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HIGH ENERGY ELECTROMAGNETIC CASCADES
IN DENSE MEDIA:
PRESENT STATUS AND PROSPECTS

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Электромагнитные ливни высоких энергий в плотной среде:
статус и перспективы

Дан краткий обзор главным образом экспериментальных исследований основных, с практической точки зрения, характеристик электромагнитных ливней, создаваемых в плотной аморфной среде при энергии более 100 МэВ. Особо подчеркнут вопрос о поиске масштабного описания этих характеристик. Вкратце обсуждается также развитие электромагнитных ливней в кристаллах и поликристаллах.

Работа выполнена в Лаборатории высоких энергий ОИЯИ, Дубна, в Институте физики Варшавского технического университета и в Институте атомной энергии, Свєрк-Отвоцк, Польша.

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High Energy Electromagnetic Cascades in Dense Media:
Present Status and Prospects

A general overview of the present experimental status is given about the main, from the practical point of view, characteristics of electromagnetic cascades created in dense amorphous and homogeneous media at energy above 100 MeV. The problem of searching for scaling description of these characteristics is emphasized. The development of electromagnetic cascades in aligned crystals and polycrystal media is briefly discussed, too.

The investigation has been performed at the Laboratory of High Energies, JINR, Dębna, as well as at the Institute of Physics, Warsaw University of Technology and at the Institute of Atomic Energy, Swierk-Otwock, Poland.

I. INTRODUCTION

As is well known an electromagnetic cascade (briefly: em. cascade or shower) arises when a high enough energy (i.e. $E > 2m_e c^2$, where $m_e c^2$ is the electron rest energy) photon or electron (or positron) enters a sufficiently heavy medium (e.g. with the atomic number $Z \geq 20-30$). Then the superposition of sequences of elementary acts of pair production and bremsstrahlung (for photons) or inversely (for electrons and positrons) occur as long as the energy of secondary (i.e. cascade's) photons exceeds $2m_e c^2$. Ionization and Čerenkov radiation, which also participate in the phenomenon, are here the main stopping factors whereas Compton scattering makes that the process is a three-dimensional one. Such is, in outline, a typical picture of the em. cascade developing within an amorphous material in the energy range of initiating particles up to ~ 100 GeV [1].

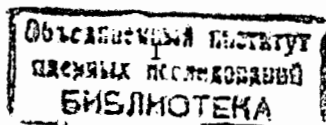
Although the process of em. cascade is known for several tens of years [1] and - being of purely electromagnetic nature (up to ~ 1 TeV) - is the simplest one as compared to other high energy cascade processes (intra- and internuclear ones), till now there is no a sufficiently reliable and quantitative approach to the problem, except a numerical modeling (using, for example, the EGS4 Code System [2]), which would make it possible to obtain information in a general form (i.e. as simple formulas) describing basic characteristics of the phenomenon, such as longitudinal and lateral profiles of showers in different materials, as well as their fluctuations and correlations. Moreover, the relevant experimental data existing in the literature are, as a rule, very scarce and fragmentary. Therefore the computer modeling of em. cascades is presently commonly considered the most comprehensive source of information about the process. But this way it is available to get a particular numerical result only, devoid of desirable and even necessary predictability. Another approach to the problem of em. cascades consists in phenomenological modeling of the phenomenon, based both on experimental data and computer simulation results, which makes it possible to obtain formulas describing basic features of cascades [1,3].

The work overviews the experimental results of investigation of em. cascades produced in dense materials by photons and electrons in a wide energy region, from ~ 100 MeV up to hundreds of GeV. The recent results and suggestions [3-5] which appeared after the review article [1] has been published are emphasized, in particular, the problem of a search for energy and material scaling property of shower profiles, as well as their dependence on cutoff energy of shower electrons. Finally, discussed is in brief the development of em. cascades in oriented crystals and polycrystals.

II. LONGITUDINAL SHOWER PROFILE

The longitudinal distribution of shower's electrons (and positrons) ionization loss, or longitudinal shower profile (LSP), which is from the practical point of view the most important characteristic of the process of em. cascade and at the same time the simplest one, was studied more frequently both experimentally and by means of computer modeling. As a result of this investigation it has been established that the LSP can be parametrized with sufficiently good accuracy by a gamma distribution function:

$$F_i(x_i | E, E_c) = a_i x_i^{a_i} \exp(-a_i x_i). \quad (1)$$



Here $x_i = t/\langle t(E) \rangle$ and t is a shower depth normalized to its average value $\langle t(E) \rangle$ at the incident particle energy E , E_c is a cut-off energy of shower's electrons and positrons, and a_i ($i=1, 2, 3$) are parameters which are determined from experimental data or results of computer modeling. Fig.1 shows a typical longitudinal profile of showers produced in liquid xenon by gamma quanta with an energy of $E_\gamma = 3375 \pm 125$ MeV when E_c is between ~ 1 MeV and ~ 1.5 MeV [3]. The experimental data are compared to the relevant results of simulation

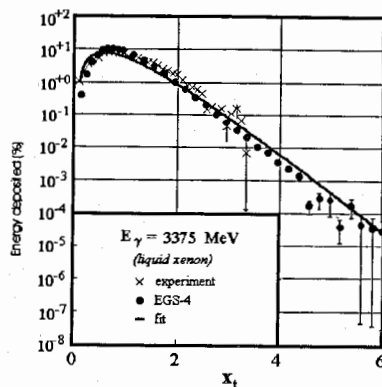


Fig.1. Longitudinal profile of showers produced in liquid xenon by gamma quanta with an energy of $E_\gamma = 3375 \pm 125$ MeV [1]. Experimental data are compared to the results of simulation with the EGS4 code [2]. Solid line represents the approximating function (1).

with the EGS4 code [2]. In the figure is also displayed the function (1) with parameters: $a_1 = 83.1 \pm 3.4$, $a_2 = 1.65 \pm 0.03$ and $a_3 = 2.62 \pm 0.03$ at $\chi^2_{35} = 35.8$ when $x_i \geq 0.15$ [3]. The energy dependence of $\langle t(E_\gamma) \rangle$ is fitted to a linear function of $\ln E_\gamma$:

$$\langle t(E_\gamma) \rangle = a_1 + b_1 \ln E_\gamma \quad (2)$$

where $a_1 = -4.84 \pm 0.09$ radiation length (r.l.) and $b_1 = 1.32 \pm 0.03$ (E_γ is in MeV). It should be noted that the linear relation between the average shower depth and the logarithm of primary particle energy (i.e. $\ln E_\gamma$ for photons) also follows from approximation B of cascade theory and from the results of modeling of showers created both by photons and electrons (see [1]).

The average shower depth contains also information about the properties of the medium in which the cascade develops and about the cut-off energy of shower electrons and positrons. This information is included mainly in the factor b_1 (2). The dependence of b_1 on the relative electron density $\rho Z/A$ for heavy shower absorbers: liquid xenon, lead and tungsten is shown in Fig.2. Fig.3 illustrates the LSP sensitivity to a change of the cut-off energy E_c in the case of showers produced in lead by electrons of energy $E = 6$ GeV [7,8].

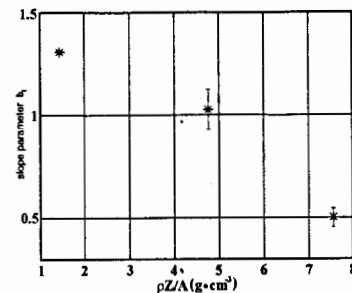


Fig.2. Dependence of the parameter b_1 of the formula (2) on the relative electron density $\rho Z/A$ [4].

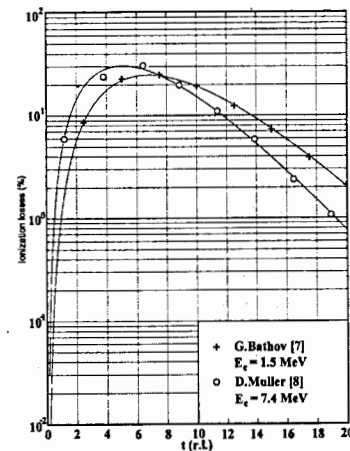


Fig.3. Longitudinal distribution of average ionization loss in em. cascades created by electrons with an energy of 6 GeV in lead at two different values of cut-off energy of cascade electrons [5].

II. TRANSVERSE SHOWER PROFILE

The transverse (or lateral) shower profile (TSP) usually means the radial, i.e. around the shower initiating particle momentum, distribution of average energy deposition at a given depth in the shower. Because of considerably increased complexity of experimental study of the TSP [1,9] the relevant information is much scantier than about the LSP. Existing data concerning the TSP, both experimental and numerical, are, unlike the case of LSP, parametrized by different analytical functions and there is no at present any commonly accepted approach to the problem. A typical example of the transverse profile of

cascades initiated in a „spaghetti”-type calorimeter by 150 GeV electrons is shown in Fig.4 [10].

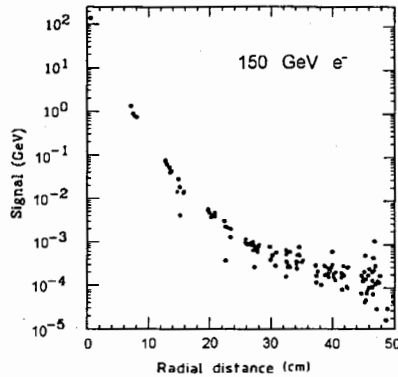


Fig.4. Transverse shower profile for 150 GeV electrons in lead [10].

This profile is summed over the shower depth. A simple examination of such distributions may suggest the two-region model of the spatial cascade's structure in which the so-called near-axis central region contains the hard component of shower particles having relatively small angular spread whereas in the peripheral cascade region where dominate mainly low-energy particles undergoing significant multiple Coulomb scattering. Consequently, some authors describe their experimental data pertaining to the TSP by a two-component function: double exponential (see [1]) or even double Gaussian [11], although one-exponential TSP parametrizations are found, too (see [1]).

Our approach, based on experimental data concerning the em. cascades created in liquid xenon by photons of energy E_γ between 100 and 3500 MeV, comes to the TSP description by means of some universal function $F(x_p)$ depending on the dimensionless radius $x_p = r / \langle p(t, E_\gamma) \rangle$

$$F(x_p) = \int_0^1 (1/s - 1/x_p) / [s^2 \sqrt{1-s^2}] \exp(-x_p/s) ds \quad (3)$$

where the scaling parameter $\langle p(t, E_\gamma) \rangle$ which, in turn, depends on the shower depth t and primary photon energy E_γ has a form $\langle p(t, E_\gamma) \rangle = \alpha + \beta(E_\gamma)t$ with $\alpha = (0.042 \pm 0.015)$ r.l., $\beta = a - b \ln E_\gamma$, $a = (75 \pm 3)10^3$ r.l., $b = (66 \pm 4)10^4$ r.l., E_γ is in MeV. Then for the TSP we have [1,3]:

$$F_r(r|t, E_\gamma) = [\pi \langle p(t, E_\gamma) \rangle^2]^{-1} F(x_p). \quad (4)$$

IV. FLUCTUATIONS

Fluctuations of the number of shower particles and their energy loss make that in each individual case of the em. cascade its longitudinal and transverse development are not strictly reflected by such average shower characteristics as the LSP and TSP at the corresponding primary energy. Instead we deal with some their concrete realizations in the form of random functions of shower depth t and radial spread r . This causes, in turn, that from the experimental point of view we loose the possibility to predict precisely, for example, what fraction A of total shower energy has been released up to a given value of t and r and, consequently, we are not able to determine this energy strictly, especially when the active volume of the detector is limited. As an illustration Fig.5 shows the normalized to unity cumulative longitudinal distribution A of ionization loss in em. cascades produced by photons of energy $E_\gamma = 3500 \pm 125$ MeV in liquid xenon. Except for the average LSP represented by the bold smooth solid line, broken thin lines in the picture display longitudinal random cumulative distributions for eight from among all 15 analyzed single events of em. cascades [12].

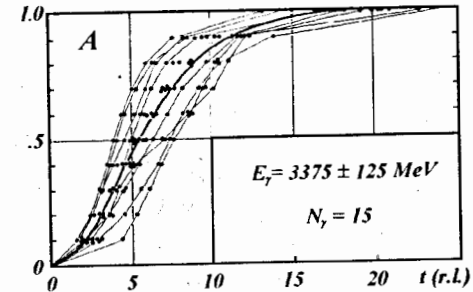


Fig.5. Cumulative longitudinal distributions A of ionization loss in em.cascades produced by photons of energy $E_\gamma = 3375 \pm 125$ MeV in liquid xenon. Solid bold line represents the average LSP. Broken thin lines display LSPs for 8 among all 15 analyzed single events of showers [12].

It is clearly seen that owing to fluctuations an individual LSP may significantly differ from the average one which is usually ascribed to every individual realization of the shower event registered partially only within limited active volume of a detector and this difference stochastically increases with decreasing the primary photon energy E_γ [1]. Therefore, a study of fluctuations in em. cascades is of utmost importance, at least from the practical point of view. Below we summarize briefly existing at present information about the fluctuations of longitudinal and transverse development of em. cascades, as well as the fluctuations of shower axis in an absorbent.

IV.1. Longitudinal fluctuations

The longitudinal fluctuations in em.cascades are the main source of errors when the energy E_γ of cascade initiating gamma quantum is determined in a detector of limited dimensions. It was found that in liquid xenon these fluctuations expressed as the rms. deviation $\sigma_r(\langle A \rangle)$ from the average fraction $\langle A \rangle$ of this energy is independent of E_γ for $E_\gamma \geq 500$ MeV and may be parametrized by the following function of $\langle A \rangle$:

$$\sigma_i(\langle A \rangle) = \langle A \rangle \{ [\alpha_i^2 + \beta_i (\gamma_i - \langle A \rangle)]^{1/2} - \alpha_i \}. \quad (5)$$

Here $\langle A \rangle$ is a function of x_i , $\langle A(x_i) \rangle = \int_0^{x_i} F_i(x_i | E, E_c) dx_i$, and $\alpha_i = (38 \pm 1)10^{-3}$, $\beta_i = (166 \pm 5)10^{-3}$, $\gamma_i = 1.01 \pm 0.01$ (see [1]). Moreover, in each individual case of a cascade the fraction A as a random number approximately obeys a normal distribution around the average $\langle A \rangle$ up to $\langle A \rangle \leq 0.7$ and a double normal law at $\langle A \rangle \geq 0.7$ [1]. Similarly the shower depths t at which fractions $A=0.1, 0.2, \dots, 0.9$ of the total shower energy are released as ionization loss also follow a normal law. Nevertheless it should be stressed that the approximation of longitudinal fluctuations in the form of formula (5) assumes that the conversion point of the photon initiating a cascade is strictly fixed as in the case of track detectors. Otherwise these fluctuations follow a Polya distribution [13].

IV.2. Transverse fluctuations

Experimental information about transverse fluctuations of ionization loss of shower's electrons in em.cascades created by gamma quanta comes, up to now, only from the 25 r.l. long Xenon Bubble Chamber of ITEP (Moscow) [14]. They has been described by means of the coefficient of variation $\sigma_p(A_p)/A_p$ of the fraction A_p of energy deposition in the transverse (but not radial!) direction of the cascade as a function of A_p at different values of photons' energy E_γ . This coefficient normalized to its maximum value $(\sigma_p(A_p)/A_p)_{\max}$ do not displays a noticeable, i.e. within the experimental error, dependence on A_p up to $A_p \cong 0.7$ and next quickly decreases to zero in all investigated region of primary photons' energy $E_\gamma = 100-3500$ MeV as shown in Fig.6.

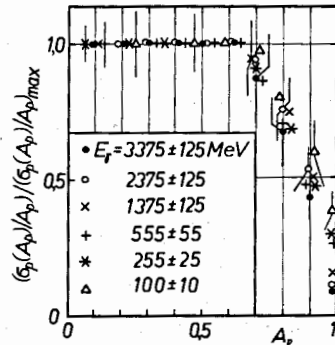


Fig.6. Relative transverse fluctuations in em. cascades produced in liquid xenon by gamma quanta of energy $E_\gamma = 100-3500$ MeV [1]. A_p is the average fraction of total ionization losses in em. cascades.

The dependence of $(\sigma_p(A_p)/A_p)_{\max}$ on E_γ was approximated by a linear function of $\ln E_\gamma$, $(\sigma_p(A_p)/A_p)_{\max} = \alpha_p - \beta_p \ln E_\gamma$, as shown in Fig.7. Here $\alpha_p = 0.48 \pm 0.2$ and $\beta_p = (48 \pm 3)10^{-3}$.

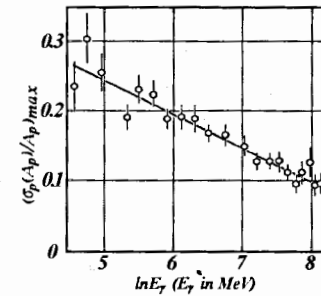


Fig.7. Dependence on energy of the maximum value of the relative rms. of the transverse spread of showers created by photons in liquid xenon [1]. Solid line displays an approximating function.

IV. Fluctuations of the shower axis

Usually the direction of flight of a primary photon is reconstructed as the so-called energy axis of the shower created by this photon (see, for example, [9]). Nevertheless in consequence of transverse fluctuations of energy deposition in showers this axis in each individual case of a cascade is no longer a straight line passing through the point of generation of the photon and the point of its conversion but it winds around this geometrical shower axis. This winding has been estimated as the average distance \bar{d} between geometrical and energy shower axes, and by means of the corresponding rms. deviation S_d . These two quantities are displayed in Fig.8 as a function of incident energy of photons in the range of $E_\gamma = 100-3500$ MeV for liquid xenon [1].

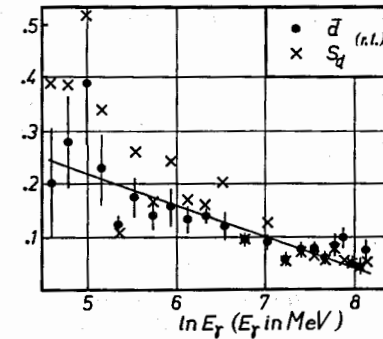


Fig.8. Energy dependence of the average distance \bar{d} between geometrical and energy shower axes, and the relevant rms. deviation S_d . Experimental data concern the em. cascades produced by gamma quanta of energy E_γ in liquid xenon [1]. Straight line is a fit for \bar{d} .

Another useful quantity describing the random walking of the energy shower axis is the maximum shower depth t_s , up to which the energy axis coincides, within the limits of experimental error, with the geometrical one. The energy dependence of average \bar{t}_s for photons of energy E_γ creating em. cascades in liquid xenon is presented in Fig.9. In this figure plotted is also the relevant rms. deviation and the approximating function for \bar{t}_s .

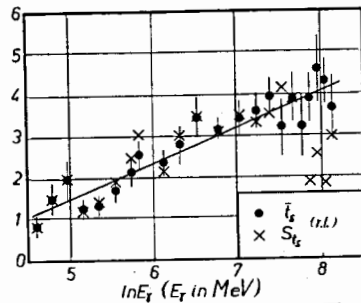


Fig.9. Energy dependence of the average maximum shower depth \bar{t}_s , within which the energy axis of showers coincides with the geometrical one [1]. Straight line displays an approximating function. Experimental data pertain to em. cascades created by gamma quanta in liquid xenon.

S_{t_s} is the relevant rms. deviation from the average value \bar{t}_s .

It is clearly seen from these figures that the higher is the energy E_γ of primary photons the closer is, on the average, the energy shower axis to the geometrical one and the longer is the average shower depth, up to which the energy axis coincides, within the limits of experimental error, with its geometrical equivalent, and, finally, the corresponding quantities depends on E_γ as $\ln E_\gamma$, at least within the range of $E_\gamma = 100-3500$ MeV.

V. CORRELATIONS

The problem of correlations in em. cascades may be formulated in different way depending on specific needs we have in mind. First of all of particular interest are such correlations between measured quantities of a shower which could be applied for registration of high energy gamma quanta in detectors of limited dimensions and/or for increasing the accuracy of the determination of energy of shower producing gamma quanta. From the practical standpoint it is important to estimate the total shower energy if, say, within a segment of thickness Δt_i at the shower depth t_i a fraction ΔE_i of ionization losses (or Čerenkov radiation) has been registered. In other words, if we have a shower detector longitudinally segmented with a step of Δt_i at the depth t_i and in certain individual event of a shower the fraction ΔE_i has been measured in a layer of thickness Δt_i , then what we can tell about its total energy and what is a confidence level of such prediction? Fig.5 may serve as an illustration of the problem under discussion. To answer this question experimental data concerning em. cascades produced in liquid xenon by photons in the energy range of $E_\gamma = 100 - 3500$ MeV have been analyzed using the standard definition of the correlation

coefficient $r_{ij} = \text{cov}(X_i, X_j) / \sigma(X_i)\sigma(X_j)$ where $X_i = \Delta A / \Delta t_i$ and $\Delta t_i = t_{i+1} - t_i$ is the thickness of a layer of liquid xenon at the shower depth t_i , in which a fraction of $\Delta A = 0.1$ of total shower energy has been released. The conclusion of this analysis is that (1) short-range correlation (i.e. where $i = j+1$) in the region of shower maximum is appreciably significant only, and (2) the correlation coefficient as a random number follows approximately a normal distribution.

Other correlations being of practical interest are those between the length t and width p in the plane of shower axis. They were studied using the experimental data for gamma quanta producing showers in liquid xenon [1] with the condition that strictly a fraction $A = 0.9; 0.8; 0.7$ and 0.6 of the total shower energy in the range $E_\gamma = 100-3500$ MeV was absorbed in a box of length t and width $2p$ when the shower axis is in the middle of this box. As a result of this investigation one can conclude that owing to the significant fluctuations of the spatial dimensions of showers it is necessary to know as well as the spatial structure of this energy deposition to estimate reliably enough the primary photon energy [1].

VI. GENERAL FEATURES OF SHOWERS

From the practical point of view it is often useful to have qualitative information about the general outline of showers, such as their relative dimensions of the longitudinal to transverse development, as well as about their relative scatter in these directions. Fig.10 presents the coefficients of variation of the longitudinal S_t / \bar{t} and transverse S_p / \bar{p} dimensions of showers initiated by gamma quanta in liquid xenon within the energy range of $E_\gamma = 100-3500$ MeV as a function of the energy E_γ [1].

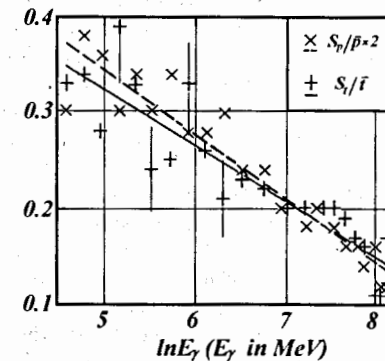


Fig.10. Coefficients of variation of the longitudinal S_t / \bar{t} and transverse S_p / \bar{p} dimensions of em. cascades produced by gamma quanta of energy E_γ in liquid xenon [1]. Straight lines represent fits to experimental data.

These coefficients are virtually the estimates of relative diffuseness of cascades in both directions. In this figure are also plotted the corresponding approximation functions. One

can notice that with increasing energy E_γ em. cascades become more and more distinct. Moreover, from the energy dependence of the ratio \bar{p}/\bar{l} of the average transverse shower dimension \bar{p} to the longitudinal shower dimension \bar{l} of these showers and the ratio S_p/S_l of corresponding rms. scatters, which are displayed in Fig.11, one can see that these showers are relatively more and more extended in the forward direction and their transverse boundaries are sharper than the longitudinal ones by a factor ~ 10 and more beginning from $E_\gamma \approx 1000$ MeV.

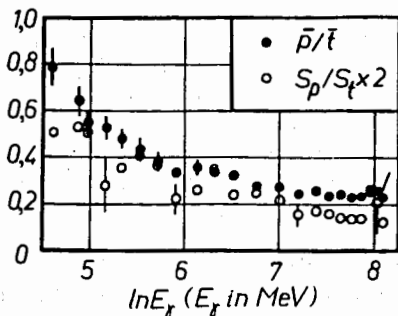


Fig.11: Ratio \bar{p}/\bar{l} of the average transverse (\bar{p}) and longitudinal (\bar{l}) dimensions of showers created in liquid xenon by gamma quanta of energy E_γ . S_p and S_l are corresponding rms. scatters [1].

VII. SHOWERS IN ORIENTED CRYSTALS AND POLYCRYSTALS

Owing to coherence and interference effects the probability of bremsstrahlung and pair production in crystals may significantly exceed the similar probability for amorphous media when a high energy particle moves in a crystal at low angle to its crystalline axes. As a consequence the shower created by this particle in the crystal develops within a length being by one or even two orders of magnitude smaller than one radiation length and, moreover, specific showers may also be produced consisting of large number of photons. Such effects occur if coherence lengths for bremsstrahlung and pair production become comparable or exceed the lattice constant of the crystal (see, for example, [15]). Experimental results concerning electromagnetic showers in aligned crystals are very scarce but seem to confirm theoretical predictions. As an illustration Fig.12 shows a difference between the longitudinal profiles of em. cascades produced by the 28 GeV electrons in oriented by the $\langle 111 \rangle$ axis a 1 mm thick tungsten crystal and in the same but disordered one [16]. It can be noticed that the maximum of the cascade profile in the oriented tungsten crystal is shifted by about 2 r.l. towards the cascade beginning with respect of the case of this sample but irradiated at an angle of $\theta = 22$ mrad to the $\langle 111 \rangle$ axis which is equivalent to amorphous medium [16].

The practical aim of the corresponding investigation is to use of crystals in gamma-astronomy and for calorimetry of gamma quanta and electrons (and positrons) at an energy of hundreds GeV [16]. In particular it has been found that the use of oriented crystals reduces the length of the calorimeter approximately by 2 r.l. at ~ 30 GeV and a unit of r.l. of

the tungsten crystal may be reduced from 3.5 to 0.86 mm [16]. In the precisely oriented lead crystal ($\sim 10^{-4}$ rad.) the length within which a shower of 1 TeV develops is of the order of magnitude of $\sim 10 \mu\text{m}$ [15]. But in practice it is difficult to orient a crystal with such a precision to every incoming particle. Nevertheless, to minimize the (longitudinal) dimension of em. showers one can use as well a polycrystalline absorbent on condition that the thickness of its granules is not less than this dimension. It was estimated that the probability for a particle to have sufficiently small angle to any of crystallites within the path of 1 r.l. is $\sim 10^{-2}$ [15].

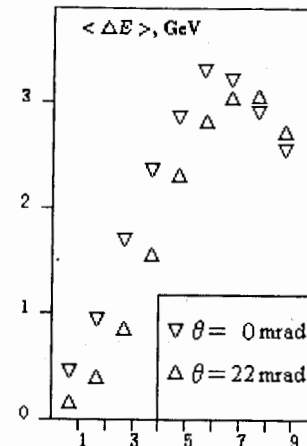


Fig.12. Longitudinal profile of em. cascades produced by 28 GeV electrons in a 1 mm thick tungsten crystal: oriented by the $\langle 111 \rangle$ axis to the electron beam ($\theta = 0$ mrad) and disordered ($0 - 22$ mrad). The cascade depth is expressed in r.l.

VIII. CONCLUSION

The phenomenon of em. cascades has been studied for several tens of years and all elementary processes making up this phenomenon, as being of electromagnetic nature, are known at the high level of exactness. Nevertheless, by now we have only as commonly accepted analytic description of average longitudinal profile of cascades developing in heavy amorphous and homogenous media. There is no such an agreement on the approximation of transverse spread of em. cascades and different ad hoc approaches to this important shower characteristic are used. Most of all unsatisfactory is our knowledge about as well fundamental feature of cascades as fluctuations and correlations, and the main relevant experimental information comes from xenon bubble chambers only.

Relatively not long ago the investigation has started, both experimental and theoretical, of the process of em. cascades in oriented crystals and polycrystal media providing interesting and useful information first of all about the longitudinal shower dimensions.

Finally, one can stress that besides practical aspects of the problem under consideration we deal with an interesting cognitive question, namely, whether and to what extent it is possible to describe basic features of em. cascades in the scaling form not only in

respect to energy of shower initiating particles but also to cut-off energy of shower particles and properties of the absorbent in which these cascades develop. This problem is interesting as well in more general aspect and may concern other multiparticle processes at high enough energies as, in particular, the process of electronuclear breeding etc.

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