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PRODUCTION RATES FOR π^+K^--, pK^-- AND $p\pi^-$-ATOMS IN INCLUSIVE PROCESSES

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1 Introduction

Measurements of the annihilation probabilities for hadron atoms allow one to determine the threshold amplitudes of the transitions of the particles forming the atoms into all possible final states. In the paper [1] the relation between the amplitude of the process

$$\pi^- \mathrm{p} \;\rightarrow\; \pi^0 \mathrm{n} \tag{1}$$

and the probability of annihilation of the π^--meson and the proton from the Coulomb bound state into π^0n was obtained. The binding energy of the pπ^--atom is 3.2 keV. At such a small energy the amplitude of process (1) can be expressed [1] through the S-wave πN-scattering lengths in the states with isospins of 1/2 and 3/2.

Similar expressions were obtained for the annihilation probabilities of the atoms formed by π^+- and π^--mesons ($A_{2\pi}$) [2, 3] and oppositely charged π- and K-mesons ($A_{\pi\mathrm{K}}$) [3]. The measurement of the annihilation probabilities for the channels

$$A_{2\pi} \;\rightarrow\; \pi^0 + \pi^0 \tag{2}$$
$$A_{\pi\mathrm{K}} \;\rightarrow\; \pi^0 + \mathrm{K}^0 \tag{3}$$

allows one to determine the amplitudes of the processes

$$\pi^+ + \pi^- \;\rightarrow\; \pi^0 + \pi^0 \tag{4}$$
$$\pi^+ + \mathrm{K}^- \;\rightarrow\; \pi^0 + \mathrm{K}^0 \tag{5}$$

by the model-independent method practically at zero energy in the center-of-mass system of initial particles. Because the amplitudes of (4) and (5) at low energies can be expressed through the differences of scattering lengths $\pi\pi(\pi\mathrm{K})$ the measurement of the probabilities of processes (2) and (3) allows one to determine the values

$$a_0 \;-\; a_2 \tag{6}$$
$$b_{1/2} \;-\; b_{3/2} \tag{7}$$

by the model-independent method. Here a_0, a_2 ($b_{1/2}, b_{3/2}$) are the $\pi\pi(\pi\mathrm{K})$ S-wave scattering lengths in the states with isospins of 0, 2 (1/2, 3/2). The values of a_0 and a_2 were calculated in Chiral Perturbation Theory (CHPT) with a precision of 5% [4, 5]. This precision has been improved to $2 \div 3\%$ [6, 7, 8].

The model-independent measurement of the $\pi\pi$-scattering lengths with a precision of several per cent allows one to test the predictions of CHPT and, consequently, our understanding of chiral symmetry breaking [9] lying in the basis of the QCD Lagrangian, which describes interactions of quarks and gluons, and effective Lagrangians, which describe interactions of physical particles. Generalized CHPT allows one to determine the quark condensate magnitude [7, 8] by using the value of $a_0 - a_2$ from precise measurement.

The πK-scattering lengths were also calculated in CHPT [10]. The model-independent measurement of these values allows one to test the concept of chiral symmetry breaking in the processes with strangeness.

In the paper [11] the relations were obtained that allow one to calculate the production probabilities of $A_{2\pi}$, $A_{\pi K}$ and any other atoms if the inclusive production cross sections of the particles forming these bound states are known. In the same paper a method was also proposed for observation and lifetime measurement of the atoms, and estimates were given for the yields of $A_{2\pi}$, $A_{\pi K}$ and other atoms in pp-collisions at a beam energy of 70 GeV and an atom emission angle of 8.4° in the lab system. The observation of $A_{2\pi}$ was carried out at the U-70 accelerator [12]. The atoms produced in pTa collisions at E_p=70 GeV were detected at an angle of 8.4° in the lab system. Later the experimental estimation of the $A_{2\pi}$ lifetime was obtained [13]. At the present time the experiment [14] on the $A_{2\pi}$ lifetime measurement and determination of the difference of $\pi\pi$-scattering lengths with a precision of 5% is under preparation at PS CERN.

The yields of $A_{2\pi}$ in proton-nuclear collisions and spectra of $A_{2\pi}$ were calculated and presented in [15] for E_p=24, 70, 450, 1000 GeV for a set of angles from 1° to 6°. In the present paper the results of similar calculations for $A_{\pi K}$, $A_{p\pi}$ (atoms formed by p and π^-) and A_{pK} (atoms formed by p and K^-) are given. The results of the calculations show that the intensities of $A_{\pi K}$, A_{pK} and $A_{p\pi}$ production are high enough for the lifetime of these atoms to be measured by the same method proposed for the $A_{2\pi}$ lifetime measurement.

Because the lifetime of $A_{p\pi}$ was measured [18] with a precision of $\sim 1\%$ this kind of measurement gives a possibility of testing the precision of the method [11] allowing determination of the lifetime of almost any hadron atoms. The experiment on measurement of the energy of the transition 2P–1S and the widths of γ-lines [16, 17] of A_{pK} is under preparation. However, the lifetime measurement of A_{pK} by the other method [11] is reasonable.

2 Basic relations

The probability of atom production is proportional to the double inclusive cross section for production of the two constituent particles of this atom, which have small relative momenta. Calculating the atom production cross sections, one should exclude the contribution to the double cross section from those constituents that arise from the decays of long-lived particles and cannot form the atom. When one or both particles in the pair come from these decays, the typical range between them is much larger than the Bohr radius of the atom. Consequently, the probability of atom production is negligible. The main long-lived sources of pions are η, η', Λ, K_s^0, $\Sigma^\pm$ and the main long-lived sources of protons are Λ, $\Sigma^\pm$. At the same time, the fraction of long-lived sources for kaons is much smaller as compared to the fraction of their short-lived sources.

The laboratory differential inclusive cross section for the atom production can be written in the form [11]

$$\frac{d\sigma_n^A}{d\vec{p}_A} = (2\pi)^3 \frac{E_A}{M_A} |\Psi_n(0)|^2 \left. \frac{d\sigma_s^0}{d\vec{p}_1\, d\vec{p}_2} \right|_{\vec{p}_1=\frac{m_1}{m_2}\vec{p}_2=\frac{m_1}{M_A}\vec{p}_A}, \tag{8}$$

where $\vec{p}_A$, E_A and M_A are the momentum, energy and mass of the atom in the lab system, respectively, $|\Psi_n(0)|^2 = p_B^3/\pi n^3$ is the atomic wave function(without regard for the strong interactions between the particles forming the atom, i.e. the pure Coulomb wave function) squared at the origin with the principal quantum number n and the orbital momentum $l=0$, p_B is the Bohr momentum of the particles in the atom, $d\sigma_s^0/d\vec{p}_1 d\vec{p}_2$ is the double inclusive production cross section for the pairs from short-lived sources (hadronization processes, ρ, ω, Δ, K^*, Σ^*, etc.) without regard for the $\pi^+\pi^-$-Coulomb interaction in the final state, $\vec{p}_1$ and $\vec{p}_2$ are the momenta of the particles forming the atom in the lab system. The momenta obey the relation $\vec{p}_1 = \frac{m_1}{m_2}\vec{p}_2 = \frac{m_1}{M_A}\vec{p}_A$ (m_1 and m_2 are the masses of the particles). The atoms are produced with the orbital momentum $l=0$, because $|\Psi_{n,l}(0)|^2 = 0$ when $l \neq 0$. They are distributed over n as n^{-3}: $W_1 = 83\%$, $W_2 = 10.4\%$, $W_3 = 3.1\%$, $W_{n\geq 4} = 3.5\%$. Note that $\Sigma_{n=1}^{\infty}|\Psi_n(0)|^2 = 1.202|\Psi_1(0)|^2$.

The double inclusive cross section without regard for the Coulomb interaction may be written in the form [19]:

$$\frac{d\sigma^0}{d\vec{p}_1 d\vec{p}_2} = \frac{1}{\sigma_{in}} \frac{d\sigma}{d\vec{p}_1} \frac{d\sigma}{d\vec{p}_2} R(\vec{p}_1, \vec{p}_2), \tag{9}$$

where $d\sigma/d\vec{p}_1$ and $d\sigma/d\vec{p}_2$ are the single particle inclusive cross sections, σ_{in} is the inelastic cross section of hadron production, R is a correlation function due to strong interaction.

The probability of particle production per interaction (yield) can be expressed through the differential cross section:

$$\frac{dN}{d\vec{p}} = \frac{d\sigma}{d\vec{p}} \frac{1}{\sigma_{in}}. \tag{10}$$

From (8), (9) and (10), after substituting the expression for $|\Psi_n(0)|^2$ and summing over n, one can obtain an expression for the inclusive yield of atoms in all S–states through the inclusive yields of positive and negative hadrons

$$\frac{d^2N_A}{dp_A d\Omega} = 1.202 \cdot 8\,\pi^2\,(\mu\alpha)^3 \frac{E_A}{M_A} \frac{p_A^2}{p_1^2 p_2^2} \frac{d^2N_1}{dp_1 d\Omega} \frac{d^2N_2}{dp_2 d\Omega} R, \tag{11}$$

where μ is the reduced mass of the atom ($\mu = \frac{m_1 m_2}{m_1+m_2}$), α is the fine structure constant, p_1 and p_2 are the momenta of positive and negative hadrons ($p_1 = \frac{m_1}{m_2}p_2$) and Ω is a solid angle.

3 Results of calculations

To obtain the yields of p, K^-–, π^+– and π^--mesons we used the computer simulation programs FRITIOF 6.0 [21] and JETSET 7.3 [22] (CERN Program Library) based on the Lund string fragmentation model. FRITIOF is a generator for hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, which makes use of JETSET for fragmentation.

For the calculation of particle yields, the simulated events were accumulated in the two-dimensional arrays depending on the emission angle and momentum of the particles with an angular bin width of 0.3° and a momentum bin width of 0.1 GeV/c. At this stage the selection of particles from long-lived and short-lived sources was performed. The particle yield distributions were parameterized by the linear combinations of elementary functions. Further, using yields only from the short-lived sources we obtained the distributions of the atom yields over the angle and momentum. The correlation coefficient due to strong interactions was found to be $R = 1.65 \pm 0.05$ [19, 20] for the pairs of $\pi^+\pi^-$-mesons whose momenta satisfy the requirement $\vec{p}_1 = \vec{p}_2$. Because R is not yet known for the pairs of π^+K^-, $p\pi^-$ and pK^-, it was assumed to be 1 in the calculations.

The results are presented in Fig.1, 2, 3, where the yields of $A_{\pi K}$, $A_{p\pi}$, A_{pK} are shown for the reaction p+Al → atom+X at the proton energies E_p = 24, 70 and 450 GeV and atom emission angles $\theta_{lab} = 1° \div 6°$ as a function of the momentum of the atom. The probability of atom production in the momentum interval Δp_A and the solid angle $\Delta\Omega$ at the emission angle θ can be estimated by multiplying the mean value of the yields in the given momentum interval by the values of this interval in GeV/c and the solid angle $\Delta\Omega$ in sr.

The yields of $A_{\pi K}$, $A_{p\pi}$, A_{pK} integrated over the momentum are shown in Fig.4 versus the emission angle.

Similar calculations for the reaction p+Ta → atom+X were performed in order to obtain the scale of A-dependence for the atom yields. In Table 1 the ratios of the atom yields integrated over the momentum for pTa to those for pAl are given as a function of the emission angle.

The comparison of the FRITIOF 6.0 results with the experimental data was made. At an energy of 24 GeV the most complete description of the yields of p, π^+, π^-, K^+ and K^- by the experimental data in the given range of emission angles and momenta is available in the paper [23]. The comparison with these data shows that the deviation of the calculated yields from the data is not larger than 20%. Thus, the precision of the calculated yields of $A_{\pi K}$, $A_{p\pi}$, A_{pK} is better than 40%.

The description of the particle yields at 24 GeV for the same angles and momenta by FRITIOF7.02, the latest version of FRITIOF, shows satisfactory agreement with the experimental data for π^+ and π^-, but big difference for p, K^+, and K^-. A large deviation of the results obtained by FRITIOF7.02 from the experimental data is observed for the proton yields. We noticed that the shapes of the proton spectra did not correspond to the shapes of the experimental yields as was the case for

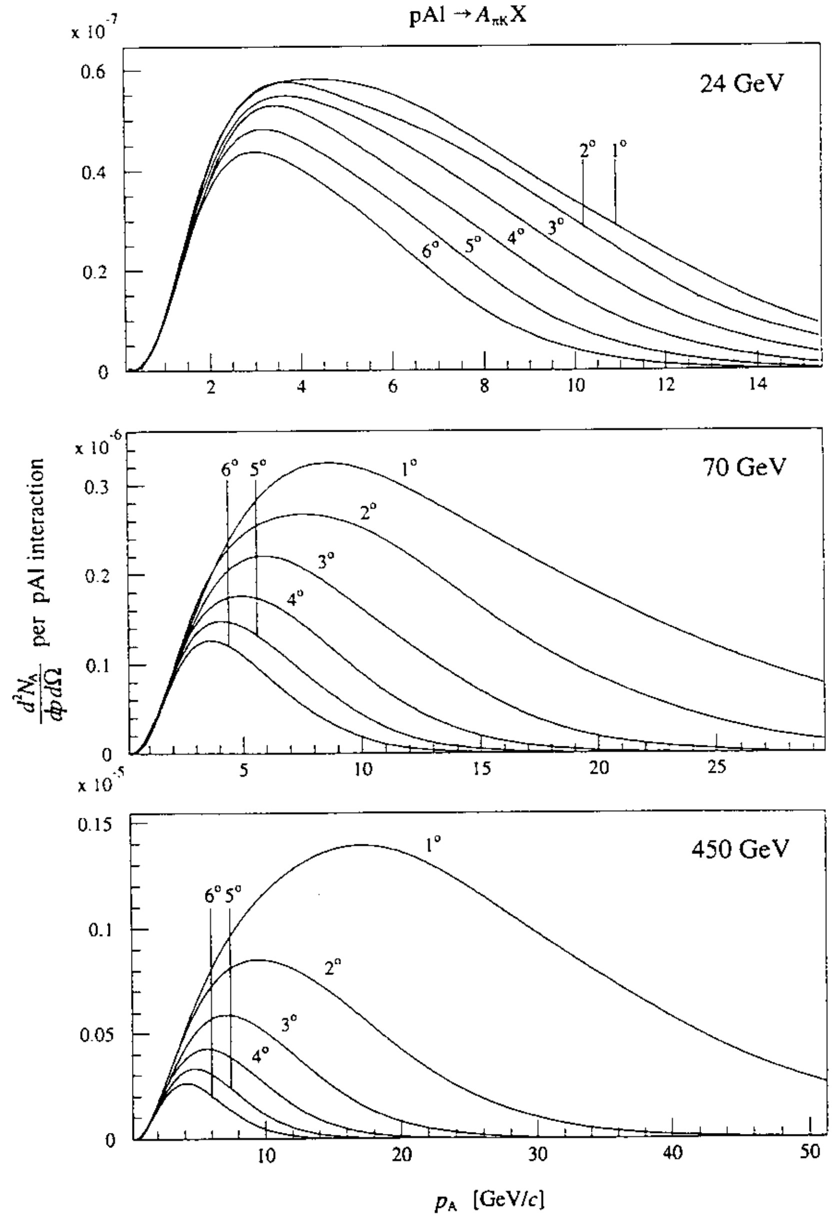

Figure 1: Yields of $A_{\pi K}$ for the reaction pAl → $A_{\pi K}X$ (per pAl interaction) at the energies E_p = 24, 70, 450 GeV and emission angles θ_{lab} = 1°, 2°, 3°, 4°, 5°, 6° as a function of the atom momentum.

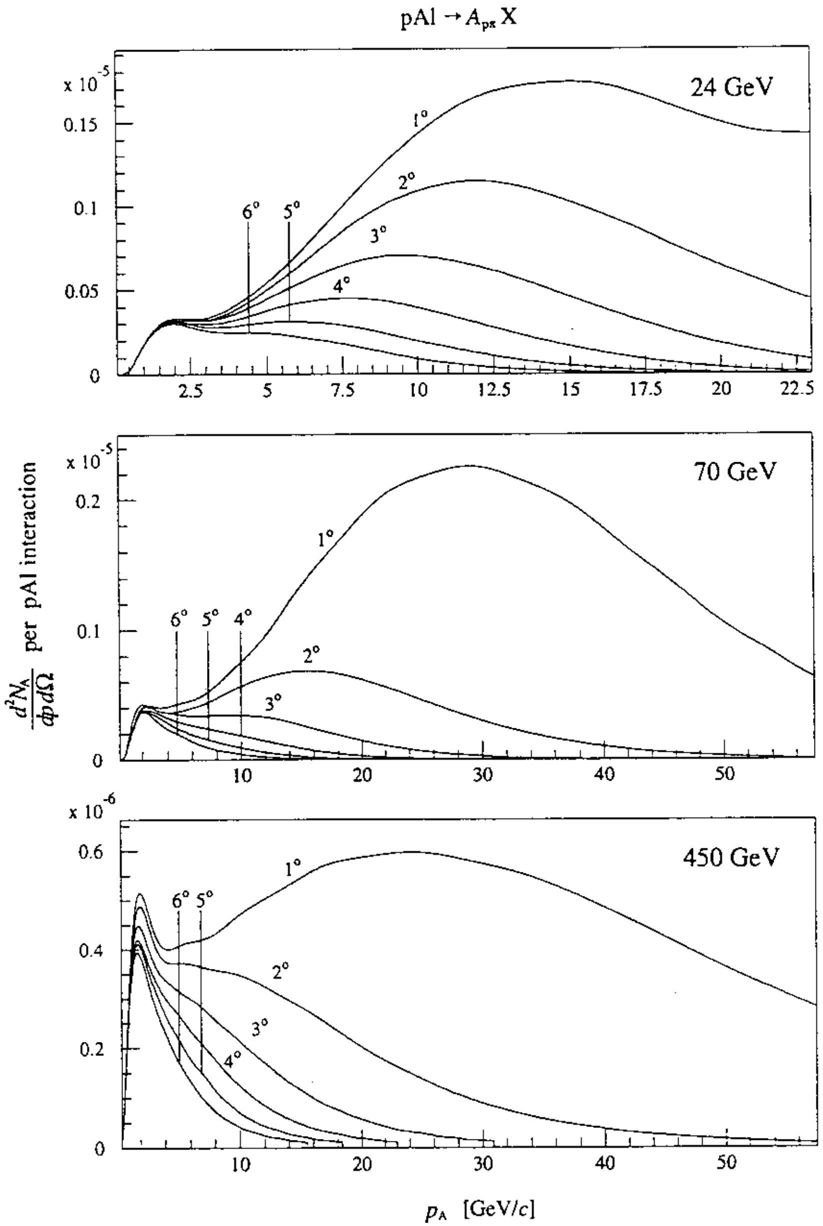

Figure 2: Yields of $A_{p\pi}$ for the reaction pAl $\to A_{p\pi}X$ (per pAl interaction) at the energies $E_p = 24$, 70, 450 GeV and emission angles $\theta_{lab} = 1°, 2°, 3°, 4°, 5°, 6°$ as a function of the atom momentum.

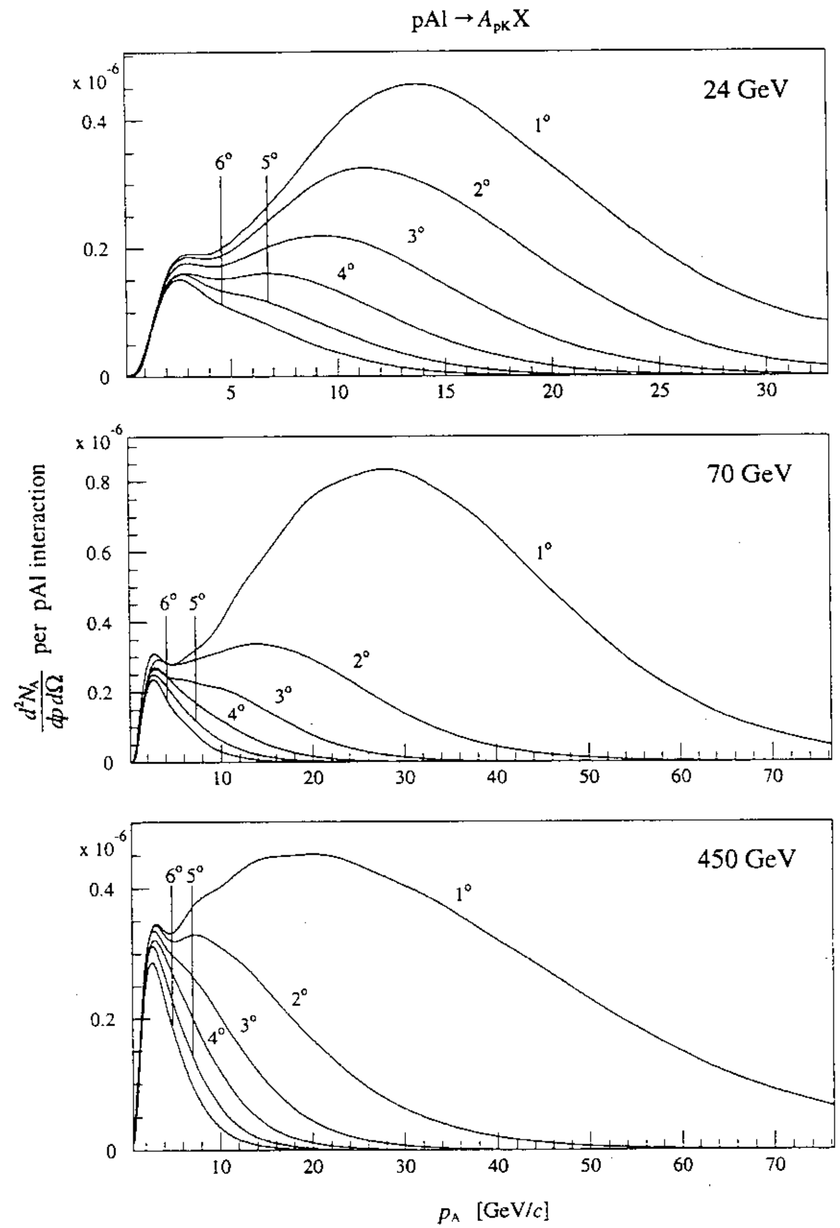

Figure 3: Yields of A_{pK} for the reaction pAl $\to A_{pK}X$ (per pAl interaction) at the energies $E_p = 24$, 70, 450 GeV and emission angles $\theta_{lab} = 1°, 2°, 3°, 4°, 5°, 6°$ as a function of the atom momentum.

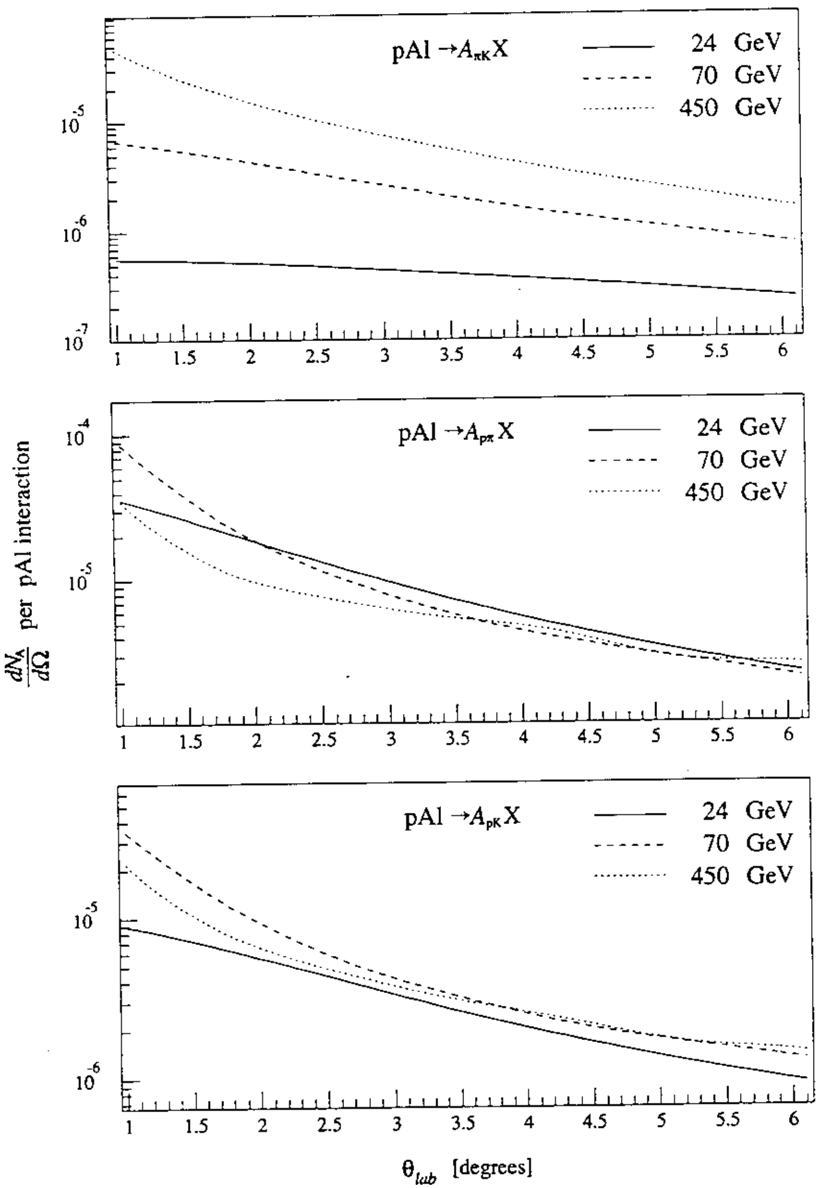

Figure 4: $A_{\pi K}$, $A_{p\pi}$, A_{pK} yields for the reaction pAl $\to$ (atom)X (per pAl interaction) at the energies E_p = 24, 70, 450 GeV integrated over the momentum versus atom emission angles.

Table 1: The ratios of atom yields integrated over the momentum for the target of Ta to those for the target of Al at the energies E_p = 24, 70, 450 GeV versus emission angles.

E_{lab}=24 GeV

	1°	2°	3°	4°	5°	6°
$A_{\pi K}$	1.10	1.16	1.19	1.20	1.25	1.30
$A_{p\pi}$	0.90	1.03	1.22	1.44	1.65	1.83
A_{pK}	0.82	0.98	1.14	1.37	1.53	1.69

E_{lab}=70 GeV

	1°	2°	3°	4°	5°	6°
$A_{\pi K}$	1.14	1.24	1.39	1.50	1.58	1.64
$A_{p\pi}$	1.13	1.68	2.06	2.27	2.36	2.39
A_{pK}	1.01	1.49	1.81	1.99	2.05	2.05

E_{lab}=450 GeV

	1°	2°	3°	4°	5°	6°
$A_{\pi K}$	1.67	1.92	2.07	2.17	2.25	2.31
$A_{p\pi}$	2.21	2.48	2.58	2.63	2.64	2.66
A_{pK}	2.19	2.51	2.63	2.68	2.70	2.74

the proton yields calculated by FRITIOF6.0. This difference in the behavior of two programs can probably be explained by the fact that FRITIOF7.02 was developed for higher energies and higher $p_\perp$ than under the conditions described. For this reason we used FRITIOF6.0 for our calculations.

We also compared the calculated particle yields with the experimental data [24] at an energy of 70 GeV. This comparison showed that agreement with the experimental data is satisfactory for the emission angles from 3° to 6°, but for the angle range of 1°–2.5° the simulated yields are higher by a factor of ~ 2 than the experimental yields.

For an energy of 450 GeV we assume that the program gives reasonable results for the calculated hadron atom yields for $p_\perp > 400$ MeV/c because this code was worked out for this energy range.

The validity of the calculation by FRITIOF6.0 for the single-particle yields was also considered in the paper [15].

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