	10
1	1	0	1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1	0	0	0
3	1	0	0	0	1	0	0	1	0	0
4	0	1	1	0	0	0	0	0	1	0
5	1	0	0	1	0	0	0	0	0	1
6	0	1	0	0	1	1	0	0	0	0
7	1	0	1	0	0	0	1	0	0	0
8	0	1	0	1	0	0	0	1	0	0
9	1	0	0	0	1	0	0	0	1	0
10	0	1	1	0	0	0	0	0	0	1
11	1	0	0	1	0	1	0	0	0	0
12	0	1	0	0	1	0	1	0	0	0
....										
....										
....										
28	0	1	1	0	0	0	0	1	0	0
29	1	0	0	1	0	0	0	0	1	0
30	0	1	0	0	1	0	0	0	0	1

supposed to coincide with the first synchropulse. As it follows from table 1, the signals at the inputs of the AND element coincide after the 30 th synchropulse. In other words, the scheme in fig.2 executes the function of digital delay. Morever, table 1 can be considered as a coding matrix H^T, where rows are changed by columns. Such a matrix has interesting properties. Two coding matrices $H^T_{30,10}$ and $H^T_{217,17}$ are shown in fig.3. As information bits are equal to zero according to the syndrome coding method, we use the theorem to evaluate the properties of the coding matrix: a linear (n, k) - code with parity-check matrix $H^T_{n,\gamma} = [h_0, h_1, h_2 \dots h_{n-1}$, where h_i are vector-columns, i = 0, 1, 2 ...n-k, with dimensions (n-k)x1 and γ = n-k parity check bits, has a maximum coding distance d when any d-1 columns of the matrix $H^T_{n,\gamma}$ are linearly independent. Therefore, for synonymous registration of two time inter-

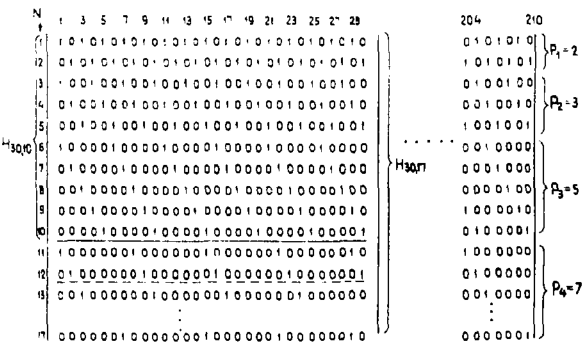

Fig.3. Coding matrix for superimposed codes.

vals with the help of the above scheme, it is necessary to have four mutually linearly independent columns in the matrix $H^T_{30,10}$ as d = 5 for t = 2. This theorem can be also interpreted as follows. For synonymous registration of two independent events (t = 2), it is necessary and enough that the modulo-2 sum or best the Bullean sum of any two columns of the coding matrix $H^T_{n,y}$ should be different. The calculations show that this condition is fulfilld except some sums. For example, if the 2-nd, 3-d, 17-th and 18-th columns of the matrix $H^T_{30,10}$ are added according to the sum rules, we obtain a similar result

2-nd column →	0101001000	17-th column →	1001001000
3-d column →	1000100100	18-th column →	0100100100
	----------		----------
	1101101100		1101101100

Since all columns of the matrix $H^T_{30,10}$ are different and the number of parity bits is equal to 10, then this matrix represents a superimposed correcting code with $4 \leq d \leq 5$. If an extended matrix is constructed by adding one more module $p_4 = 7$, then we get the coding matrix $H^T_{210,17}$ in which the sum of two arbitrary columns is different. It should be noted that adding one module increases the coding distance by one. The shortened matrix $H^T_{30,12}$ is designated as a shaded line in fig.3. The calculations show that one can get the coding matrix with d = 5 if the 12-th row of the matrix $H^T_{30,17}$ is changed for the 11-th row. Then we get an optimum coding matrix with $K_c = 30/11$ and d = 5. From simple reasonings a relation can be obtained between the number of moduli of the residual system ϕ and coding distance $d = 2\phi - 2$, or $\phi = [(d+2)]/2$, where the brackets denote the nearest integer. So, $\phi = 4$ or 5 for d = 5.

Let us consider the coding of two time intervals. For simplicity let p_1 be 3 and p_2 be 4. Assume that events are registered at the 2-nd and 4-th clock times. Then we have table 2. After 3x4 = 12 clocks the process of coding comes to an end and the code 0100011 which carries information on events at the 2-nd and 5-th clocks appears in the circular counters. This fast can be verified in the following manner. If the inputs of the counters are closed after the cycle of measurement, which equals 12 clocks of the bundle generator and then the counters are shifted, the pulses in high bits coincide after the 2-nd and 4-th clock times. These events are marked as * and #, respectively.

2. Time-to-digital converters constructed on the basis of the theory of binary BCH-codes. Using the property of BCH-co-

Table 2
Coding of two events

Triggers Times	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0
3	0	1	0	0	1	0	0
4	0	0	1	0	0	1	0
5	1	0	0	μ	0	0	1
6	0	1	0	1	1	0	0
7	0	0	1	0	1	1	0
8	1	0	0	0	0	1	1
9	0	1	0	1	0	0	1
10	0	0	1	1	1	0	0
11	1	0	0	0	1	1	0
12	0	1	0	0	0	1	1
1	0	0	1	1	0	0	1
2	1	0	0	1	1	0	0
3	0	1	0	0	1	1	0
4	0	0	1	0	0	1	1

Table 3
Coding of two events

Triggers Times	0	1	2	3	4	5	6	7
1	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0
5	0	0	0	1	0	0	0	0
6	0	0	0	0	1	0	0	0
7	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	1
10	1	0	0	0	1	0	1	1
11	1	1	0	0	1	1	1	0
12	1	1	1	0	0	1	1	1
13	1	1	1	1	0	0	0	0
14	0	1	1	1	1	1	0	0
15	0	0	1	1	1	1	1	0

des correcting t mistakes, devices can be constructed for coding t intervals during the cycle of measurement. For simplicity we consider the Galois field $GF(2^4)$ generated by the irreducible polynomial X^4+X+1 assuming that $t = 2$. Then the generated polynomial

$$g(X) = m_1(X)\,m_3(X) = (X^4+X^3+X^2+X+1)(X^4+X+1) = X^8+X^7+X^6+X^4+1.$$

The equation of carrying from the high bit to the low-order bits takes the form $X^8 = X^7+X^6+X^4+1$. Using this equation, we can determine the states of a 8-bit encoder at 15 clock times as $m = 4$ and $2^4-1=15$. The states of the 8-bit coder are given in table 3. It is supposed that the events are registered at the 2-nd and 12-th clock times. In this case we get table 3. All triggers are reset in the initial state (fig.4). The START signal initiates the bundle generator G, and the trigger T opens the AND element. At the 2-nd clock time after amplifying, the signal is registered in the 0-th bit of the coder shifted up to the 9-th clock time. At the 10-th clock time the signal is carried from the high to the low-order bit, and therefore the code 10001011 is formed. In order to determine the states of triggers of the coder after the 11-th clock time, it is necessary to shift the code 1001011 → 01000101 to the right and to add this result with the code 10001011.

$$\begin{array}{r} 01000101 \\ +\,10001011 \\ \hline 11001110 \end{array} \quad \bmod 2$$

and so on. After the 12-th clock time the second event is acciden-

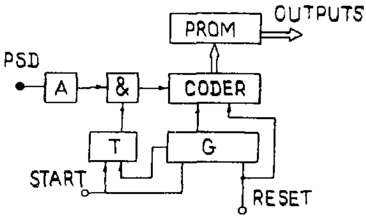

Fig.4. Block-diagram of the time-to-digital converters. PSD - position-sensitive detector, t - trigger, A - amplifier, G - bundle generator.

tally registered, and coding occurs up to the 15-th clock time as is done in the theory of binary BCH codes. It should be noted that the duration of a registered signal and the frequency of synchropulses are selected in order to provide a minimum loss of events. According to the theory of BCH-codes, the word 00111110 carries information on the time of the two events registered in the position-sensitive detector. This word can be read out in a serial or a parallel code and decoded by PROM. The efficiency of such a coding system increases with number M. So, $K_c = 1024/30$ for $M = 1024$ and $t = 3$. Besides, for complete coding it is unnecessary to use stop-signals because synchropulses are produced by the bundle generator. A principal scheme of the coder for our example is given in fig.5.

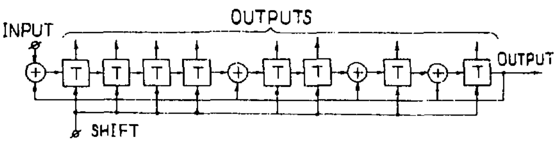

Fig.5. Principal scheme of the time-to-digital converter for two events.

2. ECONOMIC PARALLEL CODING OF WEAK ELECTRIC AND LIGHT SIGNALS

Another significant feature of the coding matrix is the possibility of constructing parallel encoders with useful properties. As is shown in /1,3-5/, if one position-sensitive detector, for example a scintillator, is set for each column and a registration channel for each row, the position and the number of ones in the coding matrix determine the structure of parallel combinational encoder (without memory) for t signals, where $t > 1$. Besides, if a superimposed code is described by a coding matrix, amplifiers-mixers can be used to calculate a syndrome code instead of modulo-2 adders. If light signals

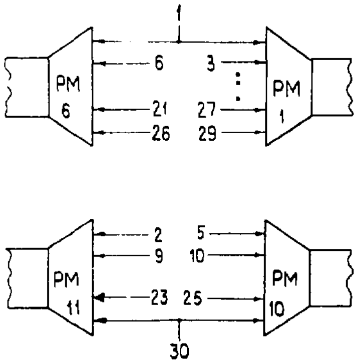

Fig.6. Scheme for calculating the 1-st, 10-th and 11-th bits of the syndrome.

are coded, photomultipliers can be used. For example, it is necessary to create a scintillation hodoscope with M=30 scintillators and multiplicity $t \le 2$. What minimum number of photomultipliers is needed? Let us consider the constructed matrix $H_{30,11}$.

$$
H_{30,11} = \left|\begin{array}{l} 101010101010101010101010101010 \\ 010101010101010101010101010101 \\ 100100100100100100100100100100 \\ 010010010010010010010010010010 \\ 001001001001001001001001001001 \\ 100001000010000100001000010000 \\ 010000100001000010000100001000 \\ 001000010000100001000010000100 \\ 000100001000010000100001000010 \\ 000010000100001000010000100001 \\ 010000001000000100000010000001 \\ 1 \ldots\ldots\ldots\ldots\ldots 30 \end{array}\right| \begin{array}{l} \text{PM} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \end{array}
$$

Scintillators →

Figure 6 shows schemes for the calculation of the 1-st. 10-th and 11-th syndrome bits. Optical fibers can be used for coding. The method of coding only for one signal is given in /14/. The

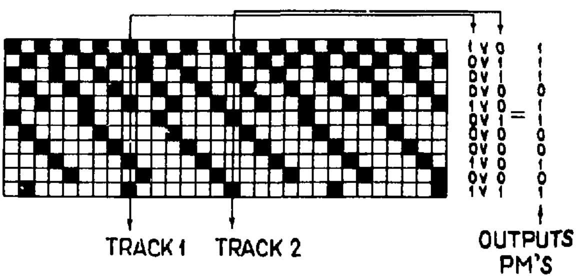

Fig.7. Coding mask-detector for t = 2 and M = 30.

coding matrix can be created as a mask, where black squares are scintillators and white squares are usual glass (fig.7).

3. SOLUTION OF THE PROBLEM OF A "GHOST" IN PIXEL DETECTORS

The use of the method of syndrome coding allows one to solve effectively the problem of registration of "ghosts" in pixel detectors or in hodoscopic calorimeters as shown in fig.8, where the number of pixels, n, is 64. The use of priority encoders for coordinate registration even for $t = 2$ does not lead to the solution of the problem, if data are read out on registers X and Y. There are two methods for solving this problem suggested by the author. The first method allows iteration codes to be used [3]. If a 2-dimensional iteration code with coding distance $d = d_1 d_2 \cdot 5$ is used, the coordinates X_1, Y_1 and X_2, Y_2 or X_1, Y_2 and X_2, Y_1 can be synonymously decoded by decoding a syndrome with the help of PROM or PLA. But it is known that iteration codes are difficult for decoding. Using the second method, n pixels are numbered as degrees of the Galois GF(2^m) field elements. If we choose the binary BCH-code with parameters $d = 5$, $m = 6$ and $N = 2^6 - 1 = 63$, the coordinates of two events can be calculated fast and simply. In detail see [3-5].

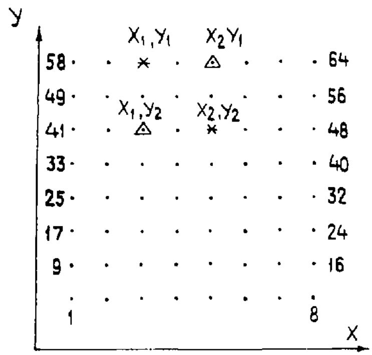

Fig.8. To the question of the identification of "ghost" by the method of syndrome coding.

4. USE OF THE THEORY OF RS-CODES

There are many experiments where the coordinates and their images of clusters should be registered. As known, in the coding theory a burst of errors β in length is determined by the error vector. Thus, a burst of errors 6 in length can look as follows:

001111100000	001000010000	001100110000
Burst 1	Burst 2	Burst 3

Similar configurations of bursts (clusters) take place in MCPD when several neighbouring position-sensitive detectors are fired from one charged particle. The author has suggested to use the theory of RS-codes for registering such events. If the length of a cluster is assumed to be $\beta \leq m$, the compression coefficient $K_c = (2^m - 1)/2t$.

Let the MDCP have $n = 60$ registration channels divided into 15 groups with 4 channels in each group. For $t = 2$ and $m = 4$ the coding matrix takes the form of [8,16].

$$
\begin{array}{r}
*0 \\ 1 \\ *2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14
\end{array}
\qquad
H^T = \left|
\begin{array}{cccc}
\alpha^0 & \alpha^0 & \alpha^0 & \alpha^0 \\
\alpha^1 & \alpha^2 & \alpha^3 & \alpha^4 \\
\alpha^2 & \alpha^4 & \alpha^6 & \alpha^8 \\
\alpha^3 & \alpha^6 & \alpha^9 & \alpha^{12} \\
\alpha^4 & \alpha^8 & \alpha^{12} & \alpha^1 \\
\alpha^5 & \alpha^{10} & \alpha^0 & \alpha^5 \\
\alpha^6 & \alpha^{12} & \alpha^3 & \alpha^9 \\
\alpha^7 & \alpha^{14} & \alpha^6 & \alpha^{13} \\
\alpha^8 & \alpha^1 & \alpha^9 & \alpha^2 \\
\alpha^9 & \alpha^3 & \alpha^{12} & \alpha^6 \\
\alpha^{10} & \alpha^5 & \alpha^0 & \alpha^{10} \\
\alpha^{12} & \alpha^9 & \alpha^6 & \alpha^3 \\
\alpha^{12} & \alpha^9 & \alpha^6 & \alpha^3 \\
\alpha^{13} & \alpha^{11} & \alpha^9 & \alpha^7 \\
\alpha^{14} & \alpha^{13} & \alpha^{12} & \alpha^{11}
\end{array}
\right| \qquad (1)
$$

In the matrix (1) the input groups are numbered by the degrees of the elements $\alpha^0 - \alpha^{14}$ which are the elements of the Galois field $GF(2^4)$ generated by the irreducible polynomial $X^4 + X + 1$. There are 15 nonzero elements: $\alpha^0 = 1000$, $\alpha^1 = 0100$ (root of the polynomial), $\alpha^2 = 0010$, $\alpha^3 = 0001$, $\alpha^4 = 1 + \alpha = 1100$, $\alpha^5 = 0101$, $\alpha^6 = 0011$, $\alpha^7 = 1101$, $\alpha^8 = 1010$, $\alpha^9 = 0101$, $\alpha^{10} = 1110$, $\alpha^{11} = 0111$, $\alpha^{12} = 1111$, $\alpha^{13} = 10011$ and $\alpha^{14} - 1001$. As assumed in the algebraic coding method of RS-codes, the nonzero component of the coordinate vector of fired position-sensitive detectors, $e(X)$, is given by a pair of elements Y_i and X_i which are the image and the coordinate of clusters, respectively. For t events we have

$$
S_l = \sum_{i=1}^{t} Y_i X_i , \qquad 1 \leq l \leq 2t . \qquad (2)
$$

For $t \leq 5$ the table method [3,17-18] can be used for the solution of eq.(2) and for $t > 5$ the well-known Peterson [8] or Chien algorithm [17]. Note that the number of events registered in MDCP should be first calculated along with the images and coordinates of clusters. The algorithm of a majority coincidence circuit is based on the property of the L_t matrix. The matrix

$$L_t = \begin{vmatrix} S_1 & S_2 & S_3 & \dots S_t \\ S_2 & S_3 & S_4 & \dots S_{t+1} \\ \cdot & \cdot & \cdot & \cdot \\ S_t & S_{t+1} & S_{t+2} & \dots S_{2t+1} \end{vmatrix} \quad (3)$$

is singular if S_i is made from t different nonzero pairs X_i, Y_i and the matrix (3) is nonsingular if S_i is made from smaller than t nonzero pairs (X_i, Y_i) /8/. In other words, the properties of the determinant L_t are such that if, e-q-, $t = 1$, then $\det L_1 \neq 0$, but all other determinants are zero. But if $t = 2$, then $\det L_1 \neq 0$, $\det L_2 \neq 0$, but $\det L_3 = 0$ and so on. For $t = 3$ we have

$$L_3 = \begin{vmatrix} S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \\ S_3 & S_4 & S_5 \end{vmatrix} \quad \begin{aligned} &\det L_1 = S_1 \\ &\mathrm{Det}\, L_2 = S_1 S_3 + S_2^2 \\ &\mathrm{Det}\, L_3 = S_1 S_3 S_5 + S_1 S_4^2 + S_2^2 S_5 + S_3^3 \end{aligned} \quad (4)$$

An efficient and fast method is given to calculate the determinant in the $GF(2^m)$ using PROM in /5/.

Continue to consider our example. Let t be 2, $X_1 = \alpha^0$ and $X_2 = \alpha^2$. In addition, $Y_1 = \alpha^7 = 1101$ and $Y_2 = \alpha^{11} = 0111$. Let us examine three cases.

a) Cluster $\alpha^{11} = 0111$ registered at position $X_1 = \alpha^2$. From (6) we have $S_1 = \alpha^{11}\alpha^2 = \alpha^{13}$, $S_2 = \alpha^{11}\alpha^4 = \alpha^{15} = \alpha^0$, $S_3 = \alpha^{11}\alpha^6 = \alpha^2$, $S_4 = \alpha^{11}\alpha^8 = \alpha^4$. In addition, form (4) we have the following relations: $\det L_1 = \alpha^{13} = 0$, $\det L_2 = \alpha^{13}\alpha^2 + \alpha^0 = 0$, $\det L_3 = \alpha^{13}\alpha^2\alpha^6 + \alpha^{13}\alpha^8 + \alpha^0\alpha^6 + \alpha^6 = 0$.

b) Assume that $X_1 = \alpha^0$, $Y_1 = \alpha^7$ and $X_2 = \alpha^2$, $Y_2 = \alpha^{11}$, i.e. $t = 2$. Then we get $S_1 = \alpha^7\alpha^0 + \alpha^{11}\alpha^4 = \alpha^5$, $S_2 = \alpha^7\alpha^0 + \alpha^{11}\alpha^4 = \alpha^9$, $S_3 = \alpha^7\alpha^0 + \alpha^{11}\alpha^6 = \alpha^{12}$ and $S_4 = \alpha^7\alpha^0 + \alpha^{11}\alpha^8 = \alpha^3$.
To find X_1 and X_2, it is necessary to solve the quadratic equation

$$X^2 + \sigma_1 X + \sigma_2 = 0. \quad (5)$$

According to the Peterson algorithm, we have

$$S_j \sigma_t + S_{j+1}\sigma_{t-1} + \dots + \dots S_{j+t} = 0. \quad (6)$$

In the specific case when $t = 2$

$$\sigma_1 = \frac{S_3 S_2 + S_1 S_2 S_4}{S_2^3 + S_1 S_2 S_3}, \quad \sigma_2 = \frac{S_2 S_4 + S_3^2}{S_2^2 + S_1 S_3}. \tag{7}$$

From eq.(2) we obtain a relation to S_j, X_i and Y_i

$$S_1 = X_1 Y_1 + X_2 Y_2 \qquad S_3 = X_1^3 Y_1 + X_2^3 Y_2. \tag{8}$$

From eqs.(7) we have $\sigma_1 = \alpha^8$ and $\sigma_2 = \alpha^2$. One can verify that for these values of σ_1, σ_2, $X_1 = \alpha^0$ and $X_2 = \alpha^2$ eq.(5) reduces to an identity. From eq.(8) we can calculate X_1, X_2, Y_1 and Y_2. A block-diagram of the special-purpose processor is given in fig.9. The groups of inputs are numbered by 0÷14; 16÷30 are amplifiers-shapers; 31÷90 the schemes of multiplication of the input symbols considered as elements of the GF(2^4) times the

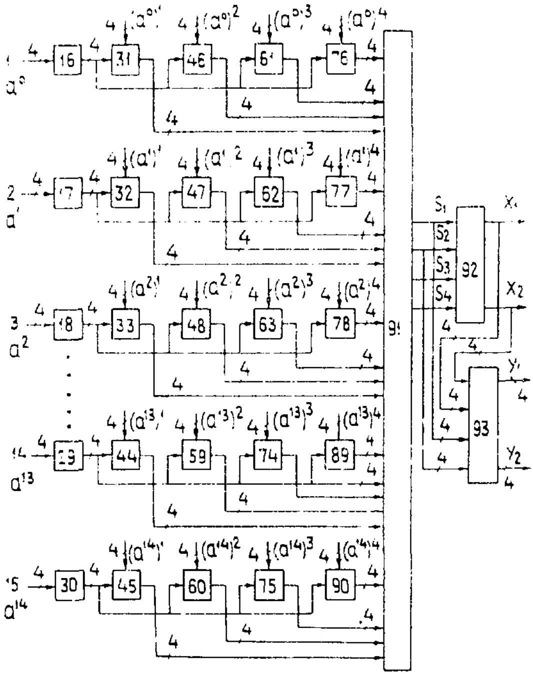

Fig.9. Block-diagram of the special-purpose processors.

constants a^0 - a^{14}. The syndrome $S_1 \div S_4$ is calculated with the help of parity checkers 91. PROMs 92 and 93 are used to calculate X_1, X_2 and Y_1, Y_2. If ECL-microcircuits are used for the creation of SP, the time of multiplication and parity checking do not exceed 6 ns. Besides, the author [19-21] has shown that in the Galois field $GF(2^m)$ such operations as multiplication of several elements and power raising can be executed fast.

CONCLUSION

Several aspects of using the syndrome coding method in nuclear electronics are described. It is shown that using the algebraic coding theory, practically new devices can be created having a number of valuable properties: time-digital converters which can code several intervals simultaneously and possess high resolution, digital delays, parallel encoders for the coding of light signals for $t > 1$ and special-purpose processors for the registration of cluster coordinates and images. The coding matrix suggested by the author for the superimposed code is close to an optimum and has higher parameters than the known ones [13].

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