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SCHEME FOR KINEMATICAL IDENTIFICATION OF EXCLUSIVE $pd \to ppn$ PROCESS AT ANKE SETUP

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1 Introduction

In this paper we describe a scheme of kinematical identification of $pd \to ppn$ reaction events. Such a scheme is supposed to be used in data processing of the $pd \to ppn$ events to be collected at the spectrometer ANKE (COSY) [1, 2, 3]. As a result of using such a scheme, we have obtained the precision parameters of the ANKE spectrometer for the process $pd \to ppn$. The approach proposed here can be easily adapted to any exclusive process.

2 Method

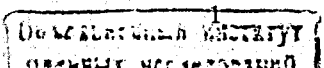
Kinematical identification of hypotheses was introduced into data processing practice more than thirty years ago [4, 5]. The purpose of this step is twofold: to check the hypothesis that a given event is of the type it is supposed to be and if it is true, to find more accurate estimates of the event parameters. For minimizing the corresponding functional the so-called Lagrange multipliers method was proposed. In this paper another technique, applicable to a wider class of problems, is used for kinematical identification [6–9].

In fig. 1 the setup layout is shown. The setup consists of a target, a backward magnet with a system of drift chambers and a forward magnet with a system of proportional chambers. Hodoscopes of scintillation counters are used to produce signals for triggering the chambers.

The process $pd \to ppn$ is fully described by the following variables: the coordinates of the interaction point and momentum vectors of two protons, the forward and backward ones. In the model used we have done some simplifications: considering that the transversal dimensions of the beam are small (~2 mm in diameter) and the beam line is practically straight in the vicinity of the target 40 cm long, the only essential parameter describing the interaction point is the coordinate along the beam line. It was generated randomly according to the uniform distribution. Another simplification was to generate the kinematics of the reaction according to its phase space. Finally, we assumed that the measurement error of coordinates for all the track detectors was 500 μm and the error for the time of flight was 1 ns[1].

The field map contained three components of the field in the nodes of the space lattice. In ref. [10] a polynomial model of the analytical representation of the magnetic field was described. All the field calculations were carried out in the framework of this model with the polynomials up to the third order. In this case 24 coefficients for any elementary volume were defined by preliminary fitting using known values in 8 corner points of the volume.

[1]Here we used a rather conservative approach and selected deliberately higher figures than one anticipated for the setup under development.



About 3000 events of the deuteron break-up were generated in which both protons crossed respectively all planes of the backward and forward detector chambers and hit the corresponding scintillation hodoscopes. The Runge-Kutta tracking was done according to the method described in ref. [11]. While tracking the particles Coulomb scattering in different elements of the setup was taken into account.

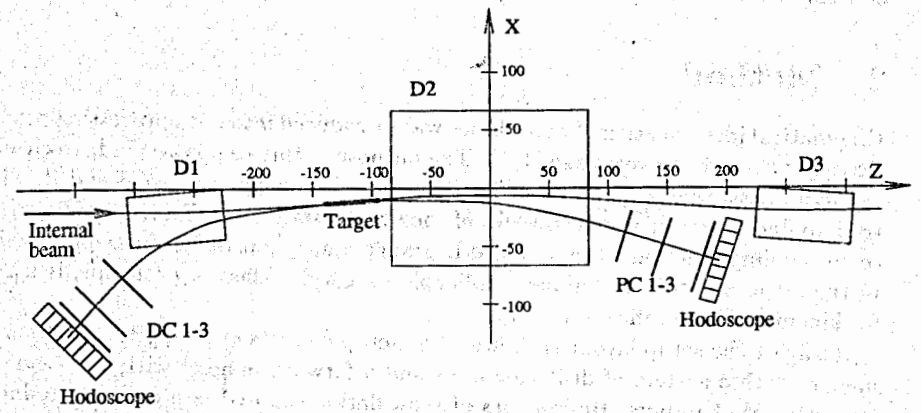


Fig. 1. Setup layout. All dimensions in cm.

A range of the event parameters was as follows:

$$\begin{array}{rcccl} -20.0 \text{ cm} & \leq & L & \leq & 20.0 \text{ cm} \\ -0.053 & \leq & x_1' & \leq & 0.390 \\ -0.027 & \leq & y_1' & \leq & 0.028 \\ 0.13 \text{ GeV/c} & \leq & P_1 & \leq & 0.51 \text{ GeV/c} \\ -0.096 & \leq & x_2' & \leq & 0.580 \\ -0.110 & \leq & y_2' & \leq & 0.100 \\ 0.56 \text{ GeV/c} & \leq & P_2 & \leq & 3.16 \text{ GeV/c} \end{array}$$

The following notations are used here: L is the coordinate of the interaction point (vertex coordinate), taken along the beam relative to the middle of the target, x_1', y_1', P_1 are the slope coefficients along the x, y axes and the momentum for the backward proton, x_2', y_2', P_2 are the same for the forward one.

These simulated events were used as the input data for the fitting program. The parameter estimates were found by minimizing the following χ^2 form:

$$\chi^2 = \sum_{i,j}(x_i^t - x_i^m) \cdot G_{ij} \cdot (x_j^t - x_j^m) + \frac{(\Delta T^t - \Delta T^m)^2}{\sigma_T^2} \qquad (1)$$

with the constraint equation:

$$[(E_b + m_p) - (E_1 + E_2)]^2 - [\vec{p}_b - (\vec{p}_1 + \vec{p}_2)]^2 = m_n^2,$$

where $x_{i,j}^t$ are the "true" coordinates of the particle hits of the track detectors, $x_{i,j}^m$ are their "measured" values, G_{ij} is the weight matrix taking into account both errors of measurement and Coulomb scattering in the general case, ΔT^t is the "true" value of the time-of-flight difference between protons in the forward and backward detectors, ΔT^m is the "measured" value of the time-of-flight difference and σ_T is the error of the time-of-flight measurement. The constraint equation is easily understandable: the missing mass of the reaction must be equal to the neutron mass; here $E_b, \vec{p}_b$ are the beam particle energy and momentum, $E_1, \vec{p}_1, E_2, \vec{p}_2$ are the energies and momenta of the forward and backward protons, m_p, m_n are the masses of the target proton and missing neutron respectively.

In order to do fitting we have to express the "true" coordinates in terms of the functions of the parameters. It is well known that the equation of motion of a particle in the magnetic field is the Lorentz equation which in the coordinate representation is equivalent to differential equations of the second order. For an inhomogenious magnetic field these equations are usually solved by the Runge-Kutta method if five initial parameters of the particle trajectory are known — two coordinates, their derivatives and the momentum. In our case there are only four initial parameters: the vertex coordinate, two derivatives and the momentum. Then the hit coordinates $x_{i,j}^t$ and ΔT^t are some regular functions of these four variables and the problem is to find these functions. Here it is done by the method described in ref. [12]. The idea of the method is to represent these functions in the form of expansion into Taylor series over the initial parameters. The maximum power of the series is defined by the condition that the approximation inaccuracy is less than other unavoidable inaccuracies. In our case the major source of the unavoidable inaccuracy is Coulomb scattering and we required that the approximation inaccuracy must be less than or comparable with it. We used the Taylor series over four variables and obtained accuracies of approximation better than the Coulomb scattering errors.

In table 1 the Coulomb scattering errors for backward and forward protons are shown. The errors were calculated for the proton momentum 0.5 GeV/c for the backward proton and 1.5 GeV/c for the forward one. The method of taking Coulomb scattering into account is described in ref. [13]. Inaccuracies due to approximations, expressed as square roots from mean quadratic deviations, are also given in table 1.

Table 1. Coordinate inaccuracies (R.M.S. in cm) caused by approximation procedure and Coulomb scattering

	Backward spectrometer			Forward spectrometer		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
Function inaccuracy	0.060	0.100	0.120	0.032	0.056	0.080
Coulomb inaccuracy	0.140	0.280	0.340	0.055	0.070	0.095

Here $\sigma_1, \sigma_2, \sigma_3$ are the function and Coulomb scattering errors for the first, second and third chambers respectively.

It is seen that the inaccuracies caused by the approximation procedure are lower than the Coulomb scattering errors.

3 Results

In fig. 2 and in the first column of table 2 we show the accuracies obtained with the model as described above. It is seen that the accuracies in momentum are ~1.0%, in interaction coordinate ~3.8 cm and in angle less than ~3.3 mrad.

In the second column of table 2 the same accuracies in the so-called "ideal" case are shown — when we assume that functions for observables are known exactly. One can see that the accuracies are ~10% better, in other words if the Runge-Kutta tracking is done right during the kinematical fit, one will get these accuracies. In these cases the weight matrix G_{ij} in formula (1) was written with taking into account both measurement and Coulomb scattering errors.

Sometimes during parameter estimation Coulomb scattering is either not taken into account at all or dealt with in an approximate manner. To clarify the influence of such simplification we have conducted a special investigation in the framework of our model. In column 3 of table 2 we give the parameter accuracies when only measurement errors were included in the matrix G_{ij}. In comparison with column 2 we obtain a ~40% decrease in accuracy. In the last column we show the accuracies when Coulomb scattering was included only in the diagonal terms of the matrix G_{ij}, i.e. all the correlation terms were neglected. It is seen that we again obtain a worse accuracy though worsening is not so big as in the previous case.

Table 2. Parameter accuracy for different cases.

	1	2	3	4
$\sigma(\Delta L)$, cm	3.79	3.58	5.11	3.70
$\sigma(\frac{\Delta p_1}{p_1})$, %	1.04	1.00	1.11	1.06
$\sigma(\frac{\Delta p_2}{p_2})$, %	0.87	0.80	1.04	0.85
$\sigma(\Delta x_1')$, mrad	3.34	3.24	3.65	3.50
$\sigma(\Delta y_1')$, mrad	1.04	0.94	1.99	1.59
$\sigma(\Delta x_2')$, mrad	2.87	2.64	3.75	2.80
$\sigma(\Delta y_2')$, mrad	0.68	0.58	0.72	0.59

Finally, in fig 3. the time of flight differences are shown for the cases when a proton (or two) from deuteron break-up is replaced by a π-meson taking the same momentum. These spectra were obtained for the same sample of events which was used during the model testing (remember that the kinematics of the process was generated according to phase space).

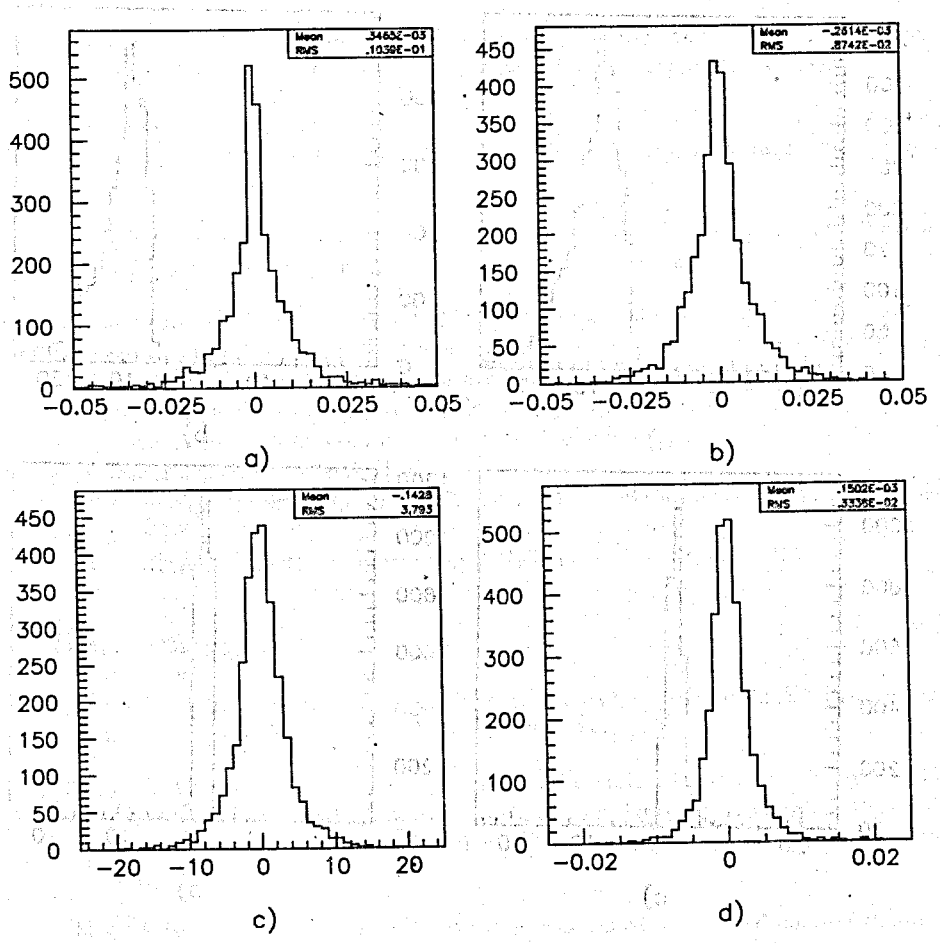


Fig. 2. Parameter accuracy in the model.
a) backward proton $\Delta p/p$, b) forward proton $\Delta p/p$,
c) vertex coordinate ΔL (in cm), d) slope coefficient $\Delta x_1'$.

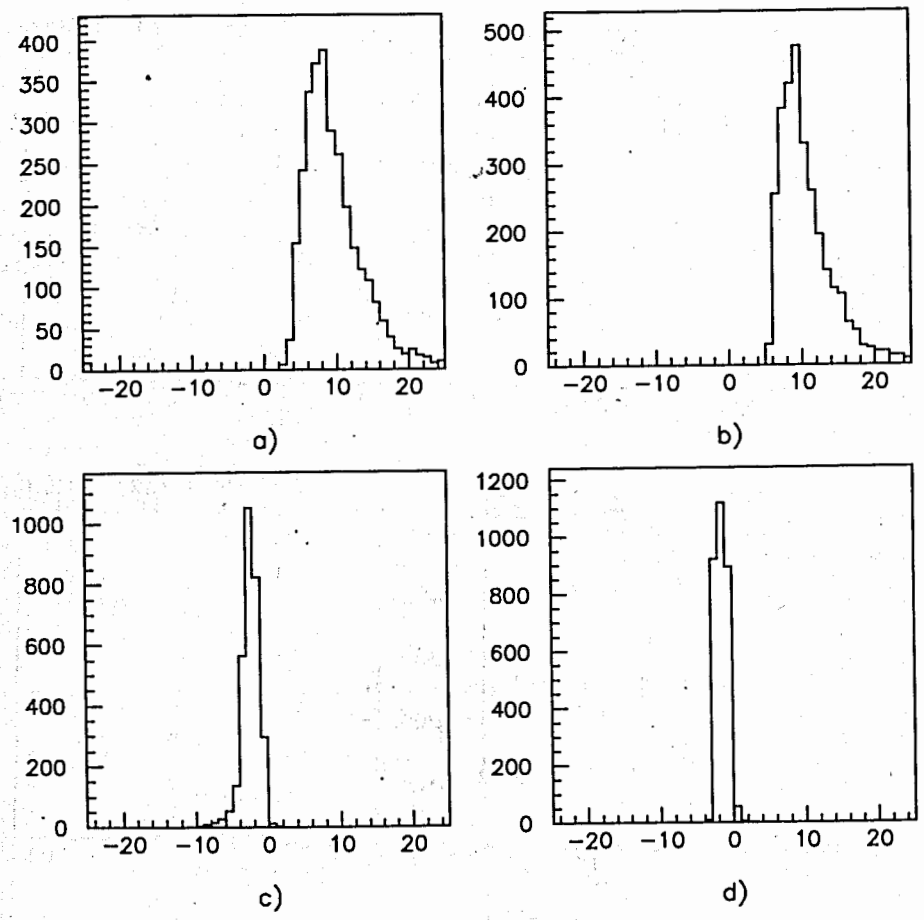


Fig. 3. Time of flight difference (in nsec) between the backward and forward particles.
a) backward — proton, forward — proton,
b) backward — proton, forward — π-meson,
c) backward — π-meson, forward — proton,
d) backward — π-meson, forward — π-meson.

4 Conclusion

Using the model data we tested a scheme for selection of the deuteron break-up reaction. Briefly, the results can be summarized as follows:

- The proposed scheme gives good estimates of parameters and may be considered as a candidate for the proposed data processing procedure.
- Usage of the approximate functions which is much quicker than using Runge-Kutta for event observables (in our model in the polynomial form), gives a ~10% decrease in accuracy.
- Coulomb scattering should be taken into account during the kinematical analysis phase. Otherwise one loses ~40% in accuracy.

The proposed scheme may be applied to any exclusive process. As for inclusive processes, minimization of the quadratic form with a correct covariation matrix and use of the Runge-Kutta method are recommended if one is going to get the maximal accuracy of estimates. Such an approach is resource-costing but the performance of modern computers is high enough to permit it in the case of experimental facilities like ANKE.

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