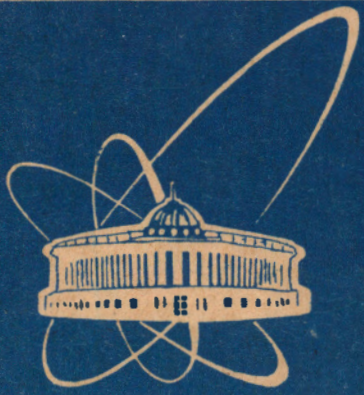

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EXTRACTION OF ASYMMETRIES AND SPIN-DEPENDENT STRUCTURE FUNCTIONS FROM POLARIZED LEPTON-NUCLEUS CROSS-SECTIONS

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1 Introduction

Deep inelastic scattering of polarized leptons on polarized nucleons has proved to be a powerful tool for studying the internal spin structure of nucleons. This structure is characterized by two spin dependent structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ entering the cross sections calculated in one-photon exchange approximation (for review of the phenomenology see [1]). This approximation assumes that the deep inelastic differential cross section of polarized leptons on polarized nucleons, σ^S, can be written as a sum of unpolarized, σ_u, and polarized, σ_p, terms:

$$\sigma^S \equiv \frac{d^2\sigma}{dxdQ^2} = \frac{d^2\sigma_u}{dxdQ^2} + \frac{d^2\sigma_p}{dxdQ^2} S_{eN}, \qquad (1)$$

where the second term depends on relative orientations of the lepton and nucleon spins, S_{eN}. The polarized part of the cross section can be separated from the unpolarized one taking the difference of cross sections with opposite orientations of spins:

$$\frac{d^2\sigma_p}{dxdQ^2} = \frac{1}{2}(\sigma^{S\uparrow} - \sigma^{S\downarrow}), \qquad (2)$$

because the unpolarized part of the cross section is assumed to be the same in $\sigma^{S\uparrow}$ and $\sigma^{S\downarrow}$ and $S_{eN} = +1(-1)$ for the same (opposite) orientation of the lepton and nucleon spins. Expression (2) is valid for measurement from pure (mononuclear) polarised targets like hydrogen, deuterium or Helium-3 . It is also valid in case of multinuclear targets containing a small fraction of polarised hydrogen (deuterium), if other nuclei are unpolarised .

In practice there are two cases of the relative spin orientations. In the first case spins of nucleons and leptons are parallel ($\Longrightarrow\rightarrow$) or antiparallel ($\Longleftarrow\rightarrow$) to each other and aligned along the beam direction, i.e. leptons and nucleons are polarised longitudinally. In the second case the leptons are polarised longitudinally ($\rightarrow$) but nucleons are polarised transversely ($\Downarrow$ and $\Uparrow$).

From the text books(see [1]) one can find that

$$\Delta\sigma^{\rightarrow} \equiv \frac{d^2\sigma^{\Longrightarrow\rightarrow}}{dxdQ^2} - \frac{d^2\sigma^{\Longleftarrow\rightarrow}}{dxdQ^2} = -\frac{4\pi\alpha^2\nu E'^2}{Q^2xE^2}[(E + E'cos\theta)MG_1 - Q^2G_2] \qquad (3)$$

and

$$\Delta\sigma^{\uparrow} \equiv \frac{d^2\sigma^{\Uparrow\rightarrow}}{dxdQ^2} - \frac{d^2\sigma^{\Downarrow\rightarrow}}{dxdQ^2} = -\frac{4\pi\alpha^2\nu E'}{Q^2xE^2} sin\theta(MG_1 - 2EG_2), \qquad (4)$$

where $\Delta\sigma^{\uparrow(\rightarrow)}$ is the cross section difference for longitudinally polarized leptons and longitudinally (transverselly) polarized nucleons. These expressions are written for the practical cases when the polarization and scattering planes are perpendicular

to each other. As one can see from (3) and (4), the measurements of $\Delta\sigma^{\uparrow}$ and $\Delta\sigma^{\rightarrow}$ provide the data to separate G_1 and G_2 [1]. Due to different contributions of G_1 and G_2 to $\Delta\sigma's$ at different scattering angles θ, it is possible to separate G_1 and G_2 from measurements of only $\Delta\sigma^{\uparrow}(\theta)$ or $\Delta\sigma^{\rightarrow}(\theta)$ using a likelihood procedure. The determination of G_1 and G_2 from (3) and/or (4) requires the absolute measurements of each cross section. Experimentally the cross sections (3) and (4) are determined from the normalized counting rates corrected for losses, acceptance and high order radiative processes. If the counting rates are high enough, i.e. statistical errors are small, the accuracy of the cross section measurements will be limited only by systematic uncertainties of normalizations and corrections.

To avoid the problems of normalizations and, to some extend, acceptance corrections, the earlier (E80/130, EMC, SMC, E143, E142) experiments [2] - [5] used another method: instead of the absolute measurements of cross section differences, (3) and (4), they performed relative measurements of the so called cross section asymmetries $A_{\parallel}$ or $A_{\perp}$

$$A_{\perp} = \left(\frac{d^2\sigma^{\rightarrow\Uparrow}}{dxdQ^2} - \frac{d^2\sigma^{\rightarrow\Downarrow}}{dxdQ^2}\right)/\left(\frac{d^2\sigma^{\rightarrow\Uparrow}}{dxdQ^2} + \frac{d^2\sigma^{\rightarrow\Downarrow}}{dxdQ^2}\right), \tag{5}$$

$$A_{\parallel} = \left(\frac{d^2\sigma^{\Rightarrow\rightarrow}}{dxdQ^2} - \frac{d^2\sigma^{\Leftarrow\rightarrow}}{dxdQ^2}\right)/\left(\frac{d^2\sigma^{\Rightarrow\rightarrow}}{dxdQ^2} + \frac{d^2\sigma^{\Leftarrow\rightarrow}}{dxdQ^2}\right). \tag{6}$$

From [1] it is seen that

$$A_{\parallel} = D(A_1 + \eta A_2), \tag{7}$$

where D and η are kinematic factors and A_1 and A_2 are virtual photon-nucleon asymmetries. In the first approximation, $A_{\parallel}$ can be related to the spin dependent structure function g_1 as follows:

$$g_1(x,Q^2) = \frac{A_{\parallel}}{D} \cdot \frac{F_2(x,Q^2)}{2x[1+R(x,Q^2)]}\left(1 + \frac{4M^2x^2}{Q^2}\right) \tag{8}$$

where F_2 and R are unpolarized structure functions, and the neclected second term in (7) is expected to be small.

So, to extract the spin dependent structure functions from the asymmetry measurements, one needs to know the unpolarized structure functions from other experiments. But in many cases these structure functions are either unknown in a definite kinematic domain or not precise enough for the purposes of the particular experiment. For example, the kinematical region of the SMC experiment [4] is extended to the low values of x up to $x \simeq 0.001$. There were no measurements of the structure function $R(x,Q^2)$ for x below ~ 0.01. So, to apply (8), the SMC

[1] relationship between g_1, g_2 and G_1, G_2 see in [1]

has to extrapolate the existing data on R to the unknown region. This can cause systematic errors [7] which are not easy to estimate.

The use of (3) and (4) does not require the unpolarized structure functions. So, measurements of g_1 and g_2 can be done within the same experiment without the data from other experiments. The other advantages of this approach will be discussed below.

The aim of this paper is to develop practical algorithms to extract the spin dependent structure function from the measured cross sections. The paper consists of 7 sections. In Sect.2 we describe an algorithm to separate σ_u and σ_p from(1). The experimental data are collected during certain periods of time. Sometimes it is necessary to analyze the data for each period separately and combine them at the end. This procedure is described in Sect.3. When σ_u and σ_p are separated, there are two ways to continue the calculations:

- either to go further and find g_1 and g_2 from (3) and/or from (4);
- or to define asymmetry as

$$A = \frac{\sigma_p}{\sigma_u} \tag{9}$$

and find g_1 and g_2 from equations similar to (8).

The first way will be the subject of a separate publication. The second way is described in Sect.4. To extract g_1 and g_2 for proton and neutron, the measurements are performed with Hydrogen, Deuterium and Helium targets. Nuclear effects should be taken into account in two last cases. The correction procedures for radiative processes and nuclear effects are given in Sect.5. Sect.6 contains tests of this approach using the Monte-Carlo and HERMES data for ^{3}He [9]. Conclusions are given in Sect.7.

2 Likelihood procedure to separate σ_u and σ_p

The cross sections (1) are small and the data to determine them are collected during long periods of time , usually within several years. In these periods the normalized counting rates could vary due to different reasons like variations of beam, P_b, and target, P_t, polarization, beam intensity, and/or target density(luminosity) and/or spectrometer efficiencies, etc. To minimize the systematic errors associated with these variations, the data are divided in samples with relatively stable experimental conditions. And again to minimize the systematics, the target polarization is frequently reversed. Inside each sample of the data we can determine the number of events $N_{ij}(x,Q^2)$ collected for a given pair (bin) of the beam (i) and target (j) polarizations:

$$N_{ij}(x,Q^2) = \sigma_u L_{ij} + \sigma_p L_{ij} P_i^b P_j^t, \qquad (10)$$

where N_{ij} is the number of events for ij-bin , L_{ij} is "luminosity" for a ij- bin, $L_{ij} = a_{ij}M_{ij}T_{ij}$, (where $a_{ij}(x,Q^2)$ is acceptance, M_{ij} is an incident flux and T_{ij} is the target density), P_i^b,(P_j^t) is the measured beam(target) polarization. For simplicity we have assumed that the event reconstruction efficiency , dead time and other corrections are known and properly accounted for. We have also assumed that the target is mononuclear. The multinuclear case will be discussed in Section 7. Our aim is to extract σ_u and σ_p from $N_{ij}(x,Q^2)$. Taking into account statistical errors of N_{ij} and polarizations, the solution of this problem is found in the framework of the maximum likelihood method. As one can see from (10), the normalized counting rates, $N_{ij}(x,Q^2)/L_{ij}$, as a function of the polarizations product $P_i^b P_j^t$ are represented by the straight line. The intersept of this line at $P_i^b P_j^t = 0$ gives σ_u and the slope gives σ_p. Since N_{ij} and $P_i^b P_j^t$ are measured with errors, each point on the $N_{ij}(P_i^b P_j^t)$ plot will be represented by a cross. The task of the likelihood procedure is to find the true product $<P_i^b P_j^t>$ and the parameters of the straight line. If fluctuations of the polarizations and $N_{ij}(x,Q^2)$ are assumed to be Gaussian, the logarithm of the likelihood functional,L, is as follows:

$$\begin{aligned} -lnL = \frac{1}{2}\sum_{i,j=1}^{m,n}(N_{ij} - \sigma_u L_{ij} - \sigma_p L_{ij} <P_i^b P_j^t>)^2/N_{ij} + \\ +(P_i^b P_j^t - <P_i^b P_j^t>)^2/\delta_{ij}^2 + \text{const}, \end{aligned} \qquad (11)$$

where m, n are numbers of P^b and P^t bins; δ_{ij} is the statistical error of $P_i^b P_j^t$ and $<P_i^b P_j^t>$ is a product of true polarizations. According to the definition of the mean value, $P_i^b P_j^t$ can be written as follows:

$$\begin{aligned} P_i^b P_j^t &= <P_i^b P_j^t> + \epsilon_{ij}, \\ \epsilon_{ij} &\sim N(0, \delta_{ij}^2), \end{aligned} \qquad (12)$$

where ϵ_{ij} is the statistical fluctuation of the polarizations product with the zero mean value and variance δ_{ij}^2.

The quantities $<P_i^b P_j^t>$, i =1, ..., m, j =1, ..., n, and σ_u, σ_p are parameters. Minimizing the functional (11) over $<P_i^b P_j^t>$, i =1, ..., m, j =1, ..., n and inserting values of $<P_i^b P_j^t>$, i =1, ..., m, j =1, ..., n corresponding to the minimum in (11), we obtain the functional

$$-lnL = \frac{1}{2}\sum_{i,j=1}^{m,n}(N_{ij} - \sigma_u L_{ij} - \sigma_p L_{ij} P_i^b P_j^t)^2/(N_{ij} + \sigma_p^2 L_{ij}^2 \delta_{ij}^2). \qquad (13)$$

Minimization of (13) gives σ_u and σ_p.

The complete matrix of statistical errors for σ_u and σ_p is calculated as the inverse matrix of the second derivative matrix of the functional (13) at the minimum.

This approach guarantees the minimal bias for σ_u and σ_p. Extracting σ_u and σ_p from $N_{ij}(x,Q^2)$ or $N_{ij}(x)$, one can have either the cross sections within a certain Q^2 bin or the cross section averaged over Q^2 of the corresponding x-bin, respectively.

3 Procedure to combine results from different samples of the data

As it has already been mentioned above, the collected data consist of several samples which could have different normalized counting rates even after appropriate corrections being done due to the known fluctuations of efficiencies, luminosity, etc. The weighted averaging is a usual procedure to combine results of different samples. But if the samples have uncontrolled fluctuations and/or drifts, the average value could have a systematic shift. To minimize this systematics, instead of averaging one can use the likelihood procedure based on certain physics assumptions.

3.1 Combined results for asymmetries

Let us assume that the collected data are divided in n samples. For each of them the equation (9) can be rewritten as follows:

$$\sigma_p^i = A\sigma_u^i, i = 1,2...n. \qquad (14)$$

where A-is the asymmetry of the physics origin and should be the same for all the samples. The cross sections σ_p^i and σ_u^i for each sample are determined by the likelihood procedure up to a certain normalization constant depending of the flux,aceeptance, etc. This constant is crossed out in Eq.(14).

Each pair of σ_u^i and σ_p^i is determined with errors δ_u^i and δ_p^i, respectively. The value of A (common for all samples) can be obtained within these errors using the likelihood procedure. The procedure logically consists of two steps: the first - to find the true value of the unpolarized cross section $<\sigma_u^i>$ and, the second - to determine the asymmetry A for all samples.

Assuming fluctuations of σ_u^i and σ_p^i to be Gaussian, the logarithm of the likelihood functional is as follows:

$$-lnL = \frac{1}{2}\sum_{i=1}^{n}(\sigma_p^i - A<\sigma_u^i>)^2/(\delta_p^i)^2 + (\sigma_u^i - <\sigma_u^i>)^2/(\delta_u^i)^2. \qquad (15)$$

According to the definition of the mean value, σ_u^i can be written as follows:

$$\sigma_u^i = <\sigma_u^i> + \epsilon_u^i, \qquad \epsilon_u^i \sim N(0,(\delta_u^i)^2). \tag{16}$$

Minimizing the functional (15) over $<\sigma_u^i>, i = 1,...,n$ and substituting values of $<\sigma_u^i>, i = 1,...,n$, corresponding to minimum in (15), we obtain the functional:

$$-lnL = \frac{1}{2}\sum_{i=1}^{n}(\sigma_p^i - A\sigma_u^i)^2/((\delta_p^i)^2 + A^2(\delta_u^i)^2). \tag{17}$$

Minimization of Eq.(17) yields the combined value for A. As the result of the likelihood procedure we obtain:

- the asymmetry A with minimal possible statistical error and bias;
- the χ^2-values;
- the values of residuals .

The analysis of χ^2 values gives the confidential level of compatibility of samples, therefore one can detect bad samples of experimental data.

3.2 Combined results for cross sections

The combined values for σ_p and σ_u can be obtained in two steps.
First, minimizing the functional

$$-lnL = \frac{1}{2}\sum_{i=1}^{n}(\sigma_p^i - A\sigma_u)^2/(\delta_p^i)^2 + (\sigma_u^i - \sigma_u)^2/(\delta_u^i)^2 \tag{18}$$

we obtain A and σ_u which are given by:

$$\sigma_u = (\sum_{i=1}^{n}\frac{\sigma_u^i}{(\delta_u^i)^2})/(\sum_{i=1}^{n}\frac{1}{(\delta_u^i)^2}) \tag{19}$$

and

$$A = (\sum_{i=1}^{n}\frac{\sigma_p^i}{(\delta_p^i)^2})/(\sum_{i=1}^{n}\frac{\sigma_u}{(\delta_p^i)^2}) \tag{20}$$

Second, knowing A and σ_u, one can find σ_p from (9). The complete matrix of statistical errors for σ_u and A are calculated as the inverse matrix of the second derivative matrix of the functional (18) at the minimum.

4 Calculation of the $g_1^n(x)$ and Γ_1^n

Knowing the asymmetry A for the particular target and spin orientations one can calculate the structure functions using the text book algorithms [1]. For the HERMES experiment a software package is written to convert $A_{\|}^{^3He}(x)$ into the $A_1^n(x)$, $g_1^n(x)$ and calculate the first moment of $g_1^n(x)$ to test the Ellis-Jaffe sum rule[10,11].

The measured Helium-3 spin asymmetry is calculated as:

$$A_{1\ meas}^{^3He}(x) = \frac{1}{D}A_{\|}^{^3He}(x), \tag{21}$$

where $A_{\|}^{^3He}(x)$ is calculated for ^{3}He-target as discribed above and D is a depolarization factor:

$$D = \frac{y(2-y)}{y^2 + 2(1-y)(1+R)}. \tag{22}$$

The factor D is calculated for a given (x, $<Q^2>$) kinematical point where $<Q^2>$ is the average Q^2 for given x-bin. The structure function R is taken from SLAC parameterization [12].

The POLRAD code [13] is used to calculate the radiative corrections. On the basis of the explicit formulae (Sect. 5) and fit (see Appendix), the RC corrected asymmetry $A_{1(n)}^{^3He}(x)$ is obtained.

Nuclear corrections are taken into account for the neutron asymmetry $A_1^n(x)$:

$$A_1^n(x) = \frac{A_{1(n)}^{^3He}(x) - (1 - f_d(x))P_pA_1^p(x)}{f_d(x)P_n}, \tag{23}$$

where dilution factor is given by the formula:

$$f_d(x) = \frac{1}{2F_2^p/F_2^n + 1}, \tag{24}$$

P_p=-0.028 and P_n=0.86 are effective nucleon polarizations in Helium-3 [14].

The result of ref.[15] is used to approximate proton spin asymmetry $A_1^p(x)$:

$$A_1^p(x) = x^{0.727}, \tag{25}$$

which has been obtained by fitting the SMC and SLAC data (χ^2/ndf=0.866). A more complicated parameterization from [16] gives the same results for $A_1^p(x)$.

The structure function $g_1^n(x)$ is obtained by

$$g_1^n(x,<Q^2>) = A_1^n(x)F_1^n(x,<Q^2>), \tag{26}$$

where $F_1^n(x, <Q^2>)$ is defined with the formula

$$2xF_1^n(x, <Q^2>) = \frac{F_2^n(x, <Q^2>)\left(1 + \frac{4M^2x^2}{<Q^2>}\right)}{1 + R(x, <Q^2>)}. \quad (27)$$

The first moment of g_1^x is obtained as a sum of three terms

$$\Gamma_1^n = \int_{0.02}^{0.8} dx g_1^n(x, Q_0^2) + \int_0^{0.02} dx g_1^n(x, Q_0^2) + \int_{0.8}^{1} dx g_1^n(x, Q_0^2), \quad (28)$$

where the second and third terms from the HERMES unmeasured regions are calculated using the extrapolation of the fit (A.1), see Appendix. Before the integral is calculated, the structure function $g_1^n(x, <Q^2>)$ (26) is reevaluated to the common value of $Q^2 = Q_0^2 = 3GeV^2$, assuming that $A_1^n(x, Q^2)$ is Q^2- independent and using parameterization [12] for F_2 and R. The account for the possible Q^2 dependence on A_1^n could be performed using several suggestions [18].

5 Radiative corrections

The interpretation of experiments on deep inelastic scattering requires to separate the Born cross section from background contributions known as radiative corrections originating from loop diagrams and processes with the emission of additional photons. Radiative events cannot be completely removed by experimental methods and that is why they have to be calculated theoretically and subtracted from the measured cross sections.

There are three basic channels for scattering (cross sections) electrons on nuclei with respect to regions of the four momentum and energy transfer: elastic, quasielastic and inelastic processes. At the Born level both ν and Q^2 (and a channel of scattering as a result) are fixed completely by the measured momentum of the scattered lepton. In case of elastic scattering ($\nu = Q^2/2M_A$, M_A is nuclear mass) the electrons are scattered on the nucleus leaving it in a ground state. Quasielastic scattering ($\nu \simeq Q^2/2M$, M is nucleon mass) corresponds roughly to direct collisions with individual nucleons inside the nuclei. The inelastic scattering occurs when the pion threshold is reached ($\nu \geq Q^2/(2M + m_\pi)$, m_π is pion mass). However at the level of radiative corrections the radiated photon removes the fixation by taking away an arbitrary amount of energy and all the channels contribute to the observed cross section. Such elastic, quasielastic and inelastic processes with radiation of a real photon are known as radiative tails from the elastic ,σ_{el}, and quasielastic σ_q peaks and from the continuous spectrum σ_{in}. The total radiative correction of the lowest order is obtained as a sum of these

contributions and the one from the effects of vacuum polarization and radiation of additional virtual photons (σ_v):

$$\sigma_{RC} = \sigma_{in} + \sigma_q + \sigma_{el} + \sigma_v. \quad (29)$$

The radiative correction $\Delta^{RC}A_1$ to the measured asymmetry (21)

$$A_1^{^3\mathrm{He}} = A_{1\,meas}^{^3\mathrm{He}} - \Delta^{RC}A_1 \quad (30)$$

can be written as follows

$$\Delta^{RC}A_1 = \frac{\sigma_0^u(\sigma_{in}^p(g_1) + \sigma_q^p + \sigma_{el}^p) - \sigma_0^p(g_1)(\sigma_{in}^u + \sigma_q^u + \sigma_{el}^u)}{\sigma_0^u((1+\delta_v)\sigma_0^u + \sigma_{in}^u + \sigma_q^u + \sigma_{el}^u)}, \quad (31)$$

where $\delta_v = \sigma_v^p/\sigma_0^p = \sigma_v^u/\sigma_0^u$. The Born, σ_0, and inelastic, σ_{in}, parts of cross sections depend on g_1, and in the last case the dependence is non-trivial. So, the equation (30) becomes dependent on $g_1(x)$ or a system of equations when the extraction of g_1 is performed for n kinematical points x_i $(i = 1, ..., n)$. Usually such a system of equations is solved with the iteration methods. The most evident iteration algorithm is as follows:

$$A_{1(n)}^{^3\mathrm{He}} = A_{1\,meas}^{^3\mathrm{He}} - \Delta_{(n-1)}^{RC}A_1, \quad n = 1, 2, \ldots, \quad (32)$$

where n is the iteration number, $A_{1\,meas}^{^3\mathrm{He}}$ is given by Eq.(21), $\Delta_{(n-1)}^{RC}$ is calculated theoretically (31) and $A_{1(0)}^{^3\mathrm{He}} \equiv A_{1\,meas}^{^3\mathrm{He}}$.

At the first step of iteration, $n = 1$, we fit the measured asymmetry $A_{1\,meas}^{^3\mathrm{He}}$ using parameterization (A.1) from Appendix. With the help of the found parameters we calculate $g_{1(0)}^n$, the correction Δ_0^{RC} and RC-corrected asymmetry $A_{1(1)}^{^3\mathrm{He}}$. At the next step of iterations, $n = 2$, $A_{1(1)}^{^3\mathrm{He}}$ is fitted with (A.1), new values are found for $g_{1(1)}^n$ and then for $\Delta_{(1)}^{RC}A_1$ and $A_{1(2)}^{^3\mathrm{He}}$. Iterations are stopped if $\mid \Delta_{(n)}^{RC}A_1 - \Delta_{(n-1)}^{RC}A_1 \mid < 0.0001$. In practice the procedure converges within 4-5 steps.

6 Tests of the method.

The method was tested using Monte-Carlo program and the 1995 HERMES data.

6.1 Monte-Carlo test

To test the proposed method, a special Monte-Carlo program has been developed. For each event at the positron beam energy 27.5 GeV the kinematical

variables (x, Q^2) are generated with $1/Q^4$ for Q^2 and flat for ν distributions within the following intervals: $x = 0.02 - 0.8$ and $Q^2 = 1. - 20.$ GeV^2. Additional cuts are applied: $\nu = 0. - 23.375$ GeV; $W^2 = 4. - 500.$ GeV ^{2}; $\theta = 0.02 - 0.3$ Rad; $y = 0. - 0.85$.

Each event is weighted with W:

$$W = 1. + P_b P_t A_{\|}(x), \tag{33}$$

$$A_{\|}(x) = D(A_1(x) f_d(x) P_n + (1. - f_d(x)) A_1^p(x) P_p) \tag{34}$$

where A_1^p is a proton spin asymmetry:

$$A_1^p(x) = 0.01902 + x^{(-0.01163)}(1 - e^{(-1.845x)}). \tag{35}$$

The structure functions R, F_2^n and F_2^p are taken from the SLAC and NMC parameterizations [12]. This program simulates the real HERMES data taking, in particular, the accelerator bursts, the values of the beam and target polarizations, dead time of the set-up and values of the luminosity monitor.

The software package of the asymmetry calculations, as discribed in Section 4, was applied to the sample of these events. Figure 1 presents the typical dependence of the normalized yield versus product of the beam and target polarizations obtained with the Monte-Carlo program. The fit by straight line of this dependence gives the values of σ_p and σ_u in units of the luminosity monitor. The calculated asymmetry and the input parameterization are presented in Figure 2. One can see that there is a good agreement between the reconstructed and input asymmetries (χ^2=3.83 for 10 points). The first moment of g_1 calculated from the reconstructed value of $g_1(x)$ coincides with the input value within less than 1 % . The ratio between $A_1^n(x)$ and input parameterization of asymmetry is shown in Figure 3. It also confirms the correctness of the procedure test. Thus, we can conclude that the method is valid and can be applied to real experimental data.

6.2 Tests using HERMES data

The method, as described in sections 3.2 and 4, has been applied to the 1995 HERMES data on Helium-3 target. The results are in good agreement with the data presented in [9].

7 Discussions

7.1 Multinuclear target

This method can be applied to multinuclear targets like SMC target [18]. It is seen from the following consideration. For the multinuclear targets the equation (10) can be written as follows:

$$N_{ij}(x, Q^2)/L_{ij} = \sum_k \sigma_u^k + \sum_k \sigma_p^k P_{ik}^b P_{jk}^t, \tag{36}$$

where index k means the certain type of the nucleus in the target. Since the value of the target polarization is high enough only for hydrogen(deuterium), but it is close to zero for other nuclei, the second term of Eq.(36) can be expressed as follows:

$$\sum_k \sigma_p^k P_{ik}^b P_{jk}^t \simeq \sigma_p^1 P_{i1}^b P_{j1}^t, \tag{37}$$

where indexes $j1, ..., jk$ and $i1, ..., ik$ mean the binning over target and beam polarizations, σ_p^1 is a polarized deep inelastic cross section of polarized leptons on hydrogen(deuterium) nuclei. The first term of Eq.(36) is expressed through dilution factor(F_d) for target [3] , which takes into account the fraction of hydrogen(deuterium) nuclei contained in the target :

$$\sigma_u^1 = F_d \sum_k \sigma_u^k, \tag{38}$$

where σ_u^1 is an unpolarized deep inelastic cross section of polarized leptons on hydrogen(deuterium) nuclei.

So, the equation (10) for polarized hydrogen(deuterium) nuclei contained in multinuclear target is written as follows:

$$N_{ij}(x, Q^2)/L_{ij} = \frac{\sigma_u^1}{F_d(x, Q^2)} + \sigma_p^1 P_i^b P_j^t, \tag{39}$$

which is similar to Eq.(10).

7.2 Unpolarised structure functions

So far we have discussed the procedure of determinaion of the spin dependent structure functions. But it is obvious that having σ_u either from the likelihood procedure or/and from $\sigma^{S1} + \sigma^{S\uparrow}$ and applying the unfolding procedure [19] to account for the acceptance and resolution , we can also determine the unpolarized structure function $F_2(x, Q^2)$. Comparison of this function with the existing precision measurements will provide additional tests for the method.

8 Conclusions

The method to separate the polarized and unpolarized parts of the measured cross sections for deep inelastic scattering of polarized leptons on polarized nucleons is proposed and tested. These parts of cross sections can be used for direct calculations of spin dependent structure functions. This method allows to extract x and Q^2 depenedences of g_1 directly, without using the F_2 and R structure functions from other experiments. To take into account the resolution and acceptance of the set-up, the method can be improved by the special unfolding procedure [19].

Appendix

The following function taken from Schaefer's parameterization [16] is used for fitting the neutron spin asymmetry measured by HERMES [9]:

$$A_1^n(x) = \frac{1}{a_0 + 3a_1}\left(a_0 f_d^0 + \frac{a_1}{9}(-16 f_u^1 + 8 f_u^0 - 2 f_d^1 + f_d^0)\right), \qquad \text{(A.1)}$$

where

$$a_0 = \frac{2x^{\alpha_u}(1-x)^{\beta_u}}{B(\alpha_u, \beta_u + 1)} - \frac{x^{\alpha_d}(1-x)^{\beta_d}}{2B(\alpha_d, \beta_d + 1)}, \qquad \text{(A.2)}$$
$$a_1 = \frac{3}{2}\frac{x^{\alpha_d}(1-x)^{\beta_d}}{B(\alpha_d, \beta_d + 1)}$$

and

$$f_u^0 = \frac{1}{1 + a_{u0}x^{\alpha_u}(1-x)^2}, \quad f_u^1 = \frac{1}{1 + a_{u0}a_{10}x^{\alpha_u}(1-x)^2}, \qquad \text{(A.3)}$$
$$f_d^0 = \frac{1}{1 + a_7 a_{u0}x^{\alpha_d}(1-x)^2}, \quad f_d^1 = \frac{1}{1 + a_7 a_{u0}a_{10}x^{\alpha_d}(1-x)^2}.$$

Parameters a_7, a_{u0} and a_{10} are considered as free ones and α_u, α_d, β_u and β_d are fixed:

$$\alpha_u = 0.588, \quad \alpha_d = 1.03, \quad \beta_u = 2.69, \quad \beta_d = 6.89. \qquad \text{(A.4)}$$

Fitted values of the parameters are as follows:

$$a_{10} = 0.88, \quad a_{u0} = 0.46, \quad a_7 = 11.64. \qquad \text{(A.5)}$$

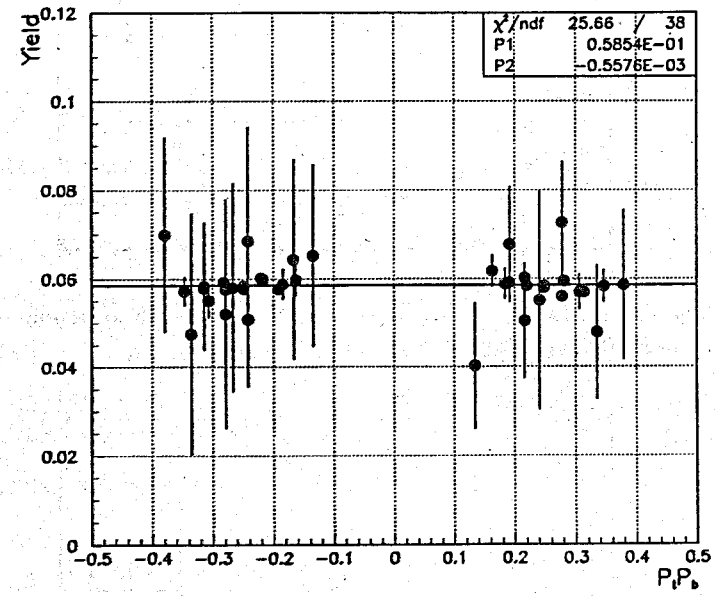

Рис. 1: The normalized yield versus a product of the beam and target polarizations at x=0.025 obtained with Monte-Carlo program discribed in section 6.1. The result of fit by a straight line is also shown (see Eq.(10), $P_1 = \sigma_u$, $P_2 = \sigma_p$ in units Knb, where K is the normalized constant for luminosity monitor).

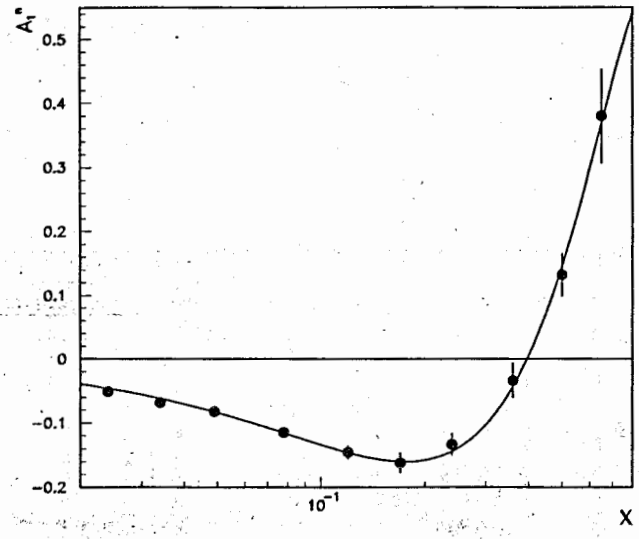

Рис. 2: The asymmetry A_1(x) calculated from the Monte-Carlo sample. Solid line is the parameterization used for generation.

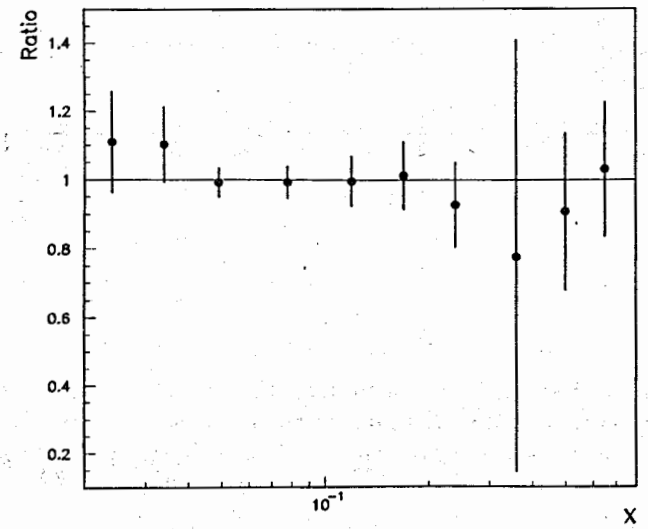

Рис. 3: The ratio (Ratio) of $A_1^n(x)$ calculated from the Monte-Carlo sample and input values of A_1^n.

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Извлечение асимметрий и спин-зависимых структурных функций
из поляризованных лептон-нуклонных сечений

Предложен новый метод извлечения асимметрий и спин-зависимых структурных функций. Метод основан на точных формулах разности сечений для глубоконеупругого рассеяния поляризованных лептонов на поляризованных нуклонах со спинами, параллельными и антипараллельными друг другу. Разработана специальная процедура минимизации функционала максимального правдоподобия, учитывающая динамические изменения поляризации пучка и мишени, а также особенности набора данных и их обработки.

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Extraction of Asymmetries and Spin-Dependent Structure Functions
from Polarized Lepton-Nucleus Cross-Sections

A new method to extract asymmetries and spin-dependent structure functions is proposed. The method is based on the exact formulae for differences of cross-sections for deep inelastic scattering of polarized leptons on polarized nucleons with spins parallel and antiparallel to each other. A special likelihood procedure taking into account dynamical variations of beam and target polarizations as well as peculiarities of data taking and processing, has been developed.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

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