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DETERMINATION OF THE INITIAL APPROXIMATION FOR THE KINEMATICAL PARAMETERS OF A CHARGED PARTICLE

ЛАБОРАТОРИЯ ВЫЧИСЛИТЕЛЬНОЙ ТЕХНИКИ
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DETERMINATION OF THE INITIAL APPROXIMATION FOR THE KINEMATICAL PARAMETERS OF A CHARGED PARTICLE

In order to obtain the optimal estimates of the kinematical parameters $P, \beta, \tan\alpha$ (P is the momentum, β is the angle between the axis OX and the tangent to the track projection in the XOY plane, α is the angle between the plane XOY and the tangent to the track) it is necessary to have their initial approximations as precise as possible without spending much computer time.

The kinematical parameters of a particle are determined from the coefficients of the parabola which approximates the track projection, taking into account the ionization losses and the nonuniformity of the magnetic field.

By integrating over the equation of motion of a charged particle in a nonuniform magnetic field with the account of the energy losses /1/

$$\frac{d\vec{P}}{dt} = \frac{e}{c}[\vec{v}, \vec{H}] + \frac{\vec{P}}{P}\frac{dP}{dt}, \tag{1}$$

($\vec{H}$ is the magnetic field, $\vec{v}$ is the velocity, e is the charge, c is the velocity of light, t is the time) one obtains an expression for the angle $\beta = \beta(s)$ in the form of expansion in s (s is the plane arc) as follows:

$$\beta(s) = \beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3, \tag{2}$$

where

$$\beta_0 = -\left(\frac{1}{2}\beta_1 s_e + \frac{1}{3}\beta_2 s_e^2 + \frac{1}{4}\beta_3 s_e^3\right),^{x)} \tag{3}$$

$$\beta_1 = \frac{d\beta}{ds}\Big|_{s=0} = \frac{1}{\cos\alpha}\,\frac{0{,}3}{P}\,H , \tag{4}$$

$$\beta_2 = \frac{1}{2}\,\frac{d^2\beta}{ds^2}\Big|_{s=0} = \frac{1}{2\cos^2\alpha}\,\frac{0{,}3}{P}\left\{H' + \frac{1}{P}\left[-H\frac{dP}{dS} + 0{,}3\,B\left(\frac{D}{\cos^2\alpha} + 2H\tan\alpha\right)\right]\right\}, \tag{5}$$

$$\beta_3 = \frac{1}{6}\,\frac{d^3\beta}{ds^3}\Big|_{s=0} = \frac{1}{6\cos^3\alpha}\,\frac{0{,}3}{P}\Big\{H'' + \frac{1}{P}\Big[-H\frac{d^2P}{dS^2} - 2H'\frac{dP}{dS} +$$

$$+ 0{,}3\left((3B'H + 4BH')\tan\alpha + \frac{1}{\cos^2\alpha}(2BD' + B'D)\right)\Big] + \frac{1}{P^2}\left[2H\left(\frac{dP}{dS}\right)^2\right] + \tag{6}$$

$$+ \frac{0{,}3}{P^2}\Big[\left(-7H\tan\alpha - \frac{3D}{\cos^2\alpha}\right)B\frac{dP}{dS} + 0{,}3\left(\frac{H}{\cos^2\alpha}\left(-D^2 + (9 - 6\cos^2\alpha)B^2\right) + 2D\tan\alpha\left(\frac{3B^2}{\cos^2\alpha} - H^2\right)\right)\Big]\Big\},$$

$$B = H_y\cos\beta - H_x\sin\beta ,$$

$$B' = \frac{dH_y}{dS}\cos\beta - \frac{dH_x}{dS}\sin\beta ,$$

$$D = H_x\cos\beta + H_y\sin\beta ,$$

$$D' = \frac{dH_x}{dS}\cos\beta + \frac{dH_y}{dS}\sin\beta ,$$

$$H = -H_z + \tan\alpha\,(H_x\cos\beta + H_y\sin\beta),$$

$$H' = -\frac{dH_z}{dS} + \tan\alpha\left(\frac{dH_x}{dS}\cos\beta + \frac{dH_y}{dS}\sin\beta\right),$$

$$H'' = -\frac{d^2H_z}{dS^2} + \tan\alpha\left(\frac{d^2H_x}{dS^2}\cos\beta + \frac{d^2H_y}{dS^2}\sin\beta\right),$$

(P – MeV/c , H – kgs , S, s – cm.).

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x) s_e is the total length of the track projection.

The equation of the particle trajectory in the coordinate system of fig. 1 can be written as follows /2/:

$$x(s) = \int_0^s \cos\beta(s)\,ds\,, \qquad y(s) = \int_0^s \sin\beta(s)\,ds\,. \tag{7}$$

One can approximate the trajectory by the parabola /2/

$$y(s) = a\,(x(s) - \ell/2)^2 + c\,, \tag{8}$$

where

$$\ell = \sqrt{x^2(s) + y^2(s)}\,. \tag{9}$$

and find the minimum of the functional

$$Q^2 = \int_0^{s_e} [\,y(s) - a(x(s) - \ell/2)^2 - c\,]^2\,ds\,.^{x)} \tag{10}$$

The conditions of the minimum for the functional Q^2 are

$$\begin{cases} \dfrac{\partial Q^2}{\partial a} = 0\,, \\ \dfrac{\partial Q^2}{\partial c} = 0\,, \end{cases} \tag{11}$$

hence

$$a = [\,s_e \int_0^{s_e} y(s)(x(s) - \ell/2)^2\,ds - \int_0^{s_e} y(s)\,ds \int_0^{s}(x(s) - \ell/2)^2\,ds\,][\,s_e \int_0^{s_e}(x(s) - \ell/2)^4\,ds - (\int_0^{s_e}(x(s) - \ell/2)^2\,ds)^2\,]^{-1} \tag{12}$$

Putting into (12) the expressions for $x(s)$ and $y(s)$ from the formula (7) one obtains

$$a = \frac{1}{2}[\beta_1 + \beta_2\,s_e + (\frac{6}{7}\beta_3 + \frac{3}{56}\beta_1^3)\,s_e^2\,]\,. \tag{13}$$

---

x) The weight of each interval "ds" is the same; it means that in this formula we use the approximation of equidistant points.

By means of the least squares method, using the experimentally measured points, one constructs the parabola of the form (8).

As the result of the approximation one obtains

$$a_e = \Big[N\sum_{i=1}^{N} y_i (x_i - \ell/2)^2 - \sum_{i=1}^{N} y_i \sum_{i=1}^{N} (x_i - \ell/2)^2\Big]\Big[N\sum_{i=1}^{N}(x_i - \ell/2)^4 - \Big(\sum_{i=1}^{N}(x_i - \ell/2)^2\Big)^2\Big]^{-1}, \qquad (14)$$

$$c_e = \Big[\sum_{i=1}^{N} y_i \sum_{i=1}^{N} (x_i - \ell/2)^4 - \sum_{i=1}^{N}(x_i - \ell/2)^2 \sum_{i=1}^{N} y_i (x_i - \ell/2)\Big]\Big[N\sum_{i=1}^{N}(x_i - \ell/2)^4 - \Big(\sum_{i=1}^{N}(x_i - \ell/2)^2\Big)^2\Big]^{-1}. \qquad (15)$$

If the measured points are spaced in the track, equidistantly enough, and the nonuniformity of the magnetic field and the relative momentum due to the ionization losses are small, then, without great error, we can assume

$$a_e = a,$$

i.e.

$$a_e = 1/2\Big[\beta_1 + \beta_2 s_e + \Big(\frac{6}{7}\beta_3 + \frac{3}{56}\beta_1^3\Big)s_e^2\Big]. \qquad (16)$$

The right part of (16) is a function of P_0. Solving this equation one derives

$$\begin{aligned} P_0 = P_e &+ \Big\{\frac{0{,}15}{\cos^2\alpha}\Big(H' R_e + \frac{1}{\cos\alpha}\frac{BD}{H} + 2B\sin\alpha\Big) - \frac{1}{2\cos\alpha}\Big(\frac{dP}{dS}\Big)\Big\} s_e + \\ &+ \frac{0{,}3}{7\cos^2\alpha}\Big\{\frac{2}{\cos^2\alpha}\frac{BD'}{H} + \frac{1}{2}\tan\alpha\frac{BH'}{H} + 3B'\tan\alpha + \frac{1}{\cos^2\alpha}\frac{B'D}{H} - \frac{5}{6}\Big(\frac{dP}{dS}\Big)\frac{H'}{H} + \\ &+ \frac{25}{9}\frac{\cos\alpha}{R_e}\Big(\frac{dP}{dS}\Big)^2\frac{1}{H} - \frac{10}{3}\frac{d^2P}{dS^2} - \frac{7}{4}\frac{1}{\cos^2\alpha}\frac{BDH'}{H^2} + \frac{R_e}{\cos\alpha}H'' - \frac{1}{R_e\cos\alpha}\frac{D^2}{H} + \\ &+ \frac{1}{R_e}\Big(\frac{2}{\cos\alpha} + \cos\alpha\Big)B^2/H - \frac{7}{4}\frac{1}{R_e\cos^3\alpha}\frac{B^2D^2}{H^3} - \frac{\tan\alpha}{R_e\cos\alpha}\frac{B^2D}{H^2} - \frac{2\sin\alpha}{R_e}D + \\ &+ \frac{3}{8}\frac{\cos\alpha}{R_e}H + \frac{5}{3R_e\cos\alpha}\frac{BD}{H^2}\Big(\frac{dP}{dS}\Big)\Big\} s_e^2, \end{aligned} \qquad (17)$$

where

$$P_e = \frac{0,3\,R_e}{\cos\alpha}\left[-H_z + \tan\alpha\,(H_x\cos\beta + H_y\sin\beta)\right],$$

$$R_e = \frac{1}{2a_e}$$

while calculating the P_0 momentum according to the formula (17) one should keep in mind that the values of $\frac{dP}{dS}$ and $\frac{d^2P}{dS^2}$ are to be taken at the real value of the P_0 momentum which is still undetermined.

One knows only the value of P_e. Therefore the following iteration procedure should be applied:

1. Calculate $\frac{dP}{dS}$ and $\frac{d^2P}{dS^2}$ for the momentum value being P_e and calculate $P_0^{(1)}$ according to formula (17).

2. Calculate $\frac{dP}{dS}$ and $\frac{d^2P}{dS^2}$ for the momentum value being $P_0^{(1)}$ and calculate $P_0^{(2)}$ according to formula (17), etc. until the following inequality is satisfied:

$$|P_0^{(i)} - P_0^{(i-1)}| / P_0^{(i)} < \epsilon,$$

where ϵ is a certain small number.

Practically, it is sufficient to make one or two iterations. The expressions for the values of the angles α and β which are contained in (17), can be obtained according to the formulae:

$$\tan\alpha = \left[N\sum_{i=1}^{N} z_i s_i - \sum_{i=1}^{N} z_i \sum_{i=1}^{N} s_i\right]\left[N\sum_{i=1}^{N} s_i^2 - \left(\sum_{i=1}^{N} s_i\right)^2\right]^{-1}; \tag{18}$$

$$\tan\beta = -\frac{\ell}{2(R_e + c_e)}. \tag{19}$$

The initial estimate of the β_0 angle at the first point (see fig. 1) can be determined more precisely according to formula (3) using the P_0 momentum value calculated according to (17).

Now one can improve the initial estimate of the dip angle ($\tan\alpha_0$) at the first point. The expansion of $\tan\alpha$ in the neighbourhood of the first point is

$$\tan\alpha = \tan\alpha \Big|_{s=0} + \frac{\partial \tan\alpha}{\partial s}\Big|_{s=0} s + \dots \qquad (20)$$

Taking into account that $\tan\alpha = \frac{dz}{ds}$, one has from (20)

$$z_i = z_0 + \tan\alpha \Big|_{s=0} s_i + \frac{1}{2}\frac{\partial \tan\alpha}{\partial s}\Big|_{s=0} s_i^2 . \qquad (20)$$

Now one can construct the functional

$$\theta^2 = \sum_{i=1}^{N} (z_i - z_{ie})^2 \qquad (22)$$

and from the minimum conditions which are

$$\frac{\partial\theta^2}{\partial z_0} = 0, \quad \frac{\partial\theta^2}{\partial \tan\alpha_0} = 0, \quad (\tan\alpha_0 = \tan\alpha\Big|_{s=0}), \qquad (23)$$

one obtains

$$\tan\alpha_0 = \overline{\tan\alpha} + [1 + (\overline{\tan\alpha})^2]^{3/2} \alpha_0' s_e , \qquad (24)$$

where

$$\alpha_0' = \frac{d\alpha}{dS}\Big|_{s=0} = \frac{0,3}{H} B ,$$

S is the space track length, and $\overline{\tan\alpha}$ is calculated according to formula (18). One can get some idea on the accuracy of the method from Table 1.

The initial estimates of the momentum and the angles were checked by means of test tracks (protons) which were generated in the 2-metre propane bubble chamber, /3/ with the account of the ionization losses and the nonuniform magnetic field, but without the account of the measurement errors and multiple scattering. The magnetic field of the chamber /4/ has a considerable nonuniformity, which is represented in Table 1 by the values of D, H and H'.

Formula (17) for the effective value of the magnetic field differs from the expression previously obtained in ref. /5/. The difference arises because we approximate the track by means of the least squares method using a great number of points, while in ref./5/ the expression for the effective value of the magnetic field is derived for the case when one determines the curvature of the track projection in the XOY plane using only three points.

The initial data for the generator ($P_0, \alpha_0, \beta_0, X_0, Y_0, Z_0$ are related to the first point of the track) and the results of the calculations P_0, $\tan \alpha_0$ and β_0 are listed in Table 1. The values P_0 and $\tan \alpha_0$ are determined according to formulae (17) and (24), respectively. The angle β_0, which is calculated in the coordinate system of the track according to formula (3), is recalculated for the coordinate system of the chamber. The tracks were generated using 21 equidistant points in each track.

From the analysis of the Table 1 it is seen that the maximum displacement in the momentum value is not larger than 2.4% and in β_0 value it is not larger than 0.009 rad., while the displacement in $\tan \alpha_0$ value is not larger than 0.008.

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TABLE I

Comparison between the generated test track parameters and their initial approximation determined according to the described method.

N	GENERATOR			characteristics of the magnetic field			determined initial approximation		
	p_o Mev/c	α_o rad	S cm	$\mathcal{D}$ kgs	H kgs	H′ kgs/cm	$p-p_o$	$\tan\alpha-\tan\alpha_o$	$\beta-\beta_o$
I	800	-3/8π	30	2,6197	-21,8443	0,1711	3,0	0,0043	0,0005
2	800	0	30	2,2262	-14,5271	0,0155	1,1	-0,0002	0,0002
3	800	3/8π	30	3,9721	-3,8471	-0,4837	-3,1	-0,0019	0,0001
4	800	-2/8π	50	3,0195	-19,0134	0,0970	-0,6	0,0034	0,0015
5	800	0	50	2,2254	-14,5302	0,0220	-1,0	-0,0001	0,0008
6	800	2/8π	50	4,7367	-8,3430	-0,1851	-2,3	0,0033	0,0006
7	800	-1/8π	80	2,6203	-16,6213	0,0624	-10,0	0,0040	0,0054
8	800	0	80	2,2289	-14,5227	0,0136	1,5	0,0008	0,0035
9	800	1/8π	80	3,9754	-11,8114	-0,0634	-0,2	0,0041	0,0015
10	800	0	100	1,8757	-14,7540	0,0046	-7,4	-0,0056	0,0094
11	2000	0	100	2,2257	-14,5297	0,0094	3,1	-0,0007	-0,0001
12	10000	0	100	2,2252	-14,5335	0,0089	7,0	-0,0001	0,0000
13	800	-3/8π	30	0,0004	-17,0109	0,1021	5,9	0,0039	0,0001
14	800	0	30	0,2009	-15,9535	-0,0724	3,6	-0,0021	0,0002
15	800	3/8π	30	1,8554	-10,0536	-0,5192	1,5	-0,0051	-0,0006
16	800	-2/8π	50	0,0008	-17,4192	0,0532	1,9	-0,0024	0,0011
17	800	0	50	0,2803	-15,6828	-0,0608	0,5	-0,0041	0,0010
18	800	2/8π	50	2,1287	-11,0105	-0,3053	9,3	-0,0066	0,0002
19	800	-1/8π	80	0,0051	-16,8903	-0,0177	-0,1	-0,0043	-0,0013
20	800	0	80	0,0119	-15,0034	-0,0675	5,4	-0,0083	0,0020
21	800	1/8π	80	0,4478	-12,4531	-0,1360	6,1	-0,0029	0,0030
22	800	0	100	0,2417	-15,3406	-0,0466	-1,0	-0,0421	0,0066
23	2000	0	100	0,0563	-14,8046	-0,0665	49,1	-0,0044	-0,0001
24	10000	0	100	0,0547	-14,8047	-0,0678	234,0	-0,0008	0,0000

(In the cases 23 and 24 the magnetic field is badly approximated by squared parabola)

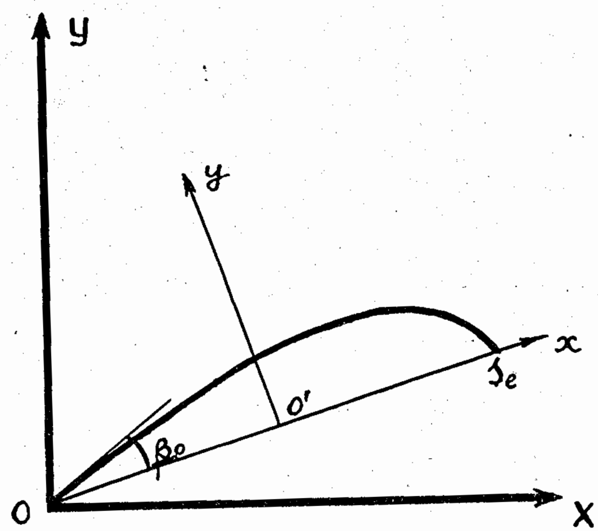

Fig. 1.