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TOTAL MULTIPLICITY DISTRIBUTIONS OF SECONDARY PARTICLES IN π^-p INTERACTIONS AT 5-40 GEV/C

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Распределение по полной множественности вторичных частиц в $\pi^- p$-взаимодействиях при 5-40 ГэВ/с

Сообщаются результаты анализа распределений по полной множественности вторичных частиц в $\pi^- p$ -взаимодействиях при 5-40 ГэВ/с. Предложена новая формула, позволяющая описать все исследуемые распределения. Показано, что распределение по множественности "истинно рожденных" частиц $n' = n - 2$ удовлетворяет гипотезе KNO скейлинга, в рассматриваемой области энергий. Предложена простая аналитическая форма универсальной KNO -функции.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

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Total Multiplicity Distributions of Secondary Particles in $\pi^- p$ Interactions at 5-40 GeV/c

The total multiplicity distributions of secondary particles in $\pi^- p$ - interactions at 5-40 GeV/c have been analysed. A new simple formula allowing the description of all the available data is proposed. It has been found that the distribution of the multiplicity $n' = n-2$ of the produced hadrons satisfies in the energy region under consideration the KNO scaling hypothesis.

The simple analytical form of the universal KNO-function has been proposed.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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I. Introduction

At the present time a hypothesis of scaling in the multiplicity distributions of secondary particles is under extensive study. This hypothesis has been formulated by Koba, Nielsen and Olesen [1]. Until now only charged particle data have been analysed from this point of view. No similar studies of the total multiplicity of secondaries, n , have been carried out yet.

The only data available on the distributions of the total number of secondaries are those at incident particle momenta of 5 [2], 10 [3], 40 [4] *GeV/c* for the reaction

$$\pi^- p \to N + n_+ \pi^+ + n_- \pi^- + n_0 \pi^0, \qquad (1)$$

where N is the nucleon, n_+, n_-, n_0 are the numbers of π^+ -, π^- -, π^0 -mesons, respectively, $n = 1 + n_+ + n_- + n_0$.

The n distribution at 5 *GeV/c* has been obtained in our previous investigation [2] by analysing partial cross sections. The majority of them have been measured directly.

In ref. [3] the distribution over the total number of particles n at 10 *GeV/c* has been obtained by assuming that the statistical isospin-independent model [5] should be valid, since partial cross sections for multiple π^0 -meson production have not been measured.

Data at 40 *GeV/c* [4] have been obtained by analysing the multiplicity distributions of charged secondaries and gamma-quanta under the assumption that the multiplicity distributions of π^0-mesons should obey the Poisson law for a fixed number of charged secondaries.

The present investigation has been aimed at analysing the characteristics of the above n distributions.

II. Study of n *Distributions for Reaction (1)*

The available experimental data on partial cross sections[6] and the statistical isospin independent model[5] allowed one to calculate similarly to the case of ref.[3], the σ_n cross sections for n particle production at 16 *GeV/c* for reaction (1).

The validity of the model[5] for the description of reaction (1) in the energy range from 1 to 16 *GeV* has been proved in ref.[7].

The values of σ_n at 5, 10, 16 and 40 *GeV/c* are given in *Fig. 1*.

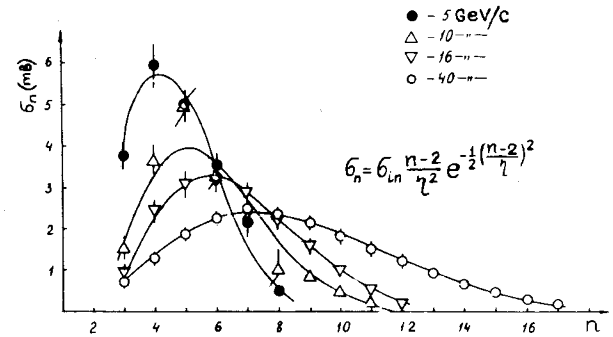

Fig. 1. Dependence of the σ_n *upon* n *;* $\chi^2/N_p = 7.8/6$*, 12.6/9, 10/10 and 16.2/16 at 5, 10, 16 and 40 GeV/c, respectively.*

Bozoki et al.[8] have analysed the experimental multiplicity distributions of charged secondaries n_{ch} and noticed that the logarithm of the integral distribution $F(n_{ch})$ (probability for production of, at least, n_{ch} charged particles) with respect to n^2_{ch} is a straight line (except for small values of n_{ch}). As has been noticed in ref.[8], this kind of dependence limits considerably the class of functions which may correspond to the differential distribution $P(n_{ch})$.

A similar analysis of integral distribution

$$F(n) = \sum_{k=n}^{n_{max}} P_k = \sum_{k=n}^{n_{max}} \sigma_k/\sigma_{in} , \qquad (2)$$

where P_k is the probability of k-particle production in the final state in reaction (1) has shown that a similar picture is observed also for n distributions. (Here σ_{in} is the total inelastic cross section of reaction (1)).

However, if one introduces the scale shift $n \to n' = n - \alpha$, it comes out that $\ln F(n)$ with respect to n'^2 is well approximated by a straight line for all n values

$$\ln F(n) = -a(n-\alpha)^2 \sim n'^2 , \qquad (3)$$

where a is an energy-dependent parameter. It has also turned out that α does not practically depend upon energy and within errors one can take $\alpha = 2$.

The results of approximation are given in *Fig. 2*. Solid lines correspond to dependence (3). The values of χ^2/N_p (N_p is the number of experimental points) are also given.

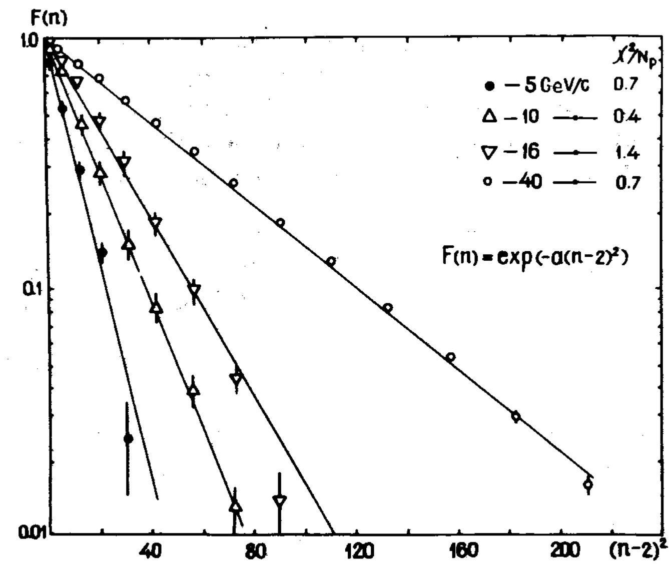

Fig. 2. Integral probability $F(n)$ *versus* $(n-2)^2$.

The function $F(n')$ is as follows:

$$F(n') = \exp(-a\,n'^2)$$

By differentiating the above function one obtains the expression for n-particle production probability

$$P_{n'} = 2an' \cdot \exp(-an'^2) \qquad (4)$$

for which the normalization condition holds:

$$\sum_{n'} P_{n'} \approx \int P_{n'}\,dn' = 1.$$

For the sake of convenience the parameter $\eta = 1/\sqrt{2a}$ is introduced instead of a, then n-particle production probability can be expressed by the formula containing one parameter only:

$$P_{n'} = \frac{n'}{\eta^2}\exp\left[-\frac{1}{2}\left(\frac{n'}{\eta}\right)^2\right]. \qquad (5)$$

The advantage of this distribution compared to other empirical formulas (Czyzewski-Rybicki /9/, Bozoki et al. /8/, E. De Wolf et al. /10/) is that its analytical form is considerably simpler and all the parameters of $n'(n)$ distributions can be easily expressed as functions of $\eta(\eta, \alpha)$.

Some of these parameters, such as the mode M_0, the average multiplicity $\langle n \rangle$, the dispersion D, the ℓ-th order central moment μ_ℓ, the second order correlation moment f_2^{nn} and the maximal value $P_n^{(max)}$ of multiplicity distribution are given in *Table 1*. Here C_ℓ are constants.

Here are also presented the values of the ratios $\langle n \rangle/D$, $M_0/\langle n \rangle$ and $\sqrt[\ell]{\mu_\ell}/\langle n \rangle$ which are constants for n' distributions.

As is seen from *Table 1*, by introducing $\eta \to \sqrt{\frac{2}{\pi}}(\langle n \rangle - 2)$ the correlation moment f_2^{nn} can be expressed in the following way:

$$f_2^{nn} = 0.273\langle n \rangle^2 - 2.06\langle n \rangle + 1.09. \qquad (6)$$

Table 1

Expressions for parameters of n' and n distributions

	n'	n
M_0	η	$\eta + \alpha$
$\langle n \rangle$	$\sqrt{\frac{\pi}{2}}\,\eta$	$\sqrt{\frac{\pi}{2}}\,\eta + \alpha$
D	$\sqrt{\frac{4-\pi}{2}}\,\eta$	
μ_ℓ	$C_\ell\,\eta^\ell$	
f_2^{nn}	$\left(\frac{4-\pi}{2}\right)\eta^2 - \sqrt{\frac{\pi}{2}}\,\eta$	$\left(\frac{4-\pi}{2}\right)\eta^2 - \sqrt{\frac{\pi}{2}}\,\eta - \alpha$
$P(n)_{max}$	$\frac{1}{\eta\sqrt{e}}$	$\frac{1}{(\alpha+\eta)\sqrt{e}}$
$\langle n \rangle / D$	$\sqrt{\frac{\pi}{4-\pi}}$	$\sqrt{\frac{\pi}{4-\pi}} + \sqrt{\frac{2}{4-\pi}}\,\frac{\alpha}{\eta}$
$M_0 / \langle n \rangle$	$\sqrt{\frac{2}{\pi}}$	$\sqrt{\frac{2}{\pi}} \cdot \frac{1+\alpha/\eta}{1+\sqrt{2/\pi}\,\alpha/\eta}$
$\sqrt[\ell]{\mu_\ell} / \langle n \rangle$	$\sqrt{\frac{2}{\pi}\,C_\ell}$	$\sqrt{\frac{2}{\pi}\,C_\ell}\left(1 + \sqrt{\frac{2}{\pi}}\,\frac{\alpha}{\eta}\right)^{-1}$

By solving the equation $f_2^{nn} = 0$ it has been found that the n distribution obeys the Poisson law at the energy corresponding to $\langle n \rangle \simeq 7$.

Note that the knowledge of the dependence of $\langle n \rangle$ and σ_{in} upon the energy allows one to determine σ_n unambiguously and, in particular, also the maximum value of the cross section σ_n^{max} at the given energy.

To clear out the degree of compatibility of formula (5) with experiment and to determine the values of the parameter at various energies we used this formula to approximate the experimental n distribution at 5, 10, 16 and 40 *GeV/c* (see, *Fig. 1*).

The values of χ^2/N_p and η are listed in *Table 2*.

Table 2

$p\ \frac{GeV}{c}$	5	10	16	40
N_p	6	9	10	16
χ^2/N_p	1.3	1.4	1.0	0.4
η	2.25 ± 0.03	3.01 ± 0.05	3.65 ± 0.06	5.28 ± 0.04
$\langle n \rangle$	4.76 ± 0.12	5.80 ± 0.16	6.60 ± 0.18	8.65 ± 0.12
D	1.36 ± 0.06	1.87 ± 0.08	2.25 ± 0.10	3.44 ± 0.06
$\frac{\eta}{\langle n \rangle-2}$	0.82 ± 0.04	0.79 ± 0.05	0.80 ± 0.04	0.80 ± 0.02
$\frac{\langle n \rangle-2}{D}$	1.91 ± 0.11	2.01 ± 0.12	2.04 ± 0.11	1.93 ± 0.04
$\langle n \rangle/D$	3.50 ± 0.12	3.11 ± 0.14	2.93 ± 0.13	2.51 ± 0.05
f_2^{nn}	-2.91 ± 0.18	-2.31 ± 0.30	-1.55 ± 0.26	3.22 ± 0.31

There is a good agreement of distribution (5) with experimental data.

Table 2 presents the values of $\langle n \rangle$, D, f_2^{nn}, $\langle n \rangle/D$, $\langle n-2 \rangle/D$ and $\eta/\langle n-2 \rangle$ for n distributions.

It is seen that the experimental values of $\eta/\langle n' \rangle$ and $\langle n' \rangle/D$ do not practically vary with energy and within errors coincide with the corresponding values in *Table 1*

$$\frac{\eta}{\langle n' \rangle} = \frac{M_0'}{\langle n' \rangle} = \sqrt{\frac{2}{\pi}} = 0.80 \qquad \langle n' \rangle/D = \sqrt{\frac{\pi}{4-\pi}} = 1.92 .$$

We have carried out the combined analysis of the dependence of the average multiplicity of all secondaries $\langle n \rangle$, charged particles $\langle n_{ch} \rangle$ and neutral pions $\langle n_0 \rangle$ in $\pi^- p$ - interactions at 5-205 *GeV/c* upon the square of c.m.s. energy s. These values are shown in *Fig. 3*,* shaded symbols are 5 *GeV/c* data [2].

As a result of the analysis it has been established that the relations $K_n = \langle n \rangle/\langle n_{ch} \rangle$ and $K_{\pi^0} = \langle n_0 \rangle/\langle n_{ch} \rangle$ do not practically depend upon energy. Their average values are 1.58±0.03 and 0.45±0.03, respectively (see, *Fig. 3*).

This means that the dependences $\langle n \rangle$, $\langle n_0 \rangle$ and $\langle n_{ch} \rangle$ upon s can be of similar analytical form (to an accuracy of the constant factor).

It is known that in some theoretical models such as statistical, hydrodynamic and thermodynamic ones, the power dependence of the average multiplicity upon s is predicted. In particular, the authors of paper [17] have obtained the dependence $\sim s^{1/3}$. By approximating the experimental data on n_{ch} in $\pi^- p$ -interactions it has been shown that this dependence agrees with experiment up to 60 *GeV*.

Here the known data on the average multiplicities were approximated by the following formula:

$$\langle n_i \rangle = K_i\, g(s/M^2)^{1/3}, \tag{8}$$

where $\langle n_i \rangle = \langle n \rangle$, $\langle n_0 \rangle$, $\langle n_{ch} \rangle$; $K_i = 1.58,\ 0.45,\ 1.,$

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* *Fig. 3* shows data on $\langle n \rangle$ at 5-40 *GeV/c* (from *Table 2*), on $\langle n_{ch} \rangle$ at 5÷205 *GeV/c* [10], on $\langle n_0 \rangle$ at 5 [2], 6.8 [11], 18.5 [12], 25 [13], 40 [14], 100 [15] and 205 [16] *GeV/c*.

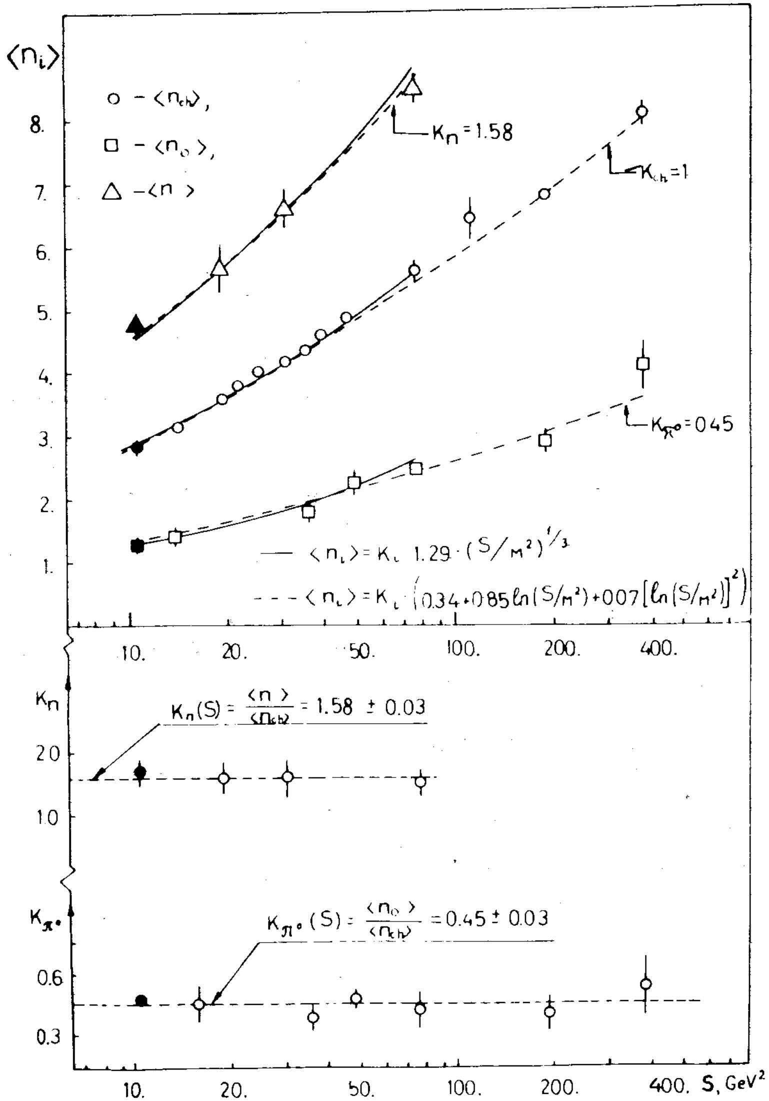

Fig. 3. Dependence of the average multiplicity of π^0 -mesons $\langle n_0 \rangle$, charged particles $\langle n_{ch} \rangle$ and all secondaries $\langle n \rangle$ and the ratios $\langle n \rangle / \langle n_{ch} \rangle$ and $\langle n_0 \rangle / \langle n_{ch} \rangle$ upon the square of energy s (c.m.s.). Shaded symbols are 5GeV/c data.

respectively. M is the proton mass, g is the normalizing parameter. It has turned out that dependence (8) agrees satisfactorily with the experiment only in the region of $s \lesssim 70\ GeV/c^2$ with $g = 1.29 \pm 0.02$ and $\chi^2/N_p = 28/16$ (see solid lines in *Fig. 3*).

Some other models (peripheral, multiperipheral ones) predict the logarithmic increase of multiplicity with respect to s in the asymptotic region. As the analysis shows, this assumption does not provide sufficient agreement with experimental data in the range of 5-205 *GeV/c*.

Therefore, for approximating these data in the above energy region (5-205 *GeV*) we used the dependence

$$\langle n_i \rangle = K_i \, (a + b \ln(s/M^2) + c \cdot \ln^2(s/M^2)), \qquad (9)$$

where a , b and c are parameters of the fit.

It has been established that this dependence describes well all the experimental data shown in *Fig. 3* (dashed curves). Here $a = 0.34 \pm 0.15$, $b = 0.85 \pm 0.16$, $c = 0.07 \pm 0.02$, $\chi^2/N_p = 20/24$. Thus, using the same function one can describe quite satisfactorily to an accuracy of the constant factor the dependence of $\langle n_{ch} \rangle$ and $\langle n_0 \rangle$ upon energy .

III. KNO Scaling for the Total Number of Secondary Particle Distributions in Reaction (1)

Koba et al [1]. have investigated the limiting behaviour of the multiplicity distribution in the region of asymptotic energies. They have shown that if Feynman's scaling is valid [18] for all the inclusive processes and for sufficiently large s (so that one can neglect $(\ln s)^{-1}$ comparing to unity), then the function $\langle n \rangle P_n$ should depend on the ratio $n/\langle n \rangle$ only:

$$\langle n \rangle P_n \to \Psi\left(\frac{n}{\langle n \rangle}\right)\left[1 + 0\left(\frac{1}{\langle n \rangle}\right)\right]. \qquad (10)$$

It is essential that the form of the function Ψ in ref. [1] is not predicted and should be determined experimentally.

The universality of the function $\langle n\rangle P_n$ means that the ratio of the distribution mode to average multiplicity and all the normalized central momenta are independent of energy:

$$\frac{M^o}{\langle n\rangle} = \mathrm{const}\,, \qquad K_\ell = \frac{\mu_\ell}{\langle n\rangle^\ell} = \mathrm{const}. \tag{11}$$

In particular, the ratio $D/\langle n\rangle$ should remain constant too.

The analysis of the multiplicity distributions of charged particles [19–21] has shown that if instead of the variable $z = \frac{n_{ch}}{\langle n_{ch}\rangle}$ one uses $z' = (n_{ch} - a)/\langle n_{ch} - a\rangle$ (where a is an energy-independent constant), the function $\langle n_{ch} - a\rangle P_{n_{ch}} = \Psi(z')$ within experimental errors does not depend upon energy starting from $\simeq 10$ *GeV/c*. The parameter a, as has been shown in ref. [20], can be interpreted as the average number of leading particles and hence, $n_{ch} = n_{ch} - a$ as the number of "really produced" charged particles. Similarly, the total number of "really produced" particles for reaction (1) is $n' = n - 2$.

Now one can obtain directly the analytical form of the $\Psi(z')$ function for the multiplicity distribution of these particles in the KNO-representation.

Indeed, substituting $\eta \to \sqrt{\frac{2}{\pi}}\langle n'\rangle$ in (5) we obtain:

$$P_{n'} = \frac{\pi}{2}\,\frac{n'}{\langle n'\rangle^2}\exp\left[-\frac{\pi}{4}\left(\frac{n'}{\langle n'\rangle}\right)^2\right]. \tag{12}$$

Note that Buras and Koba [22] using the model of local excitation have obtained the expression for the probability of secondary particle distribution. Its form coincides with that of dependence (12). However, there is a considerable difference: in the formula obtained in ref. [22] the multiplicity n is used as an argument. Besides, as has been shown in our study, the satisfactory description of experimental distribution by means of the dependence of (12) type is provided due to the use of $n' = n - a$ as an argument. Note also, that dependence (12) is, in fact, a simple consequence of the fact that experimental distributions $F(n)$ are well described by the linear dependence of (3) type.

Formula (12) in the KNO representation is as follows:

$$\langle n-2\rangle \frac{\sigma_n}{\sigma_{in}} = \frac{\pi}{2} z' e^{-\frac{\pi}{4} z'^2}. \tag{13}$$

For this function, as is seen from *Table 1*, conditions of (11) necessary for universality are fulfilled. Thus, it seems natural for the description of experimental data on the total multiplicity n to use scaling dependence (13).

Indeed, the direct check-up has shown that the approximation of the experimental dependence of the values $\langle n'\rangle / P_n'$ on z' for reaction (1) using exp. (13) is quite satisfactory. With a fixed value $a=2$ the obtained values of χ^2/N_p are 1.6, 1.5, 1.1 and 0.6 for 5, 10, 16 and 40 *GeV/c*, respectively. The result of approximation is shown in *Fig. 4*.

IV. Conclusions

1. The new (one-parameter) dependence, (5), for the description of the total multiplicity distributions in the reactions

$$\pi^- p \to N + n_- \pi^- + n_+ \pi^+ + n_0 \pi^o$$

at 5-40 *GeV/c* has been proposed.

2. The combined analysis of the experimental data on average multiplicities $\langle n\rangle$, $\langle n_0\rangle$ and $\langle n_{ch}\rangle$ in $\pi^- p$ - interactions at 5-205 *GeV/c* has been performed. It has been shown that to an accuracy of the constant factor the dependences of $\langle n\rangle$, $\langle n_0\rangle$ and $\langle n_{ch}\rangle$ upon s are well approximated by the common function, i.e., the polynomial of the second power on $\ln s$.

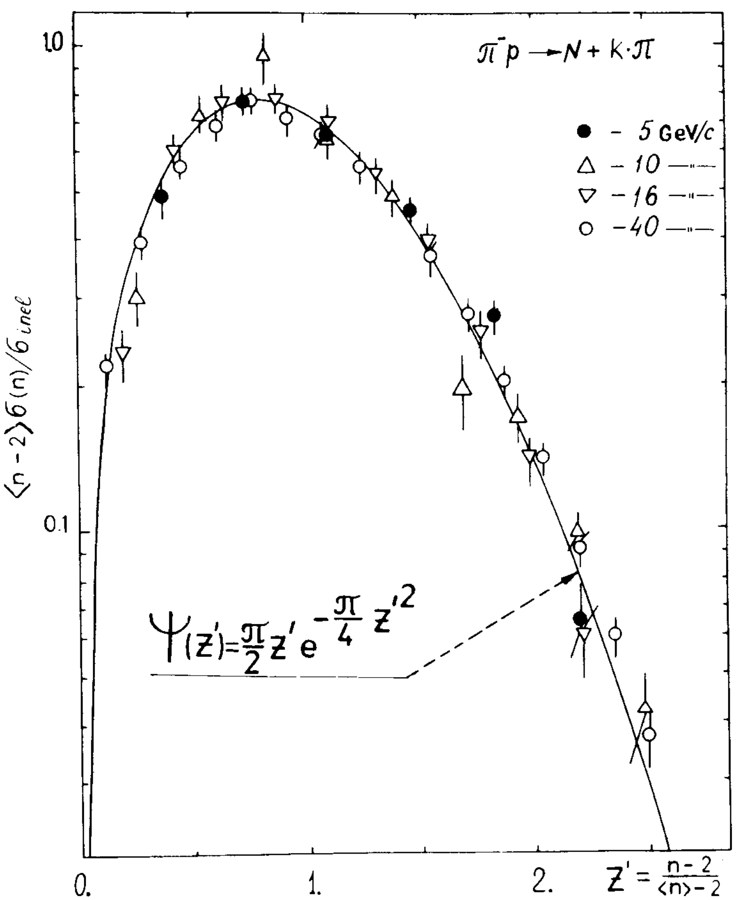

Fig. 4. Dependence $\langle n-2\rangle \sigma_n/\sigma_{in}$ *upon* $z' = (n-2)/\langle n-2\rangle$. *The values of* χ^2/N_p *are 9.6/6, 13.5/9, 11/10 and 9.6/16 at 5, 10, 16 and 40 GeV/c, respectively.*

3. The similarity of distributions on the total number of "really produced" particles n' for $\pi^- p$-interactions at 5-40 *GeV/c* has been observed in the KNO representation. A simple analytical form has been obtained for the universal KNO function:

$$\langle n'\rangle P_{n'} = \Psi\left(\frac{n'}{\langle n'\rangle}\right) = \frac{\pi}{2}\,\frac{n'}{\langle n'\rangle}\exp\left[-\frac{\pi}{4}\left(\frac{n'}{\langle n'\rangle}\right)^2\right].$$

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