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ANALYTICAL REPRESENTATION  
OF THE LONGITUDINAL HADRONIC SHOWER  
DEVELOPMENT

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One of the important questions of hadron calorimetry is the question of the longitudinal development of hadronic showers [1]. For analysing of energy depositions in various depths of calorimeters it is necessary to have the analytical representation of longitudinal shower development from the face of calorimeter. Such formula is particularly useful at designing, testing and calibration of huge calorimeter complex like in ATLAS [2] at LHC.

There is well known parameterisation of longitudinal shower development from shower origin suggested by Bock et al. [3]:

$$\frac{dE_s(x)}{dx} = k \left\{ w \left( \frac{x}{X_o} \right)^{a-1} e^{-b \frac{x}{X_o}} + (1-w) \left( \frac{x}{\lambda_I} \right)^{a-1} e^{-d \frac{x}{\lambda_I}} \right\}, \quad (1)$$

where  $X_o$  is the radiation length,  $\lambda_I$  is the interaction length,  $a$ ,  $b$ ,  $d$ ,  $w$  are parameters,  $k$  is the normalisation factor. In this work we use the values of parameters obtained in [4]:  $a = 0.786 + 0.427 \cdot \ln E$ ,  $b = 0.29$ ,  $d = 0.978$ ,  $\omega = 1.032 - 0.159 \cdot \ln E$ .

In practice, the calorimeter longitudinal segmentation is insufficient for precise determination of shower vertex position. Because of it is necessary to integrate the longitudinal profile from shower origin over the shower position:

$$\frac{dE(x)}{dx} = \int_0^x \frac{dE_s(x-x_v)}{dx} e^{-\frac{x_v}{\lambda_I}} dx_v, \quad (2)$$

where  $x_v$  is a coordinate of the shower vertex.

In given work the analytical representation of the hadronic shower longitudinal development from the calorimeter face has been derived.

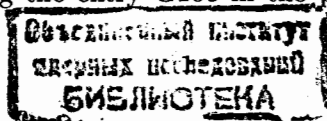
It turns out that

$$\frac{dE(x)}{dx} = N \left\{ \frac{wX_o}{a} \left( \frac{x}{X_o} \right)^a e^{-b \frac{x}{X_o}} {}_1F_1 \left( 1, a+1, \left( b - \frac{X_o}{\lambda_I} \right) \frac{x}{X_o} \right) + \frac{(1-w)\lambda_I}{a} \left( \frac{x}{\lambda_I} \right)^a e^{-d \frac{x}{\lambda_I}} {}_1F_1 \left( 1, a+1, -(1-d) \frac{x}{\lambda_I} \right) \right\}, \quad (3)$$

where  ${}_1F_1(\alpha, \beta, z)$  is the confluent hypergeometric function [5] and  $N$  is the normalisation factor. The confluent hypergeometric function in the first term of formula (3) can be calculated by using the following relation:

$${}_1F_1(1, a+1, z) = az^{-a} e^z \gamma(a, z), \quad (4)$$

where  $\gamma(a, z)$  is the incomplete gamma function [5]. This function can be calculated by using the entry G106 in the program library [6]. The



confluent hypergeometric function in the second term can be calculated by using the series development:

$${}_1F_1(1, a+1, -z) = 1 - \frac{z}{a+1} + \frac{z^2}{(a+1)(a+2)} + \dots \quad (5)$$

This series has the remarkable property: at  $a = 0$  it corresponds to the function  $e^{-z}$ . For real calorimeters with longitudinal size usually less than  $10 \lambda_I$  the value of  $z = (1-d)\frac{x}{\lambda_I}$  does not exceed 0.2. Therefore it is sufficient only 3 terms in order to calculate the function  ${}_1F_1(1, a+1, -z)$  with the precision of 0.1%.

The normalization factor is

$$N = E_{beam} / \int_0^\infty \frac{dE(x)}{dx} dx, \quad (6)$$

where  $E_{beam}$  is the beam energy. The integral in the denominator can be calculated by using the following relation [7]:

$$\int_0^\infty z^\nu e^{-\lambda z} {}_1F_1(\alpha, \gamma, kz) dz = \Gamma(\nu+1) \lambda^{-\nu-1} {}_2F_1\left(\alpha, \nu+1, \gamma, \frac{k}{\lambda}\right), \quad (7)$$

where  ${}_2F_1(\alpha, \beta, \gamma, z)$  is the hypergeometric function [5], which in our case have the very simple form:

$${}_2F_1(1, a+1, a+1, z) = \frac{1}{1-z}. \quad (8)$$

As a result we obtain

$$N = \frac{E_{beam}}{\lambda_I \Gamma(a) (w X_o b^{-a} + (1-w) \lambda_I d^{-a})}. \quad (9)$$

In Fig. 1 the calculations by formula (3) are shown and compared with the experimental data at 20 - 140 GeV obtained by using the conventional iron-scintillator calorimeter [4] and the one with innovative feature — the longitudinal orientation of the scintillating tiles [8]. The good agreement is observed.

So, now there is useful analytical formula which gives the possibility immediately to obtain the longitudinal shower energy deposition from calorimeter face.

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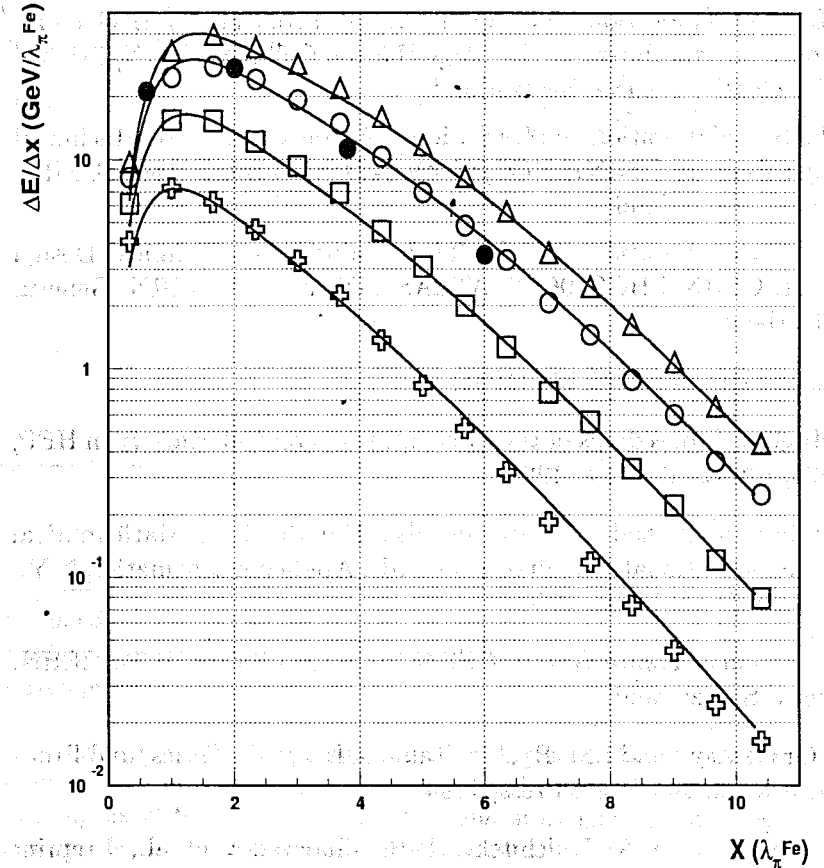


Figure 1: Longitudinal profiles of the hadron showers of 20 GeV (crosses), 50 GeV (squares), 100 GeV (open circles) and 140 GeV (triangles) energies as a function of the longitudinal coordinate  $x$  in units  $\lambda_I$  for conventional iron-scintillator calorimeter [4] and of 100 GeV (black circles) for tile iron-scintillator calorimeter [8]. The solid lines are calculations by function (3) with parameters from [4].

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