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MULTIPLE SCATTERING MATRIX WITH ENERGY LOSS

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MULTIPLE SCATTERING MATRIX WITH ENERGY LOSS

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The multiple scattering matrix[1] used for taking into account multiple scattering in the determination of the kinematic parameters of bubble chamber tracks was deduced under the assumption that the particle momentum is constant. Indeed, especially in heavy liquid bubble chambers the energy loss due to ionization and bremsstrahlung (for electrons and positrons) is sufficiently high. This results in a larger straggling of points due to multiple scattering along the track. Therefore, in the present investigation the multiple scattering matrix with energy loss was deduced.

The multiple scattering matrix is determined if one knows the multivariate distribution function $\Phi(x, \xi, \theta, p_0)$ of random displacements $\xi_1, \xi_2, \ldots, \xi_N$ of track points and random deviations at the angles $\theta_1, \ldots, \theta_N$ at the lengths $x_1, \ldots, x_N$. The function $\Phi(x, \xi, \theta, p_0)$ can be obtained from Markov's properties of the multiple scattering stochastic process[2], if one knows the probability density $P(x,\xi,\theta,p_0)$ that a particle undergoes the displacement ξ at a distance x and will move at an angle θ to the axis x (when the initial conditions are $x_0=\xi_0=\theta_0=0$ and the momentum is p_0).

The function P satisfies the equation [3]

$$\frac{\partial P}{\partial x} = -\theta \frac{\partial P}{\partial \xi} + \frac{n(x,h_0)}{\kappa^2} \frac{\partial^2 P}{\partial \theta^2}, \qquad (1)$$

where $\kappa^2 = 4m^2 X_0/E_s^2$ (E_s = 21 MeV, X_0 - is radiation length),

$$n(x,h_0) = (1+h^2(x,h_0))/h^4(x,h_0),$$

where

$$h_0 = p_0/m, \quad h(x,h_0) = p(p_0,x)/m,$$

and the function $p(p_0,x)$ determines the law of momentum variation.

The solution of eq. (1) which satisfies the initial conditions can be obtained in the following form [4]

$$P(x,\xi,\theta,h_0) = \psi(x,h_0)\exp(-\kappa^2(\beta(x,h_0)\theta^2 + 2\gamma(x,h_0)\theta\xi + \omega(x,h_0)\xi^2)),$$

where

$$\omega^{-1} = 8J_3 - 4J_2^2/J_1, \quad \gamma = -\omega J_2/J_1, \quad \beta = 1/4J_1 + \gamma^2/\omega, \quad \psi = (\kappa^2/2\pi)(\omega/J_1)^{1/2}$$

$$J_1 = \int_0^x n(t,h_0)\,dt, \quad J_2 = \int_0^x J_1(t,h_0)\,dt, \quad J_3 = \int_0^x J_2(t,h_0)\,dt.$$

Then in the same manner as in [2] one finds

$$\Phi(x_1,\xi_1,\theta_1,\ldots,x_N,\xi_N,\theta_N,h_0) = \qquad (3)$$
$$= \prod_{k=0}^{N-1} \psi_k \exp(-\kappa^2(\beta_{k+1}\Delta\theta_{k+1,k}^2 + 2\gamma_{k+1}\Delta\theta_{k+1,k}(\Delta\xi_{k+1,k} - \Delta x_{k+1,k}\theta_k) +$$
$$+ \omega_{k+1}(\Delta\xi_{k+1,k} - \Delta x_{k+1,k}\theta_k)^2)),$$

where the following notations are used $\beta_k = \beta(\Delta x_{k,k-1}, h_{k-1})$,
$\gamma_k = \gamma(\Delta x_{k,k-1}, h_{k-1})$,

$$\omega_k = \omega(\Delta x_{k-1,k}, h_{k-1}), \quad \psi_k = \psi(\Delta x_{k-1}, h_{k-1}), \quad h_k = h(x_k, h_0),$$

$$\Delta x_{k,k-1} = x_k - x_{k-1}, \quad \Delta\xi_{k,k-1} = \xi_k - \xi_{k-1}, \quad \Delta\theta_{k,k-1} = \theta_k - \theta_{k-1} .$$

Now, by definition, the multiple scattering matrix is

$$G_{ik} = \int \ldots \int \xi_i \xi_k \Phi(x_1, \xi_1, \theta_1, \ldots, x_N, \xi_N, \theta_N, h_0) d\theta_1 d\xi_1 \ldots d\theta_N d\xi_N . \quad (4)$$

For performing integration we introduce new variables

$$\xi_j = \frac{1}{2\kappa} \sum_{\ell=1}^{2j} \left((1-(-1)^\ell) \frac{x_j - x_m - \gamma_m/\omega_m}{(\beta_m - \gamma_m^2/\omega_m)^{1/2}} + (1+(-1)^\ell)\omega_m^{-1/2} \right) \lambda_\ell$$

$$\theta_j = \frac{1}{2\kappa} \sum_{\ell=1}^{j} \lambda_{2\ell-1} (\beta_\ell - \gamma_\ell^2/\omega_\ell)^{-1/2}$$

with the Jacobian

$$D(\xi_1, \theta_1, \ldots, \xi_N, \theta_N / \lambda_1, \lambda_2, \ldots, \lambda_{2N-1}, \lambda_{2N}) = (1/\kappa^{2N}) \prod_{\ell=1}^{N} (\omega_\ell \beta_\ell - \gamma_\ell^2)^{-1/2}$$

(where $m = (4\ell + 1 - (-1)^\ell)/4$).

With such transformation of variables the quadratic form in the exponent power of the multivariate distribution function is reduced to a sum of squares. Without any loss of generality we may consider that in (4) $i \le k$. Then the terms $\lambda_s \lambda_t$ in (4) with $s \neq t$ equal to zero and it is easy to verify that

$$G_{ik} = \frac{\pi^N}{2\kappa^{2(N+1)}} C_{ik} \left(\prod_{\ell=1}^{N} \psi_\ell\right)\left(\prod_{\ell=1}^{N} (\omega_\ell \beta_\ell - \gamma_\ell^2)^{-1/2}\right), \qquad (5)$$

where

$$C_{ik} = \sum_{\ell=1}^{i} (\beta_\ell/\omega_\ell + (x_i - x_\ell)(x_k - x_\ell) - (\gamma_\ell/\omega_\ell)(x_i + x_k - 2x_\ell))(\beta_\ell - \gamma_\ell^2/\omega_\ell).$$

One finds from (2')

$$\beta_\ell - \gamma_\ell/\omega_\ell = 1/4\, J_1(\Delta x_{\ell-1,\ell}, h_{\ell-1}),\ \gamma_\ell/\omega_\ell = J_2(\Delta x_{\ell-1,\ell}, h_{\ell-1})/J_1(\Delta x_{\ell-1,\ell}, h_{\ell-1})$$

$$\beta_\ell/\omega_\ell = 2J_3(\Delta x_{\ell-1,\ell}, h_{\ell-1})/J_1(\Delta x_{\ell-1,\ell}, h_{\ell-1})$$

and

$$J_1(\Delta x_{\ell-1,\ell}, h_{\ell-1}) = J_1(x_\ell, h_0) - J_1(x_{\ell-1}, h_0)$$

$$J_2(\Delta x_{\ell-1,\ell}, h_{\ell-1}) = J_2(x_\ell, h_0) - J_2(x_{\ell-1}, h_0) - \Delta x_{\ell-1,\ell} J_1(x_{\ell-1}, h_0)$$

$$J_3(\Delta x_{\ell-1,\ell} h_{\ell-1}) = J_3(x_\ell, h_0) - J_3(x_{\ell-1}, h_0) - \Delta x_{\ell-1,\ell} J_2(x_{\ell-1} h_0) - \frac{1}{2}\Delta x^2_{\ell-1,\ell} J_1(x_{\ell-1}, h_0).$$

By using the above expressions and the normalization $\psi_\ell = (\kappa^2/\pi)(\omega_\ell \beta_\ell - \gamma_\ell^2)^{1/2}$, one finds from (5) the final expression for matrix element of the scattering matrix

$$G_{ik} = (2/\kappa^2)(2J_3(x_i, h_0) + (x_k - x_i) J_2(x_i, h_0)). \qquad (6)$$

If $p = const$, $n = m^2(m^2 + p^2)/p^4$ then $J_2 = \frac{n x_i^2}{2}$, $J_3 = \frac{n x_i^3}{6}$
and

$$G_{ik} = \frac{E_s^2}{12 X_0} \frac{m^2+p^2}{p^4} x_i^2 (3x_k - x_i),$$

what coincides with familiar expression of the scattering matrix with the constant momentum.

By using (2), one can easily show that eq. (6) can be rewritten in a simpler form

$$G_{ik} = (2/\kappa^2) \int_0^{x_i} (x_i - t)(x_k - t) n(t, h_0) dt \tag{7}$$

or by replacing the variables $t = s + x_i$

$$G_{ik} = (2/\kappa^2)\left(\int_{-x_i}^{0} s^2 n(s + x_i, h_0) ds - (x_k - x_i) \int_{-x_i}^{0} s n(s + x_i, h_0) ds \right). \tag{8}$$

In the case when particle mass can be neglected compared to the momentum (which is always valid for high energy electrons), formula (8) coincides with the expression obtained in ref./5/. Here one should mention that eq. (1), from which formulas (6)- (8) for the multiple scattering matrix have been obtained, is valid, when energy loss fluctuations are much smaller than mean loss. This is always valid when a particle loses energy only due to ionization. In reducing mean energy loss due to the bremsstrahlung, formulas (6)-(8) are valid only approximately.

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