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ROBUST ESTIMATION OF TRACK PARAMETERS IN WIRE CHAMBERS

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INTRODUCTION

The experimental data obtained from particle track detectors consist of a good "useful" part, and of bad "noisy" part. In many cases the contaminated distribution of errors is

$$F(x) = (1 - \epsilon) N(x) + \epsilon G(x), \qquad (1)$$

where $N(0, \sigma^2)$ is a normal distribution, $\epsilon \in (0, 1)$ determines the background level, $G(x)$ is an arbitrary distribution.

The new "robust" theory suggests new "robust" methods of estimation for the distributions, given above. The main idea of different authors is to decrease the influence of the points, where the experimental deviations are considerable. The classics are Box (1953), Tukey (1960), Huber (1964), Andrews [1,2].

"ROBUST" is insensitive to small departures from the idealised assumptions.

Here are two different interpretations:

1) either fractionally small departures for all data points. (N is not normal distribution, $\epsilon = 0$),

2) or else fractionally large departures for a small number of data points, $\epsilon > 0$).

This interpretation, leading to the case of "outlier" points, is generally the most important for statistical procedures in high energy physics experiments.

First interesting approaches to physics applications have been done in Ososkov et al. papers [3,4,5], especially to regression parameters in models, arising in the particle track recognition problems.

The aim of this paper is to compare the possibilities of some old and robust methods to determine the linear regression parameters (and corresponding physical parameters) of charged particles for modelled and real tracks data.

METHODS

When $\epsilon = 0$ and N is a normal distribution, the optimal estimation is given by least square method (LSM). For N_0 points and M parameters we have to minimize:

$$\sum_{i=1}^{N_0}\left(\frac{y_i - y(x_i, \vec{a})}{\sigma_i}\right)^2 = \min(\vec{a}) . \tag{2}$$

After differentiation we receive M normal equations:

$$\sum_{i=1}^{N_0} \frac{(y_i - y(x_i, \vec{a}))}{\sigma_i^2} \frac{\partial y(x_i, \vec{a})}{\partial a_k} = 0 \qquad k = 1, \ldots, M . \tag{3}$$

After substitution

$$e_i = y_i - y(x_i, \vec{a}), \qquad z_i = \frac{e_i}{\sigma_i} . \tag{4}$$

The (2) is

$$\sum_{i=1}^{N_0} \frac{e_i^2}{\sigma_i^2} = \min(\vec{a}) \tag{*}$$

and the (3) is

$$0 = 1/\sigma_i \cdot z_i \cdot \frac{\partial y(x_i, \vec{a})}{\partial a_k} , \tag{5}$$

where

$$1/\sigma_i \cdot z_i = 1/\sigma_i^2 \cdot e_i = w_i e_i ,$$

then

$$w_i^{LSM} = 1/\sigma_i^2 . \tag{6}$$

In robust methods we have to minimize

$$L = \sum_{i=1}^{N_0} \rho(z_i) \tag{*'}$$

And the normal equations are:

$$0 = \sum_{i=1}^{N_0} 1/\sigma_i \cdot \psi(z_i) \cdot \frac{\partial y_i(x_i, \vec{a})}{\partial a_k} \tag{5'}$$

$$\psi(z) = \rho'(z) = \rho'\left(\frac{e}{\sigma}\right) ,$$

where

$$1/\sigma_i \cdot \psi(z_i) = 1/\sigma_i^2 \cdot \frac{\psi(z_i)}{z_i} \cdot e_i = w_i e_i \tag{6'}$$

$$w_i^R = 1/\sigma_i^2 \cdot \frac{\psi(z_i)}{z_i} .$$

We prefer to use the Tukey's bi-square robust function for our noisy distribution/3/ .

$$\psi(z) = \begin{cases} z(1 - z^2/c^2)^2 & |z| < c \\ 0 & |z| \geq c . \end{cases}$$

ESTIMATION OF THE PARAMETERS OF A STRAIGHT LINE

Let us consider the case of estimation of the parameters of tracks, measured in wire chambers. To be able to compare different estimation methods (5 and 5'), we generated a sample, consisting of 2500 tracks, by the Monte-Carlo method. The deviations of the signal from the position of the track hit in any plane are generated using an experimental distribution, measured in the experiment BIS-2/6/. In this experiment multiwire proportional chambers with spacing of 2 mm are used. The distribution, shown in Fig.1, consists of a sharp peak and long tails due to misidentification of clusters in the pattern recognition stage. The signals in the different planes are generated independently and thus no correlations between the measurements are taken into account (in the considered case the amount of matter is so small that multiple scattering and energy losses of particles are not very important).

First we consider the example of estimation of the parameters of a straight line (X=A+BZ),

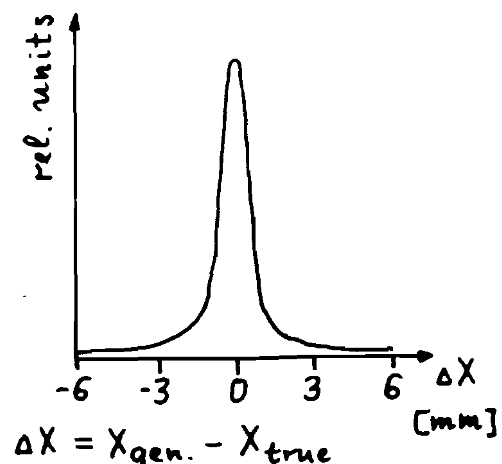

Fig. 1.

using the equidistant planes for Z from -180 to +180 cm. The results for the LSM and the method of Tukey ($c = 4, \sigma = 0.62$ mm) are shown in Table 1.

Table 1

Estimation of the parameters of a straight line ($X = A + BZ$)

Method	σ(A) [mm]	σ(B) [mm]	$\bar{N}$ iterations
LST	0.53	0.47	1
Tukey	0.36	0.36	4.2

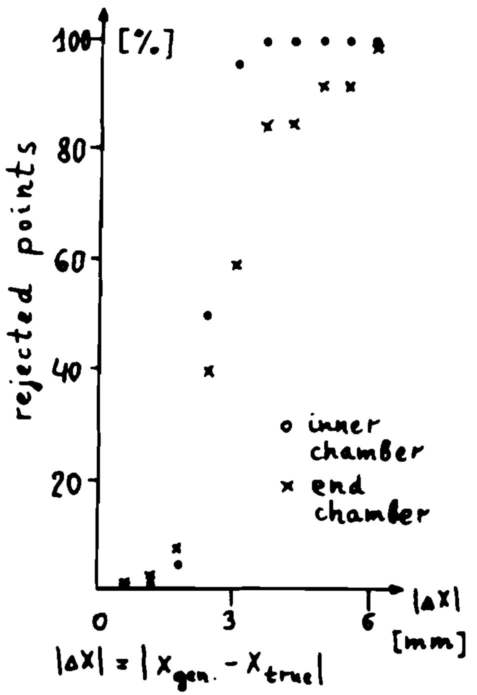

Fig. 2.

The accuracy of the robust method is clearly better than the results from the LSM. In Fig.2 the number of signals with $w < 0.1 * w_{max}$ as a function of the deviation of the signal from the true position (known in Monte-Carlo) is shown for an inner and one of the end chambers. Clearly, the robust method is performing nicely for the inner chambers (small loss of the "true" signals and total rejection for deviations more than 4σ). For the most difficult case of end chambers (extrapolation) the results are also good (the rejection power here is not so big).

ESTIMATION OF THE PARAMETERS OF MODELLED TRACKS

As a next step we consider the estimation of track parameters in the case of two blocks of wire chambers, located before and after an analyzing magnet with constant and homogeneous magnetic field. Each block contains five chambers placed equidistantly between -340 and -180 and +180 to +340 cm in Z (beam direction). The magnetic field of 1,4 T is in the Y direction with length in Z of 150 cm.

The tracks are parametrized as follows

$$X = AX + BX \cdot Z$$
$$Y = AY + BY \cdot Z$$
before or after the magnet.

From the eight parameters quoted above only five are independent due to the fact that the two semitracks belong to the same particle. The fits performed take into account these relations between the track parameters (iterative linearization procedure described in detail in /7/).

Again a sample of 2500 tracks generated by the Monte Carlo method as described in the previous paragraph is used. The momenta of the tracks are disturbed uniformly from 3 to 35 GeV/c.

Four different estimation strategies are compared:
1) LSF as described in /7/ (equal weights of all accepted signals);
2) Signals which lie outside of a given distance from the fitted track are dropped ($w = 0$) and the fit is performed again,
3) If χ^2 of the fit is not good, the signal with greatest contribution to χ^2 is dropped; the fit is performed again until an acceptable χ^2 or a maximum number of dropped signals per track is reached;
4) Robust estimation (Tukey's method with $c = 4$, $\sigma = 0.62$ mm).

The results of the fits are shown in Table 2.

Table 2

Estimation of the parameters of tracks by different methods

Method	σ (AX) [mm]	σ (BX) [mrad]	σ(AY) [mm]	σ (BY) [mrad]	$\frac{\sigma(P_z)}{P_z}$ [%]
1	2,67	1,02	0,57	0,21	6,80
2	2,40	0,94	0,43	0,16	5,92
3	2,42	0,99	0,50	0,19	5,88
4	2,17	0,87	0,42	0,16	5,24

AX, BX, AY, BY – parameters of the semitrack before the magnet, P_z – Z component of the momentum of the track.

The robust method reaches the greatest precision, especially in the XZ-projection (the most important for measurement of the momenta of tracks as they are bent in this projection).

ESTIMATION OF THE PARAMETERS OF REAL TRACKS

As a final step we tested two different methods (1 and 4) on real data from the experiment BIS-2. Because the parameters of individual tracks are not known exactly, we focussed our attention on the parameters of V^o-events (22800 decays $\lambda \to p\pi^-$ and 8500 decays $K_s \to \pi^+\pi^-$ selected from 10^7 np-interactions). We compared d (the distance of the closest approach between the two charged tracks), which should be zero in the absence of measurement errors, and the deviation of the effective mass of the two-particle system from the known particle mass (Fig. 3 and Fig. 4). It can be seen that for the robust method:

1) the distribution in d is sharpening towards zero;
2) the distributions in effective mass are sharpening towards the mass of both λ and K^o.

The price which we have to pay is doubling of the computing time for decoding the experimental information and fitting the track parameters. For a robust estimation about 4 iterations and 0,014 s per track on a NORD 100/500 system are needed.

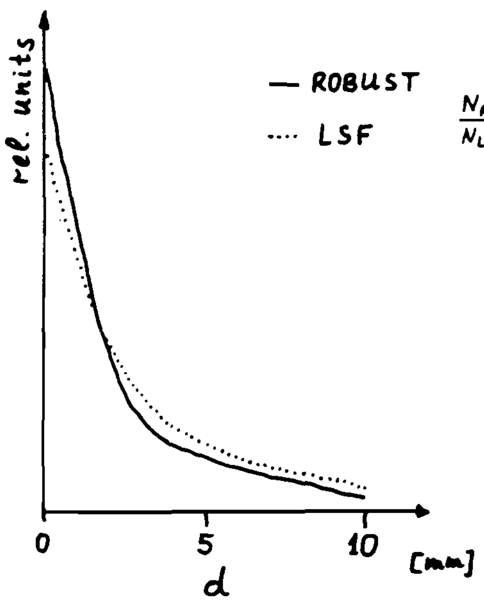

Fig. 3

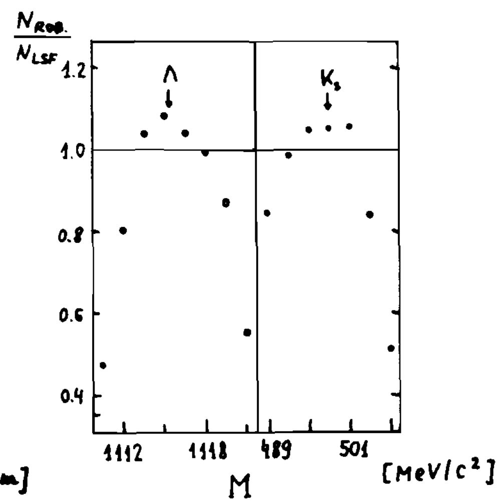

Fig.4

CONCLUSION

A variety of methods for estimating the parameters of trajectories of charged particles in wire chambers is compared. It is shown that robust estimation (Tukey's method) is superior to more standard (and widely used) methods.

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