СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E1-99-280

V.V.Glagolev, V.L.Lyuboshitz, V.V.Lyuboshitz,
N.M.Piskunov

CHARGE-EXCHANGE BREAKUP OF THE DEUTERON WITH THE PRODUCTION OF TWO PROTONS AND SPIN STRUCTURE OF THE AMPLITUDE OF THE NUCLEON CHARGE TRANSFER REACTION

1999

Глаголев В.В. и др. E1-99-280
Зарядово-обменный развал дейтрона с образованием двух протонов и спиновая структура амплитуды процесса перезарядки нейтрона

В рамках импульсного приближения обсуждается соотношение между эффективным сечением зарядово-обменного развала быстрого дейтрона $d + a \to (pp) + b$ и эффективным сечением процесса перезарядки $n + a \to p + b$. При этом учитываются эффекты тождественности протонов (ферми-статистика) и кулоновского и сильного взаимодействий в конечном состоянии. Исследуется распределение по относительным импульсам протонов, рожденных в зарядово-обменном процессе $d + p \to (pp) + n$ в направлении вперед. При переданных импульсах, близких к нулю, эффективное сечение зарядово-обменного развала быстрого дейтрона, сталкивающегося с протоном мишени, определяется только спин-флиповой частью амплитуды реакции перезарядки $n + p \to p + n$ при нулевом угле. Показано, что изучение процесса $d + p \to (pp) + n$ в пучке поляризованных (выстроенных) дейтронов позволит, в принципе, разделить два спин-зависящих члена в амплитуде реакции перезарядки $n + p \to p + n$, один из которых не сохраняет, а другой сохраняет проекцию спина нуклона на направление импульса при переходе нейтрона в протон.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 1999

Glagolev V.V. et al. E1-99-280
Charge-Exchange Breakup of the Deuteron with the Production of Two Protons and Spin Structure of the Amplitude of the Nucleon Charge Transfer Reaction

In the framework of the impulse approximation, the relation between the effective cross section of the charge-exchange breakup of a fast deuteron $d + a \to (pp) + b$ and the effective cross section of the charge transfer process $n + a \to p + b$ is discussed. In doing so, the effects of the proton identity (Fermi-statistics) and of the Coulomb and strong interactions of protons in the final state are taken into account. The distribution over relative momenta of the protons, produced in the charge-exchange process $d + p \to (pp) + n$ in the forward direction, is investigated. At the transfer momenta being close to zero the effective cross section of the charge-exchange breakup of a fast deuteron, colliding with the proton target, is determined only by the spin-flip part of the amplitude of the charge transfer reaction $n + p \to p + n$ at the zero angle. It is shown that the study of the process $d + p \to (pp) + n$ in a beam of the polarized (aligned) deuterons allows one, in principle, to separate two spin-dependent terms in the amplitude of the charge transfer reaction $n + p \to p + n$, one of which does not conserve and the other one conserves the projection of the nucleon spin onto the direction of momentum at the transition of the neutron into the proton.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 1999

1. Our purpose is to analyze the relation between the effective cross section of the peripherical charge-exchange breakup of a fast deuteron with the production of two protons:

$$d+a\to(pp)+b \tag{1}$$

and the effective cross-section of the charge transfer process

$$n+a\to p+b\ , \tag{2}$$

in which the neutron is transferred into the proton. In so doing, it is supposed that the target particles a and b with the unity charge difference are included in the same isomultiplet. In particular, we will speak concretely about the process

$$d+p\to(pp)+n\ , \tag{3}$$

taking place at a collision of a fast deuteron with the proton target.

The connection between the processes (1) and (2) was discussed partly in the series of works[1,2,3,4,5]. We will continue the study of this problem, taking into account:

a) the spin structure of the amplitude of the charge transfer reaction $n+a\to p+b$,

b) the identity of protons (Fermi-statistics effect),

c) the Coulomb and strong interactions of protons in the final state.

2. We will assume that the velocity of a projectile deuteron is large in comparison with the characteristic one of nucleons in the deuteron:

$$v\gg\sqrt{\frac{\varepsilon}{m}}\sim\frac{1}{20}\ . \tag{4}$$

Here ε is the binding energy ($\varepsilon\approx 2.3\,MeV$), m is the nucleon mass. Under the condition (4), the duration of the collision is much smaller than the characteristic period of the movement of nucleons in the deuteron and, as a result, the coordinates of the neutron and the proton in the deuteron have no time to change during the impact, and we can use the impulse approach. In accordance with the condition (4), the impulse approximation is valid, in any case, for relativistic energies.

Let the neutron, being incorporated in the deuteron, take at a collision the nonrelativistic momentum $\vec{q}$ in the rest frame of the deuteron. Then, in the framework of the impulse approximation, the wave function of the relative motion of two protons, produced in the charge-exchange process $d+a\to(pp)+b$, will have, at once after the impact, the following form:

$$\Psi(\vec{r})=\Psi_d(r)\,e^{-i\vec{q}\vec{r}/2}\ . \tag{5}$$

Here $\Psi_d(r)$ is the deuteron wave function.

The expansion of the function $\Psi(\vec{r})$ over the eigenfunctions of the two-proton Hamiltonian, taking into account the Fermi-statistics effect, gives the continuous spectrum of relative momenta of the created protons. The magnitude itself of the effective cross-section of the charge-exchange breakup of the deuteron is determined by the transitions from the deuteron spin states to the spin states of the two-proton system[3,4]. The contributions of these transitions are connected directly with the spin structure of the np charge transfer reaction.

3. The amplitude of the process $n+a\to p+b$ has the structure:

$$f(n+a\to p+b)=\left(\hat{C}(t)+\hat{\hat{B}}(t)\,\hat{\vec{\sigma}}^{(1)}\right)\hat{P}_{exch}\ , \tag{6}$$

where $t\approx\vec{q}^{\,2}$ is the 4-dimensional transfer momentum squared; $\hat{P}_{exch}$ is the exchange operator transforming the neutron into the proton and the particle a into the particle b; $\hat{\vec{\sigma}}^{(1)}$ is the Pauli operator acting between the spin states of the neutron and the proton; $\hat{C}$ and $\hat{\hat{B}}$ are the operators acting between the spin states of the particles a and b (these states will be marked later on by the index $\{3\}$).

When all the particles are unpolarized, the differential cross-section of the reaction $n+a\to p+b$ can be presented in the form

$$\frac{d\sigma}{dt}(n+a\to p+b)=\frac{d\sigma^{(nf)}}{dt}+\frac{d\sigma^{(f)}}{dt}\ , \tag{7}$$

where

$$\frac{d\sigma^{(nf)}}{dt}=\frac{1}{2j+1}\,tr_{\{3\}}\,\hat{C}(t)\,\hat{C}^{+}(t) \tag{8}$$

is the spin-nonflip part of the differential cross-section of the charge transfer reaction, which is not connected with the spin quantum number of nucleons, and

$$\frac{d\sigma^{(f)}}{dt}=\frac{1}{2j+1}\,tr_{\{3\}}\,\hat{\hat{B}}(t)\,\hat{\hat{B}}^{+}(t) \tag{9}$$

is the spin-flip part of the differential cross-section, conditioned by the presence of the nucleon spin. Here j is the spin of the particles a and b, the symbol $tr_{\{3\}}$ denotes the sum of the diagonal elements ("trace") of the operators acting in the spin space of the particles a and b.

4. Now we will consider the transitions between the spin states of the deuteron and the (pp)-system. As it is known, the neutron and the proton in the deuteron are in the triplet spin state with the total spin of 1. When the deuteron is unpolarized, then each of three spin states corresponding to the projections of the total spin onto the quantization axis z, equaling −1, 0, 1, is realized with the probability of $1/3$:

$$\left|\chi_{-1}^{(trip)}\right\rangle = \left|-\frac{1}{2}\right\rangle^{(1)} \left|-\frac{1}{2}\right\rangle^{(2)},$$

$$\left|\chi_{0}^{(trip)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle^{(1)} \left|-\frac{1}{2}\right\rangle^{(2)} + \left|-\frac{1}{2}\right\rangle^{(1)} \left|+\frac{1}{2}\right\rangle^{(2)}\right),$$

$$\left|\chi_{+1}^{(trip)}\right\rangle = \left|+\frac{1}{2}\right\rangle^{(1)} \left|+\frac{1}{2}\right\rangle^{(2)}. \qquad (10)$$

Here the index 1 is related to the spin function of the neutron, and the index 2 is related to the spin function of the spectator proton. In the process $d + a \to (pp) + b$ the system of two protons can be created in the triplet states (10) as well as in the singlet state with the zero total spin:

$$\left|\chi^{(\sin)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|+\frac{1}{2}\right\rangle^{(1)} \left|-\frac{1}{2}\right\rangle^{(2)} - \left|-\frac{1}{2}\right\rangle^{(1)} \left|+\frac{1}{2}\right\rangle^{(2)}\right). \qquad (11)$$

The two-proton system, produced in the peripherical breakup of the deuteron, can be considered as a non-relativistic one in the rest frame of the deuteron.

In accordance with the Pauli principle for identical fermions, when two protons are produced in triplet states, their coordinate wave function is antisymmetric and the orbital angular momenta have only odd values; when two protons are produced in the singlet state, the coordinate wave function is symmetric and the orbital angular momenta have only even values[6]. It is easy to see that the operator $\hat{C}(t)$ in Eq. (6), being independent of the nucleon spin, leads to the production of the *pp*-system only in the triplet states. Meanwhile, the spin-flip operator $\hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)}$ determines the transitions to both the triplet and singlet states of two protons.

5. In accordance with the above-mentioned facts, when the momentum $\vec{q}$ is transferred to the neutron in the deuteron (as a result of the charge transfer reaction $n + a \to p + b$), then the effective cross-section of the charge-exchange breakup of the unpolarized deuteron on the unpolarized target can be presented in the following form:

$$d\sigma\left(d + a \to (pp) + b\right) = \frac{1}{3(2j+1)} \times$$

$$\times\, tr_{(3)} \Bigg\{ 3\hat{C}(t)\hat{C}^{+}(t) + \sum_{\mu'}\sum_{\mu} \left\langle \chi_{\mu'}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{\mu}^{(trip)} \right\rangle \left\langle \chi_{\mu}^{(trip)} \right| \hat{\vec{B}}^{+}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{\mu'}^{(trip)} \right\rangle$$

$$\times \left| \int \Psi_d(r) e^{-i\vec{q}\vec{r}/2} \frac{1}{\sqrt{2}} \left(\varphi_{\vec{k}}^{*(-)}(\vec{r}) - \varphi_{\vec{k}}^{*(-)}(-\vec{r}) \right) d^3\vec{r} \right|^2 + \qquad (12)$$

$$+ \sum_{\mu} \left\langle \chi^{(\sin)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{\mu}^{(trip)} \right\rangle \left\langle \chi_{\mu}^{(trip)} \right| \hat{\vec{B}}^{+}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi^{(\sin)} \right\rangle \times$$

$$\times \left| \int \Psi_d(r) e^{-i\vec{q}\vec{r}/2} \frac{1}{\sqrt{2}} \left(\varphi_{\vec{k}}^{*(-)}(\vec{r}) + \varphi_{\vec{k}}^{*(-)}(-\vec{r}) \right) d^3\vec{r} \right|^2 \Bigg\} \frac{d^3\vec{k}}{(2\pi)^3} dt \;.$$

Here $\vec{k}$ is the momentum of one of the produced protons in the c.m. frame of the proton pair, coinciding practically, in the used approximation, with the rest frame of the deuteron (we assume that $|q| \ll m$, $k = |\vec{k}| \ll m$); $\varphi_{\vec{k}}^{(-)}(\vec{r})$ is the wave function of the relative motion of two interacting protons, corresponding to the scattering problem and having the asymptotics in the form of the superposition of a plane wave and a converging spherical wave[6]. Let us emphasize that Eq. (12) takes into account the Fermi-statistics effect: the antisymmetrization or symmetrization of the wave function $\varphi_{\vec{k}}^{(-)}(\vec{r})$ with respect to the substitution $\vec{r} \to -\vec{r}$ is performed in the cases of the transitions to the triplet states or the singlet state of two protons, respectively. The triplet states $\left|\chi_{\mu}^{(trip)}\right\rangle$ and $\left|\chi_{\mu'}^{(trip)}\right\rangle$ ($\mu.\mu' = \pm 1, 0$) are described by Eqs.(10), and the singlet state $\left|\chi^{(\sin)}\right\rangle$ is described by Eq.(11).

Due to the properties of the Pauli matrices, the following relations for the matrix elements of the operator $\hat{\vec{B}}(t)\,\hat{\vec{\sigma}}^{(1)}$ are valid:

$$\left\langle \chi_{+1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{+1}^{(trip)} \right\rangle = \left\langle \chi_{-1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{-1}^{(trip)} \right\rangle = \hat{B}_z(t);$$

$$\left\langle \chi_{0}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{+1}^{(trip)} \right\rangle = \left\langle \chi_{-1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{0}^{(trip)} \right\rangle = \frac{1}{\sqrt{2}}\left(\hat{B}_x(t) + i\hat{B}_y(t)\right);$$

$$\left\langle \chi_{0}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{-1}^{(trip)} \right\rangle = \left\langle \chi_{+1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{0}^{(trip)} \right\rangle = \frac{1}{\sqrt{2}}\left(\hat{B}_x(t) - i\hat{B}_y(t)\right);$$

$$\left\langle \chi_{0}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{0}^{(trip)} \right\rangle = \left\langle \chi_{+1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{-1}^{(trip)} \right\rangle = \qquad (13a)$$

$$= \left\langle \chi_{-1}^{(trip)} \right| \hat{\vec{B}}(t)\hat{\vec{\sigma}}^{(1)} \left| \chi_{+1}^{(trip)} \right\rangle = 0;$$

$$\left\langle \chi^{(\sin)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi_{+1}^{(trip)} \right\rangle = -\left\langle \chi_{-1}^{(trip)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi^{(\sin)} \right\rangle = -\frac{1}{\sqrt{2}}\left(\hat{B}_x(t) + i\hat{B}_y(t)\right)$$

$$\left\langle \chi^{(\sin)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi_{-1}^{(trip)} \right\rangle = -\left\langle \chi_{+1}^{(trip)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi^{(\sin)} \right\rangle = \frac{1}{\sqrt{2}}\left(\hat{B}_x(t) - i\hat{B}_y(t)\right);$$

$$\left\langle \chi^{(\sin)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi_{0}^{(trip)} \right\rangle = \hat{B}_z(t). \qquad (13b)$$

Here z is the spin quantization axis, the axes x and y are perpendicular to the axis z.

As a result,

$$tr_{\{3\}} \sum_{\mu}\sum_{\mu'} \left\langle \chi_{\mu'}^{(trip)} \right| \hat{B}(t)\,\hat{\sigma}^{(1)} \left| \chi_{\mu}^{(trip)} \right\rangle \left\langle \chi_{\mu}^{(trip)} \right| \hat{B}^{+}(t)\hat{\sigma}^{(1)} \left| \chi_{\mu'}^{(trip)} \right\rangle = 2\,tr_{\{3\}}\hat{B}(t)\hat{B}^{+}(t)\ ;$$

$$tr_{\{3\}} \sum_{\mu} \left\langle \chi^{(\sin)} \right| \hat{B}(t)\hat{\sigma}^{(1)} \left| \chi_{\mu}^{(trip)} \right\rangle \left\langle \chi_{\mu}^{(trip)} \right| \hat{B}^{+}(t)\hat{\sigma}^{(1)} \left| \chi^{(\sin)} \right\rangle = tr_{\{3\}}\hat{B}(t)\,\hat{B}^{+}(t)\ , \qquad (13c)$$

and, in accordance with Eqs. (8) and (9), the effective cross-section of the charge-exchange process $d + a \to (pp) + b$ for unpolarized primary particles is expressed through the spin-nonflip and spin-flip parts of the charge transfer reaction $n + a \to p + b$:

$$d\sigma\left(d + a \to (pp) + b\right) =$$
$$\left\{\left[\frac{d\sigma^{(nf)}}{dt}(n + a \to p + b) + \frac{2}{3}\frac{d\sigma^{(f)}}{dt}(n + a \to p + b)\right] \times\right.$$
$$\times\left|\int \Psi_d(\vec{r})\, e^{-i\vec{q}\vec{r}/2}\, \frac{1}{\sqrt{2}}\left(\varphi_{\vec{k}}^{*(-)}(\vec{r}) - \varphi_{\vec{k}}^{*(-)}(-\vec{r})\right) d^3\vec{r}\right|^2 + \qquad (14)$$
$$+\frac{1}{3}\frac{d\sigma^{(f)}}{dt}(n + a \to p + b) \times$$
$$\left.\times\left|\int \Psi_d(\vec{r})\, e^{-i\vec{q}\vec{r}/2}\, \frac{1}{\sqrt{2}}\left(\varphi_{\vec{k}}^{*(-)}(\vec{r}) + \varphi_{\vec{k}}^{*(-)}(-\vec{r})\right) d^3\vec{r}\right|^2\right\} \frac{d^3\vec{k}}{(2\pi)^3}\, dt\ .$$

In the case of the flight of the two-proton system in the forward direction, the longitudinal transfer momentum, connected with the increase of the effective mass of two nucleons at the transition $d \to pp$, is small as compared with the reverse radius of the deuteron, and we can take the value $\vec{q} = 0$, or $t = 0$. Then the contribution from the transitions to the triplet states of two protons into the effective cross-section of the process disappears, because the wave function of two protons in the triplet states is antisymmetrized with respect to the substitution $\vec{r} \to -\vec{r}$, whereas the deuteron wave function is symmetric with respect to this substitution. With this, we have

$$d\sigma\left(d + a \to (pp) + b\right)\Big|_{t=0} = \frac{2}{3}\frac{d\sigma^{(f)}(n + a \to p + b)}{dt}\Big|_{t=0} \times$$
$$\times\left|\int \Psi_d(r)\, \varphi_{\vec{k}}^{*(-)}(\vec{r})\, d^3\vec{r}\right|^2 dt\, \frac{d^3\vec{k}}{(2\pi)^3} \qquad (15)$$

Thus, the effective cross-section of the charge-exchange breakup of the unpolarized deuteron on the unpolarized target in the forward direction is proportional to the spin-flip part of the differential cross-section of the charge transfer process at the zero angle.

6. Now let us integrate the effective cross-section of the deuteron breakup $d + a \to (pp) + b$ over the proton momentum $\vec{k}$ in the c.m. frame of the proton pair. The completeness condition for the wave functions of the continuous spectrum, describing the relative motion of the protons, is as follows:

$$\frac{1}{(2\pi)^3}\int \varphi_{\vec{k}}^{*(-)}(\vec{r})\, \varphi_{\vec{k}}^{*(-)}(\vec{r}')\, d^3\vec{k} = \delta^3(\vec{r} - \vec{r}')\ ,$$
$$\frac{1}{(2\pi)^3}\int \varphi_{\vec{k}}^{*(-)}(\vec{r})\, \varphi_{\vec{k}}^{*(-)}(-\vec{r}')\, d^3\vec{k} = \delta^3(\vec{r} + \vec{r}')\ . \qquad (16)$$

Taking into account these relations, we obtain[4]

$$\frac{d\sigma\left(d + a \to (pp) + b\right)}{dt} = \left(\frac{d\sigma^{(nf)}(n + a \to p + b)}{dt} + \frac{2}{3}\frac{d\sigma^{(f)}(n + a \to p + b)}{dt}\right) \times$$
$$\times\left(1 - F(t)\right) + \frac{1}{3}\frac{d\sigma^{(f)}(n + a \to p + b)}{dt}\left(1 + F(t)\right) =$$
$$= \frac{d\sigma^{(nf)}(n + a \to p + b)}{dt}\left(1 - F(t)\right) + \frac{d\sigma^{(f)}(n + a \to p + b)}{dt}\left(1 - \frac{1}{3}F(t)\right)\ , \qquad (17)$$

where

$$F(t) = \int \left(\Psi_d(r)\right)^2 e^{-i\vec{q}\vec{r}}\, d^3\vec{r}$$

is the deuteron formfactor. When $t = 0$ ($\vec{q} = 0$), then the formfactor $F(t) = 1$. In this case we have the simple relation

$$\frac{d\sigma\left(d + a \to (pp) + b\right)}{dt}\Big|_{t=0} = \frac{2}{3}\frac{d\sigma^{(f)}(n + a \to p + b)}{dt}\Big|_{t=0}\ . \qquad (18)$$

It should be stressed that the last result remains valid also when the contribution of the deuteron D-state is taken into account (in the previous formulae we have neglected this contribution).

7. Let us consider in detail the process $d + p \to (pp) + n$ in the forward direction. The amplitude of the charge transfer reaction $n + p \to p + n$ at the zero angle can be presented in the following general form:

$$\hat{f} = \left[c_0 + c_1\left(\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(3)} - \left(\hat{\vec{\sigma}}^{(1)}\vec{l}\right)\left(\hat{\vec{\sigma}}^{(3)}\vec{l}\right)\right) + c_2\left(\hat{\vec{\sigma}}^{(1)}\vec{l}\right)\left(\hat{\vec{\sigma}}^{(3)}\vec{l}\right)\right]\hat{P}_{exch}\ , \qquad (19)$$

where $\vec{l}$ is the unity vector directed along the neutron momentum. In this case the operator $\hat{\hat{B}}$ in Eq. (6) is described by the formula

$$\hat{\hat{B}}(0)=\left[c_1\left(\hat{\vec{\sigma}}^{(3)}-\vec{l}\left(\hat{\vec{\sigma}}^{(3)}\vec{l}\right)\right)+c_2\vec{l}\left(\hat{\vec{\sigma}}^{(3)}\vec{l}\right)\right] . \qquad (20)$$

In so doing, the spin-flip part of the differential cross-section of the np charge transfer reaction in the forward direction is given by the expression

$$\frac{d\sigma^{(f)}}{dt}(n+p\to p+n)\Big|_{t=0}=\frac{1}{2}tr_{(3)}\hat{\hat{B}}(0)\hat{\hat{B}}^+(0)=2|c_1|^2+|c_2|^2 . \qquad (21)$$

Then, in accordance with Eq.(15), the effective cross-section of the charge-exchange breakup of the unpolarized fast deuteron on the unpolarized proton (hydrogen) target, in the forward direction, is as follows:

$$d\sigma(d+p\to(pp)+n)\Big|_{t=0}=\frac{2}{3}\left(2|c_1|^2+|c_2|^2\right)\times$$

$$\times\left|\int\Psi_d(r)\varphi_{\vec{k}}^{*(-)}(\vec{r})d^3\vec{r}\right|^2 dt\frac{d^3\vec{k}}{(2\pi)^3} . \qquad (22)$$

Now, let us assume that the deuteron is polarized and its spin state is characterized by the spin density matrix $\hat{\rho}^d$. The proton of the target is supposed to be unpolarized as before. It is not difficult to show that if the deuteron D-state is not taken into account, the contribution to the effective cross-section of the process $d+p\to(pp)+n$ in the forward direction is provided only by the diagonal elements of the density matrix $\rho^d_{+1,+1}$, $\rho^d_{0,0}$, $\rho^d_{-1,-1}$, corresponding to the definite spin projections onto the direction of the deuteron momentum $\vec{l}$. As a result, we can write

$$d\sigma(d+p\to(pp)+n)\Big|_{t=0}=$$

$$=\frac{1}{2}tr_{(3)}\left\{\sum_{\mu=0,\pm1}\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_\mu^{(trip)}\rangle\langle\chi_\mu^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle\,\rho^d_{\mu,\mu}\times\right. \qquad (23)$$

$$\left.\times 2\left|\int\Psi_d(r)\,\varphi_{\vec{k}}^{*(-)}(\vec{r})d^3\vec{r}\right|^2\right\}\frac{d^3\vec{k}}{(2\pi)^3}dt .$$

It follows from Eqs. (13a) and (20) that:

$$\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_{+1}^{(trip)}\rangle=-\frac{c_1}{\sqrt{2}}\left(\hat{\sigma}_x^{(3)}+i\hat{\sigma}_y^{(3)}\right),$$

$$\langle\chi_{+1}^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle=-\frac{c_1^*}{\sqrt{2}}\left(\hat{\sigma}_x^{(3)}-i\hat{\sigma}_y^{(3)}\right),$$

$$\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_{-1}^{(trip)}\rangle=\frac{c_1}{\sqrt{2}}\left(\hat{\sigma}_x^{(3)}-i\hat{\sigma}_y^{(3)}\right),$$

$$\langle\chi_{-1}^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle=\frac{c_1^*}{\sqrt{2}}\left(\hat{\sigma}_x^{(3)}+i\hat{\sigma}_y^{(3)}\right),$$

$$\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_0^{(trip)}\rangle=c_2\hat{\sigma}_z^{(3)},$$

$$\langle\chi_0^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle=c_2^*\hat{\sigma}_z^{(3)} . \qquad (24)$$

In so doing, the contributions of the transitions from the triplet states of the deuteron to the singlet state of the proton pair are as follows:

$$\frac{1}{2}tr_{(3)}\left\{\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_{+1}^{(trip)}\rangle\langle\chi_{+1}^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle\right\}=$$

$$=\frac{1}{2}tr_{(3)}\left\{\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_{-1}^{(trip)}\rangle\langle\chi_{-1}^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle\right\}=|c_1|^2 ,$$

$$\frac{1}{2}tr_{(3)}\left\{\langle\chi^{(\sin)}|\hat{\hat{B}}(0)\hat{\vec{\sigma}}^{(1)}|\chi_0^{(trip)}\rangle\langle\chi_0^{(trip)}|\hat{\hat{B}}^+(0)\hat{\vec{\sigma}}^{(1)}|\chi^{(\sin)}\rangle\right\}=|c_2|^2 . \qquad (24a)$$

Thus, taking into account the normalization condition

$$\rho^d_{+1,+1}+\rho^d_{0,0}+\rho^d_{-1,-1}=1 ,$$

we have

$$d\sigma(d+p\to(pp)+n)\Big|_{t=0}=2\left(|c_1|^2+\left(|c_2|^2-|c_1|^2\right)\rho^d_{0,0}\right)\times$$

$$\times\left|\int\Psi_d(r)\varphi_{\vec{k}}^{*(-)}(\vec{r})d^3\vec{r}\right|^2 dt\frac{d^3\vec{k}}{(2\pi)^3} . \qquad (25)$$

When $c_2=c_1$, Eq. (19) for the amplitude of the charge transfer reaction $n+p\to p+n$ gives

$$\hat{f}=c_1\left(\hat{\vec{\sigma}}^{(1)}\hat{\vec{\sigma}}^{(3)}\right)\hat{P}_{exch} , \qquad (26)$$

and the effective cross-section of the charge-exchange breakup of the deuteron on the unpolarized target in the forward direction does not depend on its spin state. When $c_2\neq c_1$, the dependence on the longitudinal tensor polarization of the deuteron appears:

$$d\sigma(d+p\to(pp)+n)\Big|_{t=0}=2\left(\frac{2|c_1|^2+|c_2|^2}{3}+T_{20}\left(|c_1|^2-|c_2|^2\right)\right)\times$$

$$\times\left|\int\Psi_d(r)\varphi_{\vec{k}}^{*(-)}(\vec{r})d^3\vec{r}\right|^2 dt\frac{d^3\vec{k}}{(2\pi)^3} , \qquad (27)$$

where

$$T_{20}=\frac{1}{3}\left(1-3\,\rho^d_{0,0}\right) . \qquad (28)$$

We see that the study of the process $d+p\to(pp)+n$ in a beam of the polarized (aligned) deuterons on the unpolarized hydrogen target allows one, in principle, to separate two spin-dependent terms in the amplitude of the charge transfer reaction $n+p\to p+n$, one of which (being proportional to c_1) does not conserve and the other one (being proportional to c_2) conserves the projection of the nucleon spin onto the direction of momentum at the transition of the neutron into the proton.

8. The study of the charge-exchange breakup of the polarized deuteron on the polarized proton in the forward direction allows one to obtain the additional information about the spin structure of the amplitude of the charge transfer reaction $n+p \to p+n$ at the zero angle, including the relative phase of the amplitudes c_1 and c_2. Let the spin state of the target proton be described by the density matrix

$$\hat{\rho}^p = \frac{1}{2}\left(1 + \vec{P}^p \hat{\vec{\sigma}}^{(3)}\right), \tag{29}$$

where $\vec{P}^p$ is the polarization vector of the proton. When both the deuteron and the proton are polarized, we should write instead of Eq. (23):

$$d\sigma(d+p \to (pp)+n)|_{t=0} = \\ = tr_{(3)}\left[\hat{\rho}^p \sum_\mu \sum_\nu \langle \chi_\mu^{(trip)} | \hat{\vec{B}}^+(0)\hat{\vec{\sigma}}^{(1)} | \chi^{(\sin)} \rangle \langle \chi^{(\sin)} | \hat{\vec{B}}(0)\hat{\vec{\sigma}}^{(1)} | \chi_\nu^{(trip)} \rangle \rho^d_{\nu,\mu} \times \right. \\ \left. \times 2\left| \int \Psi_d(r) \varphi_{\vec{k}}^{*(-)}(r) d^3\vec{r} \right|^2 \right] \frac{d^3\vec{k}}{(2\pi)^3} dt , \tag{30}$$

where $\rho^d_{\nu,\mu} = \rho^{d*}_{\mu,\nu}$ are diagonal $(\nu = \mu)$ and non-diagonal $(\nu \neq \mu)$elements of the deuteron density matrix $\hat{\rho}^d$.

The simple calculations with using Eqs. (24) and (29) lead to the following expression for the differential cross-section:

$$d\sigma(d+p \to (pp)+n)|_{t=0} = 2\left[\frac{2|c_1|^2 + |c_2|^2}{3} + T_{20}\left(|c_1|^2 - |c_2|^2\right) - |c_1|^2 \left(P_\parallel^d P_\parallel^p\right) - \right. \\ \left. - \mathrm{Re}\, c_1 c_2^* \left(\vec{P}_\perp^p \vec{P}_\perp^d\right) + \mathrm{Im}\, c_1 c_2^* \left(\vec{T}^d \left[\vec{P}^p \vec{l}\,\right]\right)\right] \times \left| \int \Psi_d(r) \varphi_{\vec{k}}^{*(-)}(r) d^3\vec{r} \right|^2 \frac{d^3\vec{k}}{(2\pi)^3} dt . \tag{31}$$

Here $P_\parallel^p = \vec{P}^p \vec{l}$ and $\vec{P}_\perp^p = \vec{P}^p - \vec{l}\left(\vec{P}^p \vec{l}\right)$ are the longitudinal and transversal components of the polarization vector of the proton, $P_\parallel^d = \vec{P}^d \vec{l}$ and $\vec{P}_\perp^d = \vec{P}^d - \vec{l}\left(\vec{P}^d \vec{l}\right)$ are the corresponding components for the deuteron, $\vec{T}^d$ is the average value of the vector operator

$$\hat{\vec{T}}^d = \hat{\vec{s}}\left(\hat{\vec{s}}\,\vec{l}\right) + \left(\hat{\vec{s}}\,\vec{l}\right)\hat{\vec{s}} ,$$

where $\hat{\vec{s}} = \{\hat{s}_x, \hat{s}_y, \hat{s}_z\}$ is the operator of the deuteron spin. In the coordinate system $\{x, y, z\}$ with the axis z being parallel to the direction $\vec{l}$ of the deuteron momentum, the deuteron polarization parameters are expressed through the elements of the spin density matrix of the deuteron in the following form:

$$P_\parallel^d = P_z^d = \rho^d_{1,1} - \rho^d_{-1,-1} ,\quad P_x^d = \sqrt{2}\,\mathrm{Re}\left(\rho^d_{1,0} + \rho^d_{-1,0}\right),$$

$$P_y^d = -\sqrt{2}\,\mathrm{Im}\left(\rho^d_{1,0} - \rho^d_{-1,0}\right); \\ T_z^d = \rho^d_{1,1} + \rho^d_{-1,-1} ,\quad T_x^d = \sqrt{2}\,\mathrm{Re}\left(\rho^d_{1,0} - \rho^d_{-1,0}\right), \\ T_y^d = -\sqrt{2}\,\mathrm{Im}\left(\rho^d_{1,0} + \rho^d_{-1,0}\right). \tag{32}$$

Let us note that when the deuteron is longitudinally polarized $\left(\rho^d_{0,0} = \rho^d_{1,0} = \rho^d_{-1,0} = 0\right)$, then, independently of the magnitude of the amplitude c_2,

$$d\sigma(d+p \to (pp)+n)|_{t=0} = 2|c_1|^2\left(1 - P_\parallel^d P_\parallel^p\right)\left|\Psi_d(\vec{r})\varphi_{\vec{k}}^{*(-)}(\vec{r}) d^3\vec{r}\right|^2 \frac{d^3\vec{k}}{(2\pi)^3} dt . \tag{33}$$

At $P_\parallel^d P_\parallel^p = 1$ the effective cross-section is equal to zero. In this case the charge-exchange breakup of the deuteron in the forward direction is forbidden due to the conservation of the projection of the angular momentum onto the momentum direction (this projection is equal to 3/2 in the initial state and to 1/2 in the final state, because the angular momentum of the (pp)-system is zero).

Using the transversally polarized deuteron and proton, it is possible to determine the phase difference for the amplitudes c_1 and c_2.

9. At the flight of the two-proton system in the forward direction, the distribution over the proton momentum in the deuteron rest frame, taking into account the Coulomb and strong interaction of the protons in the final state, is described by the integral factor

$$G(k) = \left| \int \Psi_d(r) \varphi_{\vec{k}}^{*(-)}(\vec{r})\, d^3\vec{r} \right|^2 . \tag{34}$$

It should be noted that, when neglecting the contribution of the deuteron D-state, the distribution over the direction of the momentum is isotropic, independently of the deuteron polarization. In the calculations we will use the Hulthen expression for the normalized S-wave deuteron wave function having the correct asymptotic behavior[7]:

$$\Psi_d(r) = \frac{1}{\sqrt{2\pi(\rho - d)}} \frac{e^{-r/\rho} - e^{-\alpha r/\rho}}{r} . \tag{35}$$

Here $\rho = 1/\sqrt{m\varepsilon} \approx 4.31\ fm$ is the radius of the deuteron, $d = 1.7\ fm$ is the effective radius of the low-energy neutron-proton interaction, $\alpha = 6.25$ (see, for example, [8]). If the final state interaction is not taken into account (as in the papers[3,4]), we would have:

$$\varphi_{\vec{k}}^{(-)}(\vec{r}) = e^{i\vec{k}\vec{r}} ;$$

$$G_0(k) = \frac{8\pi}{\rho - d}\rho^4 \left[\frac{1}{1+(k\rho)^2} - \frac{1}{\alpha^2 + (k\rho)^2}\right]^2 . \tag{36}$$

However, this expression is incorrect at sufficiently small $k\ (\leq 1/\rho)$.

In order to estimate the contribution of the Coulomb and strong interactions of two protons, it is possible to use the approximate formula for the wave function of the relative motion of two charged particles, which is valid outside the region of the nuclear force action at the distances $r \ll a_B$, $r \lesssim 1/k$, where a_B is the Bohr radius (for two protons $a_B = 57.5\ fm$; thus, $\rho \ll a_B$). This formula is[9,10]:

$$\varphi_{\vec{k}}^{(-)}(r) = \sqrt{A_c(k)}\, e^{i\delta(k)} \left(e^{i\vec{k}\vec{r}} + \frac{f_c^*(k)}{r} (\cos kr - iA_c(k) \sin kr) \right). \tag{37}$$

Here

$$A_c(k) = \frac{2\pi / k a_B}{\exp(2\pi / k a_B) - 1} \tag{38}$$

is the Coulomb (Gamov) factor taking into account the Coulomb repulsion of the protons,

$$\delta(k) = \arg \Gamma\left(1 + \frac{i}{k a_B}\right),$$

$$f_c(k) = \frac{f_0}{1 + \frac{1}{2} d_0 k^2 f_0 - \frac{2 f_0}{a_B} h(k) - ikA_c(k) f_0}, \tag{39}$$

$$h(k) = \sum_{n=1}^{\infty} \frac{1}{n\left(n^2 (k a_B)^2 + 1\right)} - C + \ln(k a_B), \tag{40}$$

where $C = 0.577\ldots$ is the Euler constant.
In so doing, $f_c(k)$ is the effective amplitude of the strong proton interaction, renormalized by the Coulomb interaction, f_0 is the scattering length ($f_0 = 7.8\ fm$), d_0 is the effective radius for the low-energy scattering of two protons ($d_0 = 2.8\ fm$).

As a result, we obtain

$$G(k) = \frac{8\pi\rho^4}{\rho - d} A_c(k) \left| \frac{1 + \frac{f_c^*(k)}{\rho} - ikf_c^*(k) A_c(k)}{1 + (k\rho)^2} - \frac{1 + \frac{\alpha f_c^*(k)}{\rho} - ikf_c^*(k) A_c(k)}{\alpha^2 + (k\rho)^2} \right|^2. \tag{41}$$

Then the expression (22) for the effective cross-section of the charge-exchange breakup of the unpolarized deuteron on the unpolarized proton target in the forward direction can be presented in the following form:

$$d\sigma(d + p \to (pp) + n)\big|_{t=0} = \frac{2}{3}\left(2|c_1|^2 + |c_2|^2\right) Q(x) x^2\, dx\, dt, \tag{42}$$

where $x = k\rho$ ($k = 45.8\,x\ MeV/c$),

$$Q(x) = \frac{1}{2\pi^2 \rho^3} G(k). \tag{43}$$

According to Eq.(31), in the case of the polarized deuteron and the polarized proton one should write in Eq. (42)

$$2|c_1|^2 + |c_2|^2 + 3T_{20}\left(|c_1|^2 - |c_2|^2\right) - 3|c_1|^2 \left(P_\parallel^d P_\parallel^p\right) - 3\,\mathrm{Re}\, c_1 c_2^* \left(\vec{P}_\perp^d \vec{P}_\perp^p\right) +$$
$$+ 3\,\mathrm{Im}\, c_1 c_2^* \left(\vec{T}^d \left[\vec{P}^p \vec{l}\right]\right)$$

instead of

$$2|c_1|^2 + |c_2|^2.$$

The formulae (41)-(43) are applicable at the values $x \le 1.5$.

The momentum distributions $Q(x)$ and $Q_0(x) = \frac{1}{2\pi^2\rho^3} G_0(k)$ (the latter does not include the contribution of the final state interaction) are given in Fig. 1.

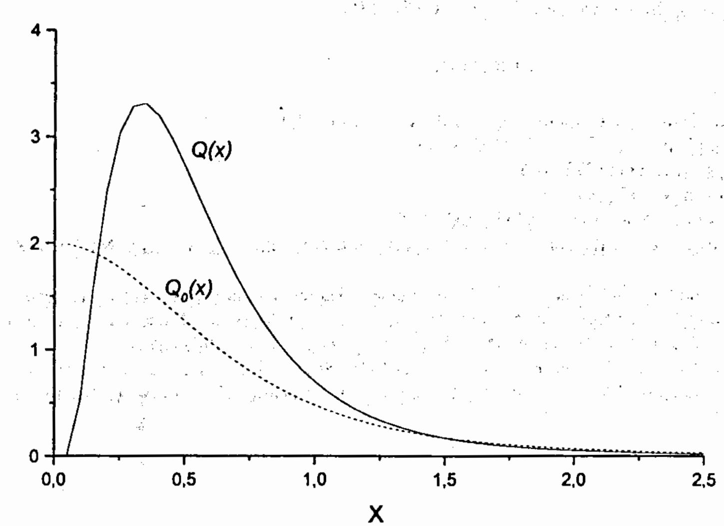

Figure 1

10. Summary

a) In the framework of the impulse approach, the relation between the effective cross-section of the charge-exchange breakup of a fast deuteron $d + a \to (pp) + b$ and the effective cross-section of the charge transfer process $n + a \to p + b$ is considered.

b) It is shown that the study of the process $d + p \to (pp) + n$ in a beam of the polarized (aligned) deuterons on the unpolarized proton target in the forward direction allows one to separate two spin-dependent terms in the amplitude of the charge transfer reaction $n+p \to p+n$ at the zero angle, one of which does not conserve and the other one conserves the projection of the nucleon spin onto the direction of momentum at the transition of the neutron into the proton. The expression for the effective cross-section of the charge-exchange breakup of the polarized deuteron at its collision with the polarized proton, containing the additional dependence on the phase difference of these terms and the deuteron polarization parameters, is obtained.

c) The distribution over relative momenta of the protons, being produced in the charge-exchange process $d+p \to (pp)+n$ in the forward direction, is investigated with taking into account the effects of the proton identity (Fermi-statistics) and of the Coulomb and strong interactions of two protons in the final state.

This work was supported in part by the Russian Foundation of Fundamental Investigations (grants 97-02-16699 and 98-02-16915).

References

[1] Pomeranchuk I. . Doklady Academii Nauk USSR, v. 78 (1951) 249
[2] Lapidus L.I. Zh. Eksp. Teor. Fiz., v. 32 (1957) 1437
[3] Dean N.W. Phys. Rev., D5 (1972) 1661
[4] Dean N.W. Phys. Rev., D5 (1972) 2832
[5] Bugg D.V., Wilkin C., Nucl. Phys., A467 (1987) 575
[6] Landau L.D., Lifshitz E.M., Kvantovaya mekhanika (Quantum Mechanics), Moscow, Nauka (1989) §§ 62, 136
[7] Hulthen L., Sugavara M., in "Structure of Atomic Nuclei", Handbuch der Physik, Springer, Berlin, v. 39 (1957), (Russian transl. "Stroyenie atomnogo yadra", IIL, Moscow (1959), part I, chapter 4, section 13)
[8] Lyuboshitz V.L. Yad. Fiz., v. 48 (1988) 1501 (Sov. J. Nucl. Phys., v. 48 (1988) 956)
[9] Lednicky R., Lyuboshitz V.L. Yad. Fiz., v. 35 (1982) 1316 (Sov. J. Nucl. Phys., v. 35 (1982) 770)
[10] Lednicky R., Lyuboshitz V.V., Lyuboshitz V.L. Yad. Fiz., v. 61 (1998) 2161 (Phys. At. Nucl., v.61 (1998) 2050)