

# сообщЕНия ОБЪЕДИНЕННОГО ИнсТИтУТА яДеРНЫХ ИсслЕдОВАНИЙ 

## $98-29$

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$B_{s}^{0} \rightarrow D_{s}^{-} \alpha_{1}^{+}\left(D_{s}^{-} \rightarrow \phi \pi^{-}, D_{s}^{-} \rightarrow K^{* 0} K^{-}\right)$-DECAY CHANNEL IN THE ATLAS $B_{s}^{0}$-MIXING STUDIES

[^0]
## 1 Introduction

The main features of the $B_{s}$-mixing studies were explained in previous notes $[1,2]$ (see also references cited therein), so we don't need to repeat them here. Instead we will try to give arguments that $B_{s}$-mixing phenomenon is indeed worthy to be studied.

The famous Russian painter Casimir Malevich said a long time ago: "The object in itself is meaningless ... the ideas of the conscious mind are worthless". We would like to choose his great painting "The black square", which is reproduced below, as a starting point of our introduction.


But from this starting point it is possible to go to the very different directions depending from one's imagination. So let us imagine the following picture behind the black square [3]:
"A cat is penned up in a steel chamber, along with the following diabolical device: in a Geiger counter there is a tiny bit of radioactive substance, that perhaps in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left the entire system to itself for one hour, one would say that the cat still lives if meanwhile no atom has decayed. The state vector $\mid \Psi>$ of the entire system would express this by having in it the living and the dead cat mixed or smeared out in equal parts."

But this is of course nonsense, at least from cat's point of view!
We have reminded Schrödinger's cat old story here in order to give an impression that although we all became familiar with particle mixing, because the superposition principle lies on a very background of quantum mechanics, this phenomenon is by no means obvious or trivial property of reality.

But what is strange and queer at the macrophysics level can still appear as the most common thing at the microphysics level. It seems that even our existence is based on particle mixing as will be explained below.

One of very important characteristics of elementary particle is its mass. We can get some insight about its origin from the following simple trick. The propagator of a massive fermion can be represented in such a way

$$
\frac{1}{\hat{p}-m}=\frac{1}{\hat{p}}+\frac{1}{\hat{p}} m \frac{1}{\hat{p}}+\frac{1}{\hat{p}} m \frac{1}{\hat{p}} m \frac{1}{\hat{p}}+\cdots
$$

or graphically

where a single line represents the propagator of a massless particle. So things look like as if the massless particle is propagating through some medium and the mass emerges
as a result of friction or interaction with this environment. But what is the medium the particle interacts with? A (massless) fermionic particle can have the following interaction with some scalar field $\mathcal{L}_{i n t}=g \bar{\psi} \psi \varphi$ :

$$
g\rangle---\varphi
$$

If now the self interactions of this scalar field are such that it doesn't disappear in a vacuum state and develops a nonzero vacuum expectation value $<\varphi>$, when it is convenient to expand $\varphi=<\varphi>+\varphi^{\prime}$, where $\varphi^{\prime}$ corresponds to the physical scalar partic:les (excitations over the vacuum) and $\langle\varphi\rangle$ just gives the medium (the vacuum) where all of us are living. Now because of this decomposition of $\varphi$ the fermion-scalar interaction splits into two parts:


The second diagram represents an emission of the real scalar quantum and the first one generates the fermion mass $m=g<\varphi\rangle$.

But if, for example $d$-quark can emit a scalar particle without changing its flavour, why can't it do this with changing the flavour? We know that the fiavour is not always conserved, so the following interaction is not excluded:


But now the first term gives $d-s$ mixing! As a result our initial $d$ and $s$ fields are no longer mass eigenstates (the states with definite mass), instead their time development in the rest frame is described by the matrix Schrödinger's equation ( $\hbar=1$ ):

$$
\begin{gathered}
i \frac{\partial}{\partial t}\binom{d}{s}=\left(\begin{array}{cc}
m_{d} & m_{d s} \\
m_{d s} & m_{s}
\end{array}\right)\binom{d}{s} \equiv \\
\left(\begin{array}{cc}
\cos \theta_{c} & \sin \theta_{c} \\
-\sin \theta_{c} & \cos \theta_{c}
\end{array}\right)\left(\begin{array}{cc}
\tilde{m}_{d} & 0 \\
0 & \tilde{m}_{s}
\end{array}\right)\left(\begin{array}{cc}
\cos \theta_{c} & -\sin \theta_{c} \\
\sin \theta_{c} & \cos \theta_{c}
\end{array}\right)\binom{d}{s}
\end{gathered}
$$

where $\tan 2 \theta_{c}=\frac{2 m_{d}}{m_{s}-m_{d}}$ and $\tilde{m}_{d}, \tilde{m}_{s}$ mass eigenvalues are defined from the equations $\pi n_{d}=$ $\tilde{m}_{d} \cos ^{2} \theta_{c}+\tilde{m}_{s} \sin ^{2} \theta_{c}, m_{s}=\tilde{m}_{d} \sin ^{2} \theta_{c}+\tilde{m}_{s} \cos ^{2} \theta_{c}$. It is obvious that the corresponding eigenvectors (the physical $d$ and $s$ quarks) are

$$
\binom{\tilde{d}}{\tilde{s}}=\left(\begin{array}{cc}
\cos \theta_{c} & -\sin \theta_{c} \\
\sin \theta_{c} & \cos \theta_{c}
\end{array}\right)\binom{d}{s} \rightarrow \begin{aligned}
& \tilde{d}=\cos \theta_{c} d-\sin \theta_{c} s \\
& \tilde{s}=\sin \theta_{c} d+\cos \theta_{c} s
\end{aligned}
$$

This particle mixing has one important observable consequence. If initially the weak transitions were possible only within the $(u, d)$ or $(c, s)$ pairs, now the intergeneration
transitions $\tilde{u} \leftrightarrow \bar{s}$ and $\tilde{c} \leftrightarrow \tilde{d}$ are also possible because, for example, the physical $s$ quark contains both $d$ and $s$ bare fields $\tilde{s}=\sin \theta_{c} d+\cos \theta_{c} s$. Thus $\bar{u} \rightarrow \tilde{s}$ transition is proportional to $\sin \theta_{c}$ - sine of the so called Cabibbo angle.

But we have three quark-lepton generations. So after the mixing the weak transitions are possible between any up and any down quarks. The amplitudes of these weak transiions are convenient to express as a $3 \times 3$ unitary matrix. This Kobayashi-Maskawa matrix is a generalization of the Cabibbo angle and reveals a remarkable hierarchical structure [4]

$$
\left(\begin{array}{ccc}
V_{u d} & V_{u s} & V_{u b}  \tag{1}\\
V_{c d} & V_{c s} & V_{c b} \\
V_{t d} & V_{t s} & V_{t b}
\end{array}\right) \approx\left(\begin{array}{ccc}
1-\frac{1}{2} \lambda^{2} & \lambda & A \lambda^{3}(\rho-i \eta) \\
-\lambda & 1-\frac{1}{2} \lambda^{2} & A \lambda^{2} \\
A \lambda^{3}(1-\rho-i \eta) & -A \lambda^{2} & 1
\end{array}\right)
$$

Here $\lambda=\sin \theta_{c} \approx 0.22$ is a small quantity. So the intergeneration weak transitions are suppressed and this suppression is more strong for not neighboring generations.

If $\eta \neq 0$, the Kobayashi-Maskawa matrix is complex and violates $C P$. It is commonly believed today that this $C P$-violation is an important ingredient in baryon-antibaryon asymmetry generation in the universe [5] and so the source of our very existence.
So far we were talking about particle mixing at quark level. But quarks are confined into hadrons and we can study quark-mixing only indirectly via hadron-mixing. $B$-meson system is very promising in this respect: because of a large mass of the $b$-quark we can enjoy an asymptotic freedom advantage of QCD and calculate strong interaction corrections, unlike, for example, $K$-meson system.

In the Standard Model the $B_{d}-\bar{B}_{d}$ mixing originates from the following diagram (and from the second one there intermediate up-quark and $W$ lines are interchanged)

$u_{i}$ stands for any up quark. So

$$
B_{d}-\operatorname{mixing} \sim \sum_{i, j} \lambda_{i} \lambda_{j} I\left(m_{i}, m_{j}\right)
$$

where $\lambda_{i}=V_{u_{i} b} V_{u_{i} d}^{*}$ and $I\left(m_{i}, m_{j}\right)$ represents the loop integral. This integral diverges quadratically. But this divergence is harmless because the unitarity of the KobayashiMaskawa matrix ensures its cancellation in the sum: the unitarity means $\sum \lambda_{i}=0$, therefore

$$
\sum_{i, j} \lambda_{i} \lambda_{j} I\left(m_{i}, m_{j}\right)=\sum_{i, j} \lambda_{i} \lambda_{j}\left[I\left(m_{i}, m_{j}\right)-I\left(0, m_{j}\right)-I\left(m_{i}, 0\right)+I(0,0)\right]
$$

and these subtractions greatly improve the convergence. For example, in case of $t$-cuiark contribution, these subtractions lead to the replacement

$$
\frac{1}{\left(k^{2}-m_{t}^{2}\right)^{2}} \longrightarrow \frac{1}{\left(k^{2}-m_{\ell}^{2}\right)^{2}}-\frac{2}{k^{2}\left(k^{2}-m_{t}^{2}\right)}+\frac{1}{k^{2} k^{2}}=\frac{m_{t}^{4}}{k^{2} k^{2}\left(k^{2}-m_{t}^{2}\right)}
$$

From this expression it is also clear that in fact just $t$-quark contribution is dominant for $B$-mixing, because of its extraordinary large mass.

When ARGUS made his measurement of the $B_{d}$-mixing [6], nobody thought that. $t$ quark is so massive. So the result of this measurement appeared as a big surprise. We can even say that $t$-quark was discovered by ARGUS, because the large $B_{d}$-mixing, observel by ARGUS, is very difficult to explain without the existence of the $t$-cuark with mass $>100 \mathrm{GeV}$.

We can infer from the above given diagram that even larger mixing is expected in $B_{s}$-system:

$$
\frac{B_{\mathrm{s}}-\text { mixing }}{B_{d}-\operatorname{mixing}} \sim\left|\frac{V_{t s}}{V_{t d}}\right|^{2} \sim \frac{1}{\lambda^{2}\left\{(1-\rho)^{2}+\eta^{2}\right\}} \sim \frac{1}{\lambda^{2}} \sim 25
$$

We see also that the relative magnitude of $B_{*}$ and $B_{d}$ mixings measures $(1-\rho)^{2}+\eta^{2}$ - one side of the notorious unitarity triangle [7]. It is worthwhile to mention that this ratio is, to a great extent, free from hadronic uncertainties, which arise when we ask how chark and antiquark from the above given $B$-mixing diagram really form $B$-meson.

To summarize, the $B_{s}$-mixing studies are interesting, because they reveal a very fundamental underlying phenomenon - the generation of particle masses and mixing angles via the Higgs mechanism, the least understood thing in the Standard Model. Because of heaviness of the $b$-quark and asymptotic freedom of QCD, the theory gives very definite predictions about expected $B_{s}$-mixing, hampered only from uncertainties due to our inability to solve QCD in the confinement region. But these uncertainties are also, to a certain extent, under control [7]. The theoretical predictions involves such a fundamental property as the unitarity of the Kobayashi-Maskawa matrix (the existence of only three generations). Any deviation between the theory and experiment can lead to significant change of our present day picture of the elementary particle world (recall the $B_{d}$-mixing story). The forthcoming ATLAS experiment sensitivity to the $B_{s}$-mixing covers the Stamdard Model prediction range [8]. So it will either give one more conformation of the theory or will open a window into a physics beyond the Standard Model.

## $2 \quad D_{s}^{-} \rightarrow K^{* 0} K^{-}$decay channel for $D_{s}$ reconstruction

To observe the $B_{s}^{0}$-mixing in a real experiment like ATLAS and extract the corresponting $x_{s}$ parameter, which characterizes the $B_{s}-\bar{B}_{s}$ oscillation frequency, you need to reconstruct $B_{s}$ meson and determine its decay vertex with great precision. Two of the $B_{s}$ dec:ay channels were considered for this goal up to now: $B_{s}^{0} \rightarrow D_{s}^{-} \pi^{+}[1]$ and $B_{s}^{0} \rightarrow D_{s}^{-} a_{1}^{+}[2]$. For the second channel $D_{s}^{-} \rightarrow \phi \pi^{-}, \phi \rightarrow K^{+} K^{-}$decay mode was used for the $D_{s}^{-}$reconstruction. It was mentioned in $\{1,2]$ that other decay chamels of $D_{s}^{-}$can be also used to increase signal statistics. In the present note we consider $D_{s}^{-} \rightarrow K^{00} K^{-}, K^{* 0} \rightarrow K^{+} \pi^{-}$ decay mode as one of the possibilities:


As it is clear from the Table 1 below, this decay chamel is quite promising if compared to the previously used $D_{s}^{-} \rightarrow \phi \pi^{-}$.

Table 1.
Brandhing ratios and sigmal statistices for $B_{s}^{0} \rightarrow D_{s}^{-} a_{1}^{+}(1260)$.

| Parameter | Value | Comment. |
| :---: | :---: | :---: |
| $\begin{aligned} & L\left[c m^{-2} s^{-1}\right] \\ & t[s] \\ & \sigma(b \bar{b} \rightarrow \mu X)[\mu l] \end{aligned}$ | $\begin{gathered} 10^{33} \\ 10^{7} \\ 2.3 \end{gathered}$ | $\begin{aligned} & y_{T}^{n}>6 G r V / c \\ & \left\|\eta^{\prime \prime}\right\|<2.2 \end{aligned}$ |
| $N(b \bar{b} \rightarrow \mu X)$ | $2.3 \times 10^{10}$ |  |
| $\overline{\operatorname{Br}}\left(b \rightarrow B_{*}^{0}\right)$ | 0.112 |  |
| $\operatorname{Br}\left(B_{s}^{0} \rightarrow D_{s}^{-} a_{1}^{+}\right)$ | 0.000 |  |
| $\operatorname{Br}\left(a_{1}^{+} \rightarrow \rho^{0} \pi^{+}\right)$ | $\sim 0.5$ | . |
| $\operatorname{Br}\left(0^{0} \rightarrow \pi^{-} \pi^{+}\right)$ | $\sim 1$ |  |
| $\operatorname{Br}\left(D_{s}^{-} \rightarrow \phi \pi^{*}\right)$ | 0.036 |  |
| $\operatorname{Br}\left(\phi \rightarrow K^{+} K^{-}\right)$ | 0.491 |  |
| $\operatorname{Br}\left(D_{s}^{-} \rightarrow K^{+0} \mathrm{~K}^{-}\right)$ | 0.034 |  |
| $\operatorname{Br}\left(K^{-0} \rightarrow K^{+} \pi^{-}\right)$ | $\sim 0.65$ |  |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 136000 | $\bar{D}_{s}^{*} \rightarrow \infty \pi^{-}$ |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 170800 | $D_{*}^{-} \rightarrow h^{+11} h^{*}$ |

Event generation and reconstruction procedures are similar to ones considered in [2]. All other general parameters, like impact parameter resolution for smearing and transverse momentum resolution, are also the same as in [2] and can be found there.

Contrary to the $D_{s}^{-} \rightarrow \phi \pi^{-}$case, where $D_{s}^{-}$peak was clearly seen in the invariant mass distribution of the three properly charged particles, assuming that two of them are $K$-mesons and one is pion, now $D_{s}^{-}$peak is not seen (Fig. 1), perlaps because $K^{* 0}$ is too wide as compared to $\phi$. Thus the combinatorial background from the signal cents alone is already able to hide the $D_{s}$ and the reader is left with the sentence [9] "if you shonld see the word "buffalo" written on a cage containing an elephant, don't believe your cyes". Although when $D_{s}^{-}$meson is reconstructed from its true decay products the resulting invariant mass resolution, shown in Fig. 2, is almost the same as for the $D_{s}^{-} \rightarrow \dot{\phi} \pi^{-}$ mode. Another picture in Fig. 2 shows $K^{\circ 0}$, reconstructed from its true decay products.

The resolution in the $B_{s}$-decay proper time (Fig. 3a) $\sigma_{\tau} \approx 0.061 p s$ is practically the same as for the $D_{s}^{-} \rightarrow \phi \pi^{-}$mode. The corresponding $B_{s}$-decay length resolution in the transverse plane is $\approx 100 \mu m$ and the relevant distribution is shown in Fig. 3b.

We expect that signal to background situation when using $D_{s}^{-} \rightarrow K^{00} K^{-}$mode will be similar to what was considered in [2]. Compare for example Fig. 4 from [2] and from this work, which describes a possible background from the $B_{d}^{0} \rightarrow D_{s}^{-} a_{1}^{+}$decay when $D_{s}^{-}$ is reconstructed via $D_{s}^{-} \rightarrow \phi \pi^{-}$or $D_{s}^{-} \rightarrow K^{* 0} K^{-}$modes respectively.

The main reason which allowed a good signal to background separation in [2] was the fact that $D^{-}$and $B_{d}^{0}$ masses are shifted from the $D_{s}^{-}$and $B_{s}^{0}$ masses by about 100 MeV . But this equally applies to the $D_{s}^{-} \rightarrow K^{\bullet 0} K^{-}$case also, because our cut on the $D_{s}^{-}$ invariant mass doesn't change very much. The only problem which can arise is a large $K^{* 0}$ width (as compared to the $\phi$-meson width) and therefore the cut on the invariant mass of $K^{* 0}$ should be considerably loose. At present we are not aware of a background for which this circumstance will play a crucial role.

Acceptance and analysis cuts are summarized in Table 2. As it is known [10], a second level trigger is necessary to reduce an event rate, which is still too high after the first level trigger (the tag-mion). For the $B_{s} \rightarrow D_{s}^{-} \pi^{+}, D_{s}^{-} \rightarrow \phi \pi^{-}$mode the problem was studied in [10] and it was shown that some loose cuts on the invariant masses of $\phi$ and $D_{s}$ candidates can be used for this purpose. The resulting trigger efficiency appeared to be [8] about 0.54 . For the $B_{s} \rightarrow D_{s}^{-} a_{1}^{+}, D_{s}^{-} \rightarrow K^{* 0} K^{-}$channel, discussed in this note, the second level trigger should be specially investigated, of course. However we expect that the similar mass cuts on $D_{s}$ and $K^{* 0}$ candidates will work in this case also and a $50 \%$ trigger efficiency should be a safe estimate.

Table 2.
Number of signal events from $B_{s}^{0} \rightarrow D_{s}^{-} a_{1}^{+}(1260)$ channel expected in ATLAS after 1 year ( $10^{7} \mathrm{~s}$ ) of operation at $10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$.

| Parameter | Value | Comment |
| :---: | :---: | :---: |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 136600 | $D_{s}^{--} \rightarrow \phi \pi^{-}$ |
| $N\left(K^{+} K^{--} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 170800 | $D_{s}^{--} \rightarrow K^{+0} K^{-}$ |
| $\begin{aligned} & \text { Cuts: } \\ & p_{T}>1 \mathrm{GeV} / \mathrm{c} \\ & \|\eta\|<2.5 \end{aligned}$ |  |  |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 9015 (6.6\%) | $D_{s}^{-} \rightarrow \phi \pi^{-}$ |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 9910 (5.8\%) | $D_{s}^{-} \rightarrow K^{\circ 0} K^{-}$ |
| $\begin{aligned} & \Delta \varphi_{\pi \pi}<35^{\circ} \\ & \Delta \theta_{\pi \pi}<15^{\circ} \\ & \left\|M_{\pi \pi}-M_{\rho}\right\|<192 \mathrm{MeV} / \mathrm{c}^{2}( \pm 3 \sigma) \\ & \left\|M_{\pi \pi \pi}-M_{a_{1}+}\right\|<300 \mathrm{MeV} / \mathrm{c}^{2} \end{aligned}$ |  |  |
| $\begin{aligned} & \Delta \varphi_{K K}<10^{\circ} \\ & \Delta \theta_{K K}<10^{\circ} \\ & \left\|M_{K K}-M_{\phi}\right\|<20 \mathrm{MeV} / \mathrm{c}^{2} \\ & \left\|M_{K K}-M_{D-}\right\|<15 \mathrm{MeV} / \mathrm{c}^{2} \end{aligned}$ |  | $D_{s}^{-} \rightarrow \phi \pi^{-}$ |
| $\begin{aligned} & \Delta \varphi_{K \pi}<20^{\circ} \\ & \Delta \theta_{K \pi}<10^{\circ} \\ & \left\|M_{K \pi}-M_{K \cdot 0}\right\|<80 \mathrm{MeV} / \mathrm{c}^{2} \\ & \left\|M_{K K \pi}-M_{D_{3}}\right\|<20 \mathrm{MeV} / \mathrm{c}^{2} \\ & \hline \end{aligned}$ |  | $\bar{D}_{s}^{-} \rightarrow \mathrm{K}^{\circ 0} \mathrm{~K}^{-}$ |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 6830 (5.0\%) | $D_{s}^{-} \rightarrow \phi \pi^{-}$ |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$ | 6830 (4.0\%) | $D_{s}^{-} \rightarrow K^{* 0} K^{-}$ |
| $\begin{aligned} & D_{s}^{-} \text {vertex fit } \chi^{2}<12.0 \\ & a_{1}^{+} \text {vertex fit } \chi^{2}<12.0 \\ & B_{s}^{0} \text { proper decay time }>0.4 \mathrm{ps} \\ & B_{s}^{0} \text { impact parameter }<55 \mu \mathrm{~m} \\ & B_{s}^{0} p_{r}>10.0 \mathrm{GeV} / \mathrm{c} \\ & \hline \end{aligned}$ |  |  |
| N( $\left.K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$after cuts | 4100 (3.0\%) | $\bar{D}_{s}^{-} \rightarrow \phi \pi^{-}$ |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$after cuts | 4780 (2.8\%) | $D_{s}^{-} \rightarrow K^{00} K^{-}$ |
| Lepton identification Hadron identification Trigger efficiency Mass cut $\pm 2 \sigma$ | $\begin{gathered} 0.8 \\ (0.95)^{6} \\ 0.5 \\ 0.95 \\ \hline \end{gathered}$ |  |
| $N\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-} \pi^{+}\right)$reconstructed | 1240 (0.9\%) | $D_{s}^{-} \rightarrow \phi \pi^{-}$ |
| N $\left(K^{+} K^{-} \pi^{-} \pi^{+} \pi^{-\pi^{+}}\right)$reconstructed | 1330 (0.8\%) | $D_{s}^{-} \rightarrow K^{* 0} K^{-}$ |

As we see, about 2570 reconstructed $B_{s}^{0}$ mesons are expected for $10^{4} p b^{-1}$ integrated luminosity from $B_{s}^{0} \rightarrow D_{s}^{-} a_{1}^{+}$channel when both of $D_{s}^{-} \rightarrow \phi \pi^{-}$and $D_{s}^{-} \rightarrow K^{00} K^{-}$ modes are used for $D_{s}^{-}$reconstruction. To them we should add $3640 B_{s}^{0}$ mesons from $B_{s}^{0} \rightarrow D_{s}^{-} \pi^{+}$channel $[8]$. So in total we expect 6210 reconstructed $B_{s}^{0}$ mesons per $10^{4} p b^{-1}$ integrated luminosity when all as yet considered decay modes are used.

## 3 Peak amplifier

To extract the oscillation frequency, from M.C. or experimental asymmetry distributions the so called amplitude fit method is useful [11]. Here we describe some refincments of this method.

In the amplitude fit method an asymmetry distribution $A(t)$ is fitted with the casine function $A_{f i t} \cos \left(x_{s} t / \tau\right)$ in which $x_{s}$ is fixed and $A_{f i t}$ is the only free parameter. Reperating the fit for different values of $x_{s}$, we get $A_{f i t}\left(x_{s}\right)$ distribution. This distribution is peaked at $x_{s}$ which corresponds to the true value of the oscillation frequency.

The peak position in the $A_{f i t}\left(x_{s}\right)$ distribution can be found with the help of the recently suggested "quantum" peak finding algorithm [12]. The idea of this algorithm is based on the property of small quantum balls to penetrate narrow enough obstacles. So if such a ball is placed on the edge of some potential wall it will find its way down to the potential wall bottom even if the potential wall is distorted by statistical fluctuations.

Let us introduce instead of continuous $x_{s}$ some discrete parameter $i$, say through $N_{i}=A_{f i t}(i / 2)$. The transformation $A_{f i t}\left(x_{s}\right) \rightarrow u\left(x_{s}\right)$, which we call peak amplifier, is defined for selected discrete values of $x_{s}$ as follows

$$
\begin{equation*}
u(i / 2) \equiv u_{i}, u_{i+1}=\frac{P_{i, i+1}}{P_{i+1, i}} u_{i}, \sum u_{i}=1 . \tag{2}
\end{equation*}
$$

And $P_{i, i \pm 1}$ transition probabilities are determined by the initial $N_{i}$ spectrum [12]

$$
\begin{equation*}
P_{i, i \pm 1}=A_{i} \sum_{k=1}^{2} \exp \left[\frac{N_{i \pm k}-N_{i}}{\sqrt{\sigma_{i \pm k}^{2}+\sigma_{i}^{2}}}\right], \tag{3}
\end{equation*}
$$

$A_{i}$ normalization constant being defined from the $P_{i, i-1}+P_{i, i+1}=1$ condition. $\sigma_{i}$ is a standard deviation (error) of $N_{i}$ as determined by the cosine fit.

If now we apply this peak amplifier to the data after the amplitude fit we get the probability distributions shown in Fig. 5 (for $x_{s}=30$ ) and in Fig. 6 (for $x_{s}=45$ ). As we see, the peak amplifier enables a clear determination of $x_{s}$ from the amplitude fit. spectrum.

## $4 \quad x_{s}$ sensitivity range

To estimate the ATLAS sensitivity range for the $x_{s}$ measurement, the analogous procedure was used as in [8].

Amplitude fit is applied to the asymmetry distribution generated by Monte-Carlo program. The input parameters of this program, such as signal to backgromed ratio,
$B_{s}^{0}$ lifetime, proper-time resolution and dilution factors are the same as in [8], with the exception of the number of signal events, which was taken to be 6210 .

The amplitude fit spectrum is further transformed using the peak amplifier transformation as described above. In the resulting $u\left(x_{s}\right)$ spectrum the mean value of $r_{s}$ and its standard deviation is calculated considering u( $\left(x_{s}\right)$ as a probability density. The "experiment" is considered as sucecssful if the measured $x_{s}$ value (mean value of $x_{s}$ according to the $u\left(x_{s}\right)$ distribution) is within two standadd deviations from the true $x_{s}$-value defined in the Nonte-Carlo program.

For cach $x_{s}$ point 1000 such "experiments" were generated and the fraction of the successful "experiments" was calculatex. The highest value of $x_{s}$. for which this fraction is above $95 \%$, is considered as a sensitivity limit for the ATLAS experiment. This limit was fomed to be about $x_{s}^{m a s}=42.5$. This is almost the same number as found in [8]. In fact the peak amplifier method doesmen give a significant inerease for the sonsitivity liunt hut it allows a more acemate six determination, as is indicated by Fig. 6. becanse the mobability peak is much more narrow.

Fig. 7 shows the distribution of the $x_{s}$ values, found by the peak amplifier method or 1000 "experiments", generated with the "true" $x_{s}=42.5$.

## 5 "Where is the beginning of the end that comes at the end of the beginning?"

So we are at the end of our investigation. Our main condusions are

- $D_{s}^{-} \rightarrow K^{\circ 0} K^{-}$mode mables a two fold increase in the signal statistices for the $B_{s}^{s} \rightarrow D_{s}^{-} a_{1}$ decay chamel.
- the ATLAS experiment can reach a sensitivity limit for $r_{s}$ as high as $r_{s}^{m a r}=42.5$ with the certainty
- $D_{s}^{-} \rightarrow K^{* 0} K^{-}$mode can be used also for the $B_{s} \rightarrow D_{s}^{-} \pi^{+}$chamel. If the same increase in signal statistics is assumed, the total mumber of reconstroeted $B_{s}$ cernts can reach $10^{4}$ por $10^{4} p b^{-1}$ integrated lmminosity. According to costimates from [ B ] this will mean is sensitivity limit for $x_{\mathrm{s}}$ about 40 .


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## Farewell

When we began this investigation our intention was to write a vivid and joyful story about. $B_{s}$-mixing, some mixture of science and art (the art part is greatly reduced in this preprint. version for technical reasons. For complete version see ATLAS Internal Note PHYS-NO115, http://atlasinfo.cern.ch/Atlas/GROUPS/PHYSICS/NOTES/notcs.htmi). Only scientific framework appeared to us as too narrow to embrace the beauty of life, because [13] "All things are full of labour: man cannot utter it: the eye is not satisfied with secing, nor the ear filled with hearing.".

Unfortunately Sasha Bamikov suddenly died at the end of last year and we are forced to end this project without him.

Farewell Sasha! Let this article be a small thing that remains after you in this world as your memory.


Figure 1: Invariant mass distributions of three charged particle combinations in signal events, assuming $2 K+\pi$ (a) or $3 \pi$ combination (b) .


Figure 2: Invariant mass distributions of reconstructed $K^{* 0}$ and $D_{s}^{-}$events.


Figure 3: Proper time (a) and transverse radins (b) resolntions for the reconstroted $B_{\text {. }}$. decay vertex.


Figure 4: Six particle invariant mass distribution corresponding to the $B_{s}^{0}$ meson. Dashed line - expected upper limit for background from $B^{0}$ decay.


Figure 5: amplitude $A\left(x_{s}\right)$ and probability $u\left(x_{s}\right)$ for $x_{s}=30$.


Figure 6: amplitude $A\left(x_{s}\right)$ and probability $u\left(x_{s}\right)$ for $x_{s}=45$.


Figme 7: The distribution of the $x_{s}$ values, found by the peak amplificer method. for 1000 "experiments", generated with the $x_{s}=42.5$.

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