

# ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ 

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## HBT MEASUREMENT OF THE EXPANSION VELOCITY OF PION PRODUCTION VOLUME

The GIBS Collaboration

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[^0]1. INTRODUCTION. It is known that identical pions, emitted from their production volume $\Delta r_{i}$ in size, must be connected by quantummechanical interference correlations, which are significant for particles with close momenta $\Delta p_{i} \cdot \Delta r_{i} \sim \hbar[1]$. Moreover, the correlation function, which is a distinction between the real two-particle spectrum and the spectrum in where the interference correlations are "of $f$ " somehow,

$$
\begin{equation*}
C(q) \equiv \frac{d^{4} \sigma / d^{4} q}{\left(d^{4} \sigma / d^{4} q\right)^{o f f}} \tag{1}
\end{equation*}
$$

contains information on both the size of the production volume and the duration of pion emission $[2,3]$. Here $q=\left(q_{0}, \vec{q}\right) \equiv p_{1}-p_{2}$ is the 4-momentum difference for two pions $(p=(E, \vec{p}))$.

If pions are independently emitted from the production volume of space-time shape $\rho(r)$, where $r=(t, \vec{r})$ is the 4 -point of pion emission, then the correlation function is equal to $[4,5](\hbar=1)$ :

$$
\begin{equation*}
C(q)=1+\left|\int \rho(r) \exp (i q \cdot r) d^{4} r\right|^{2} \tag{2}
\end{equation*}
$$

Here the 4 -point of emission is the point, where the pion had been produced or significantly rescattered last time before it left the production volume, when it can be considered as a free particle. Equality (2) is correct only if the pion momentum is not correlated with the 4 -point of its production [6]. As will be shown later, in our experiment this condition is not fulfilled for the complete $\pi^{-}$sample.

The Gaussian distribution was used to approximate the space-time shape of the production volume or its elements in their rest frames:

$$
\begin{equation*}
\rho(r)=\frac{1}{(2 \pi)^{2} R_{h} R_{v} R_{\|} T} \exp \left(-\frac{r_{h}^{2}}{2 R_{h}^{2}}-\frac{r_{v}^{2}}{2 R_{v}^{2}}-\frac{r_{\|}^{2}}{2 R_{\|}^{2}}-\frac{t^{2}}{2 T^{2}}\right) \tag{3}
\end{equation*}
$$

where $R_{h}$ and $R_{v}$ are the perpendicular to the beam and to one another root-mean-square radii of the pion production volume (horizontal and ver-
tical relative to the ground); $R_{\|}$is the one parallel to the beam; and $T$ is the root-mean-square dispersion of the pion emission time. Certainly, $R_{h}$ and $R_{v}$ must be equal. Following eq. (2), one obtains:

$$
\begin{equation*}
C(q)=1+\lambda \exp \left(-q_{h}^{2} R_{h}^{2}-q_{v}^{2} R_{v}^{2}-q_{\|}^{2} R_{\|}^{2}-q_{0}^{2} T^{2}\right) \tag{4}
\end{equation*}
$$

Here the empirical factor $\lambda$ "taking into account" various physical and methodical causes for the distortion of $C(q)$ was introduced [5, 7]. In actuality, we used only 3 -dimension approximations, joining the variables:

$$
\begin{equation*}
\dot{q}_{\perp}^{2} \equiv q_{h}^{2}+q_{v}^{2} \quad \text { and } \quad q_{\Sigma}^{2} \equiv q_{v}^{2}+q_{\|}^{2} \tag{5}
\end{equation*}
$$

If the production volume moves relative to the observation frame with velocity $\beta$ along the reaction axis, then after the usual Lorentz transformations $q_{\|} \rightarrow \gamma\left(q_{\|}-\beta q_{0}\right)$ and $q_{0} \rightarrow \gamma\left(q_{0}-\beta q_{\|}\right)$, correlation function (4) can be rewritten in the form:

$$
\begin{equation*}
C(q)=1+\lambda \exp \left\{-q_{\perp}^{2} R_{\perp}^{2}-\gamma^{2}\left(q_{\|}-\beta q_{0}\right)^{2} R_{\|}^{2}-\gamma^{2}\left(q_{0}-\beta q_{\|}\right)^{2} T^{2}\right\} \tag{6}
\end{equation*}
$$

where $q_{i}$ are given in the observation frame, while $R_{\perp}, R_{\|}$and $T$ are determined in the rest frame of the production volume. The value of $\beta$, if unknown, can be obtained along with other free parameters by fitting experimental data by means of approximation (6) [8].

The correlation peak $(C(q)-1)$ is not symmetrical in the $q_{0}-q_{\|}$plane relative to these axcs in case of a moving source (6). The peak is stretched relative to these axes at an angle depending on the source velocity: the $q_{0}-q_{\|}$correlation results from the $t-r_{| |}$correlation (see also [9]). This information is lost when data are fitted with a symmetrical function (e.g., (4)).

Likewise, if the production volume moves relative to the observation frame with velocity $\beta$ across the reaction axis, horizontally to the ground,
then:

$$
\begin{equation*}
C(q)=1+\lambda \exp \left\{-q_{\Sigma}^{2} R_{\Sigma}^{2}-\gamma^{2}\left(q_{h}-\beta q_{0}\right)^{2} R_{h}^{2}-\gamma^{2}\left(q_{0}-\beta q_{h}\right)^{2} T^{2}\right\} \tag{7}
\end{equation*}
$$

The parameters of the production volume can be determined using pions chosen from any bound part of phase space, because correlation function (2) depends only on the difference of pion 4-momenta (if there is no position-momentum correlation). For example, if a pion source consists of two fireballs, which have different velocities and emit pions with partly overlapping rapidity spectra, then one can measure the size of each of the fireballs, selecting for the interferometry analysis only pions from a distant, non-overlapping part of the overall rapidity spectrum [4].

It has recently been realized [10] that the combination of two methods, namely obtaining the parameters of the source using only a part of the pion spectrum and the extraction of the velocity of the source from fitting, provides a unique approach to reveal the nonstationarity of the overall pion source. Such nonstationarity will manifest itself in different values of $\beta$ obtained in the interferometry analysis of pions from various kinematic intervals. In the present paper we apply this approach to the experimental data. Compared to our previous paper [11], we measure here the velocities of different elements of the $\pi^{-}$production volume both along and across the reaction axis.
2. EXPERIMENTAL DATA AND THEIR FITTING. The experimental film material was obtained on a $4.4 \cdot A \mathrm{GeV} / \mathrm{c}^{24} \mathrm{Mg}$ beam of the Dubna synchrophasotron using a $4 \pi$ spectrometer SKM- 200 - GIBS including a two-meter streamer chamber placed in a magnetic field of 0.9 T [12]. A $1.2 \mathrm{~g} / \mathrm{cm}^{2} \mathrm{Mg}$ target was placed inside the chamber sensitive volume. The streamer chamber was triggered only in case of central [12] Mg-Mg interac-
tions, i.e. if stripping neutrons, protons and other beam nuclear fragments do not hit a forward cone of $\sim 2.4^{\circ}(\sim 4 \mathrm{msr})$, which corresponds to a stripping nucleon transverse momentum of $\sim 180 \mathrm{MeV} / \mathrm{c}$. The antistripping counters were placed at a distance of 6 m downstream the target beyond $2 \mathrm{~m} \cdot$ of the 0.9 T magnetic field, and so particles, which were softer than stripping ones, scarcely hit them. These central interactions accounted for $\sim 4 \cdot 10^{-4}$ of all inelastic $\mathrm{Mg}-\mathrm{Mg}$ interactions.

In the denominator of correlation function (1), we used the mixed two-particle spectrum of $\pi^{-}$pairs, where each $\pi^{-}$is randomly chosen from different events (with the same $\pi^{-}$multiplicity) [3]. The number of mixed pairs was chosen to be 10 times larger than that of real pairs. The real pairs, as well as the mixed ones having $Q_{i n v}^{2} \equiv q_{\perp}^{2}+q_{\|}^{2}-q_{0}^{2}<10 \mathrm{MeV}$, were not used because of possible measurement errors due to track interchange on different film projections. In this connection, Gamow's correction [13] becomes negligible (but it was introduced). The pairs having the measurement errors of $Q_{i n v}$ larger than 10 MeV were not used either.

In the laboratory system, the accuracy of track measurement is about $1 \%$ for pion momenta and 5 mrad for angles. This corresponds to an uncertainty of $2 \div 6 \mathrm{MeV}(\mathrm{MeV} / \mathrm{c})$ in relative momenta $q_{i}$ in the $\mathrm{Mg}-\mathrm{Mg}$ c.m.s. The multiple scattering in the target contributes an additional error of up to $5 \mathrm{MeV}(\mathrm{MeV} / \mathrm{c})$ in $q_{i}$. The overall uncertainty in $q_{i}$ is of the order of magnitude or less than the bin width $(10 \mathrm{MeV} / \mathrm{c}(\mathrm{Mev}))$ that was used at fitting (see below). And the bin width is much smaller than the interference peak width ( $\sigma \sim 40 \mathrm{MeV} / \mathrm{c}(\mathrm{MeV})$ ). About $10 \%$ of $\pi^{-}$were lost, mainly due to absorption in the target and limited measurability of vertical tracks. We used in the analysis $470000 \pi^{-}$pairs (120 000 pions

## out of 14000 events).

The method of maximum likelihood was used to obtain the parameters of the pion source $[14,15]$ ). 3-dimensional histograms of $q_{\perp}, q_{\|}, q_{0}$ (or $q_{\Sigma}$, $q_{h}, q_{0}$ ) were individually plotted for the number of real and mixed pion pairs with $10 \mathrm{MeV} / \mathrm{c}$ ( MeV ) bins inside an interval of $-200 \div 200 \mathrm{MeV} / \mathrm{c}$ ( MeV ). The contents of each non-empty bin of the histogram of mixed pairs was multiplied by $C(q)$ for a given set of parameters $R_{\perp}^{2}, R_{\|}^{2}, T^{2}$ (or $R_{\Sigma}^{2}, R_{h}^{2}, T^{2}$ ), as well as $\beta$ and $\lambda$. Assuming this value as a Poisson distribution mean value, the probability of the number of pairs, contained in the same bin of the real pair histogram, was calculated. The product of probabilities for all bins was maximized relative to these parameters by the FUMILI program. The value of $\lambda^{2}$ was calculated by the same method as $\chi_{P M L}^{2}$ in [15].
3. RESULTS FOR THE COMPLETE SAMPLE OF $\pi^{-}$. The results of fitting our data using approximation (4), (5) in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame for the complete sample of pions are the following: $\lambda=1.02 \pm 0.04 ; R_{\perp}=3.2 \pm 0.1$ $\mathrm{fm} ; R_{\|}=3.3 \pm 0.1 \mathrm{fm} ; T \sqrt{-1}=2.3 \pm 0.3 \mathrm{fm} / \mathrm{c} ; \lambda^{2}=1.15$. It should be noted that the extracted sign of $T^{2}$ is negative (opposite to the sigus of $R_{\perp}^{2}$ and $R_{\|}^{2}$ ). This eliminates the possibility to interpret the parameter as the emission time squared. Consequently, expressions (3)-(7) lose their original physical meaning for the complete $\pi^{-}$sample.

Stability of the sign of $T^{2}$ to different variations of data processing was checked at different histogram bounds and bin widths; for other means of mixed pair choice and another maximm likelihood method [16]; for different parts of event ensemble and under different additional conditions (see also [11]). Fig. 1 presents the projections on different axes $q_{i}$ of differ-
ent layers of correlation function (1). The curves are obtained using the mixed pion pair ensemble and approximation (4), (5) with the foregoing parameters. One can see that $C\left(q_{0}\right)$ increases with increasing $q_{0}$ providing $\vec{q}^{2} \equiv q_{\|}^{2}+q_{\perp}^{2} \simeq$ const (Fig.1d) which illustrates the negative sign of $T^{2}$ in (4). Without this condition, $C\left(q_{0}\right)$ decreases with increasing $q_{0}$ (Fig.1c), because $C\left(q_{0}\right)$ decreases with increasing $|\vec{q}|$, and $\left|q_{0}\right|$ is always smaller than $|\vec{q}|$ since $q_{0}=(\vec{u} \cdot \vec{q})$, where $\vec{u}=\left(\vec{p}_{1}+\vec{p}_{2}\right) /\left(E_{1}+E_{2}\right)$.

In other nucleus-nucleus experiments (under different conditions), $T^{2}$ was found to be positive or equal to 0 ; in some cases it was not included in approximation or its value was restricted to be positive at fitting (see [17]-[23] and references there). Our result, $R_{\perp}^{2} \approx R_{\|}^{2} \approx-T^{2}$, is found to be close to a one-parameter relativistic invariant approximation often used (beginning with [1]) to describe elementary, lepton and hadron, interactions $C^{\prime}(q)=1+\exp \left(-R_{i n v}^{2} Q_{i n v}^{2}\right)$. The opposite sign of $T^{2}$ was predicted [24] for quickly expanding sources and experimentally obtained in an explicit form for $e^{+} e^{-}$interactions [25] (see also [16, 26]).
4. RESULTS FOR THE BOUND PARTS OF PHASE SPACE. Figs. 2 and 3 present the parameters of the pion production volume obtained for different pion subsets moving with different rapidities $y_{\text {subset }}$ along (full circles) and across (open circles) the reaction axis. The longitudinal subsets contain pions having an energy of $E<200 \mathrm{MeV}$ in the reference frames moving with rapidities $y_{\|}=0 ; \pm 0.5 ; \pm 1.0$ in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame along the reaction axis. The transverse subsets contain pions with $E<200 \mathrm{MeV}$ in the reference frames moving with transverse rapidities $y_{h}= \pm 0.25 ; \pm 0.75$; -1.25 in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame across the reaction axis, horizontally to the ground.

The points are obtained by fitting the data using approximations (6) and (7) in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame $\left(Y_{\text {source }}=\frac{1}{2} \ln [(1+\beta) /(1-\beta)]\right)$. Within the experimental errors, the same points were obtained by fitting in the rest frame of each subset. The close points were obtained for the vertical direction (not shown). The results are close to the ones obtained in our previous paper [11], where the longitudinal pion subsets were chosen in another way.

The rapidity of the source does not have to depend on the used pion subset and should be equal to 0 in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame, if the $\pi^{-}$momentum is independent of the emission point. The observed dependence shows that pions from different parts of phase space are emitted from different elements of the source that move relative to each other. This implies that equation (2) is not correct for the complete pion sample in our experiment. Certainly, the notion "element" is conventional here, because each element of the production volume may not be quite stationary either. However, one can see that the emission time becomes real here ( $T^{2}>0$, true, with large errors) in contradistinction to the result for the complete $\pi^{-}$sample.

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Fig.1. Projections on different axes $q_{i}$ of different layers of correlation function (1) for the complete $\pi^{-}$sample. The curves are obtained using approximation (4), (5) with the resulting parameters. One can see that $C\left(q_{0}\right)$ increases with increasing $q_{0}$ on condition $|\vec{q}| \simeq$ const (Fig.d). This illustrates the negative sign of $T^{2}$ in (4).


Fig.2. Parameters of the pion production volume elements corresponding to different pion subsets (see the text) moving with different rapidities $y_{\text {subset }}$ along (full circles) and across (open circles) the reaction axis in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame. The points are obtained by fitting the data using approximations (6) and (7). Here the parameter $T^{2}>0$.


Fig.3. Rapidities $Y_{\text {source }}$ of the pion production volume elements corresponding to different pion subsets (see the text) moving with different rapidities $y_{\text {subset }}$ along (full circles) and across (open circles) the reaction axis in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame. The points are obtained by fitting the data using approximations (6) and (7) in the $\mathrm{Mg}-\mathrm{Mg}$ rest frame. Solid line is $Y_{\text {source }}=y_{\text {subsel }}$.

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Аникина МХХ. и др.
Измерение скорости расширения области генерации пионов методом НВТ корреляиий

С помощью нового метода обработки экспериментальных данных при исследовании интерференционных корреляций тождествениых частиц получено прямое доказательство нестационарности объема генерации $\pi$-мезонов в центральных $\mathrm{Mg}-\mathrm{Mg}$-взаимодействиях при импульсе $p_{\text {лаб. }}=4,4$ ГэВ/с на нуклон. В лорени-преобразование выражение, аппроксимируюиее иитерфереиционный пик, в качестве свободного параметра включена скорость источиика пар коррелированных $\pi^{-}$-мезонов. Эта́ скорость растет с ростом скорости подансамбля $\pi^{-}$-мезонов, выбранных для, интерферениионного анализа. Скорость источника является новой экспериментальын измеримой величиной иитерференционного анализа, наряду с пространственно-временными размерами этого источника.

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Anikina M.Kh. et al.
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HBT Measurement of the Expansion Velocity of Pion Production Volume

Using a new method of data processing in the investigation of interference correlations of identical particles, a direct proof of nonstationarity of the $\pi^{-}$ production volume in central $\mathrm{Mg}-\mathrm{Mg}$ collisions at a beam momentum of $p_{\text {лаб. }}=4.4 \mathrm{GeV} / \mathrm{c}$ per nucleon wasobtained. The velocity of a source of correlated $\pi$ meson pairs is introduced as a free parameter in the Lorentz transforined approximation formula of the interference peak. This velocity increases with increasing the velocity of a subset of $\pi$, mesons which are chosen for the interference analysis. The source velocity is the new experimentally measurable physical variable of interference analysis, apart from space-time sizes of the source.

The investigation has been performed at the Laboratory of High Energies, JINR.

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