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ESTIMATION OF $\pi^+\pi^-$-ATOM PRODUCTION RATE IN HIGH ENERGY COLLISIONS

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1. Introduction

In all processes with emission of opposite charge particles production of Coulomb bound states (atoms) of the two particles is possible. The production of $\pi^+\pi^-$–atoms (and other hadronic atoms) in inclusive processes was considered in [1]. Also, a method of their observation and lifetime measurement was proposed.

The lifetime of $\pi^+\pi^-$–atoms ($A_{2\pi}$) is determined by the charge-exchange process $\pi^+\pi^- \to \pi^0\pi^0$. The lifetime τ of the 1S state of $A_{2\pi}$ and difference of the S–wave $\pi\pi$–scattering lengths $|a_0 - a_2|$ with isospin values 0 and 2 are connected with exact relation [2].

The $\pi\pi$–scattering lengths belong to those parameters that can be calculated in Chiral Perturbation Theory [3, 4, 5] most precisely. For a_0 and a_2 the chiral theory gives the values with precision of 5% [4]. As can be inferred from the relation between τ and $|a_0 - a_2|$, measurement of τ with 10% precision would allow finding the value of $|a_0 - a_2|$ in a model independent way [6] with 5% precision.

The lifetime of the $A_{2\pi}$ is very short ($\tau = (3.7 \pm 0.3) \cdot 10^{-15}$ s from Chiral Perturbation Theory, $c\tau = 1.1\mu$m) and so in paper [1] the method was proposed to observe atoms by detecting $\pi^+\pi^-$–pairs from their break-up in the target where they are produced[1].

In the target placed on a proton beam $\pi^+\pi^-$–pairs in a free state (free pairs) and $\pi^+\pi^-$–atoms are produced. The atoms either annihilate into $\pi^0\pi^0$–pairs or break up (are ionized) into $\pi^+\pi^-$–pairs (atomic pairs) at $A_{2\pi}$ interaction with the target. The number of the atomic pairs depends on the lifetime and momemtum of the atoms, their break-up cross section and target thickness.

For the atomic pairs the value of the relative momentum q in the $\pi^+\pi^-$ c.m. system is less than 3 MeV/c with probability $\sim$ 95% and therefore the pions have approximately equal energies $E_+ \approx E_-$ in the lab system and a small opening angle. In the region of small relative momenta the numbers of the atomic pairs and the free pairs are of the same order.

The number of the atomic pairs can be found by subtracting the number of free pairs from the total number of the pairs in the region of small relative momenta [8]. It is possible to obtain [8] the distribution of the free pairs

[1]In [7] an experiment is proposed to observe $A_{2\pi}$ and determine its lifetime by measuring the ratio between branching ratios for $A_{2\pi}$ decay into two photons and into two π^0 mesons.

over relative momentum from the experimental distribution of accidental $\pi^+\pi^-$–pairs, because both distributions are proportional to the product of the single inclusive production cross section of π^+ and π^- mesons [9]

The number of the atoms produced in a target is strictly related to the number of free pairs with low q and can be obtained from the distribution of free pairs with a formula described below.

The measurement of the ratio between the number of the atomic pairs and the full number of the produced atoms or the measurement of the number of the atomic pairs as a function of the target thickness allows determining the atom lifetime.

The method proposed in [1] have recently allowed observation of $\pi^+\pi^-$–atoms at the Serpukhov accelerator U–70 in the inclusive process pTa $\rightarrow A_{2\pi}X$ at $E_p = 70$ GeV [8]. Also, estimation of the $A_{2\pi}$ lifetime was obtained [10].

In this paper we present numerical calculations of $A_{2\pi}$ yields for the energies of the proton beam 24, 70, 450 and 1000 GeV for the targets of Al, Be, Cu, Ag and Ta, which may be useful for planning the experiment on $A_{2\pi}$ lifetime measurement.

2. Basic relations

The exact formula for the production cross section of the atoms in the inclusive processes was obtained in [1]. The probability of $A_{2\pi}$ production is proportional to the double inclusive cross section of π^+ and π^- production with small relative momenta.

In calculation of the $A_{2\pi}$ production cross section, it is necessary to exclude the contribution of π^+ and π^- arising from decays of long-lived particles . When one or both pions in pairs are coming from such particle decays, a typical range between the pions is much larger than the Bohr radius of $A_{2\pi}$ ($r_B = 387$ fm). So the probability of $A_{2\pi}$ production in these cases is negligibly small. The main sources of such pion pairs are decays of η, η', Λ, K^0_s, $\Sigma^\pm$.

The laboratory differential inclusive cross section of $A_{2\pi}$ production may be written in the following form [1]

$$\frac{d\sigma_n^A}{d\vec{p}_A} = (2\pi)^3 \frac{E_A}{M_A} |\Psi_n(0)|^2 \left. \frac{d\sigma_s^0}{d\vec{p}_1\, d\vec{p}_2}\right|_{\vec{p}_1=\vec{p}_2=\vec{p}_A/2}, \qquad (1)$$

where $\vec{p}_A$, E_A and M_A are the momentum, energy and mass of the $\pi^+\pi^-$–atom in the lab system respectively, $|\Psi_n(0)|^2 = p_B^3/\pi n^3$ is the atomic wave function squared at the origin with the principal quantum number n and the orbital moment $l = 0$, p_B is the Bohr momentum of π in $A_{2\pi}$, $d\sigma_s^0/d\vec{p}_1 d\vec{p}_2$ is the double inclusive production cross section for $\pi^+\pi^-$–pairs from short-lived sources (hadronization processes, ρ, ω, Δ, K^*, Σ^*, etc.) without taking into account $\pi^+\pi^-$ Coulomb interaction in the final state, $\vec{p}_1$ and $\vec{p}_2$ are the π^+ and π^- momenta in the lab system. The momenta of π^+ and π^-–mesons obey the relation $\vec{p}_1 = \vec{p}_2 = \frac{1}{2}\vec{p}_A$.

The atoms are produced with the orbital momentum $l = 0$, because $|\Psi_{n,l}(0)|^2 = 0$ at $l \neq 0$. Deviation of $|\Psi_n(0)|^2$ from the pure Coulomb wave function does not exceed 0.1% [11, 12]. The $A_{2\pi}$ are distributed over n according to n^{-3}: $W_1 = 83\%$, $W_2 = 10.4\%$, $W_3 = 3.1\%$, $W_{n\geq4} = 3.5\%$. Note that $\Sigma_{n=1}^{\infty}|\Psi_n(0)|^2 = 1.202|\Psi_1(0)|^2$.

The double inclusive cross section without taking into account $\pi^+\pi^-$ Coulomb interaction may be written in the form [9]:

$$\frac{d\sigma^0}{d\vec{p}_1 d\vec{p}_2} = \frac{1}{\sigma_{in}} \frac{d\sigma}{d\vec{p}_1} \frac{d\sigma}{d\vec{p}_2} R(\vec{p}_1, \vec{p}_2), \qquad (2)$$

where $d\sigma/d\vec{p}_1$ and $d\sigma/d\vec{p}_2$ are the single particle inclusive cross sections, σ_{in} is the inelastic cross section of hadron production, R is a correlation function due to strong interaction. In the case of the particles satisfying the requirement $\vec{p}_1 = \vec{p}_2$ the correlation function was found to be $R = 1.65 \pm 0{,}05$ [9, 13].

The probability of particle production per interaction (yield) can be expressed through the differential cross section:

$$\frac{dN}{d\vec{p}} = \frac{d\sigma}{d\vec{p}} \frac{1}{\sigma_{in}}. \qquad (3)$$

From (1), (2) and (3), after substituting the expression for $|\Psi_n(0)|^2$ and summing over n, one can obtain an expression for the inclusive yield of the $\pi^+\pi^-$–atoms in all S–states through the inclusive yields of π^+ and π^-–mesons

$$\frac{d^2N^A}{dp_A d\Omega} = 1.202\, \pi^2 \frac{m_\pi{}^2\alpha^3}{2} \frac{E_A p_A^2}{p_1^2 p_2^2} \frac{d^2N_1}{dp_1 d\Omega_1} \frac{d^2N_2}{dp_2 d\Omega_2} R, \qquad (4)$$

where α is the fine structure constant, p_1, p_2 are the momenta of π^+ and π^- mesons ($p_1 = p_2$), Ω is a solid angle.

3. Results of calculations

To obtain yields of π^+ and π^-–mesons we have used the computer simulation programs FRITIOF 6.0 [14] and JETSET 7.3 [15] (CERN Program Library) based on the Lund string fragmentation model of strong interactons. FRITIOF is a generator for hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, which makes use of JETSET for fragmentation.

For calculations of pion yields the simulated events were accumulated in a two-dimensional array depending on the emission angle and momentum of the particles with the angular bin width 0.3°, and the momentum bin width 0.1 GeV/c. At this stage selection of pions from long-lived and from short-lived sources was performed. Further, using yields only from the short-lived sources the distributions of the $\pi^+\pi^-$–atom yields over the angle and momentum were obtained. Then distributions were smoothed.

The results are presented in Fig.1 where the yields of $\pi^+\pi^-$–atoms are shown for the reaction pAl $\rightarrow A_{2\pi}X$ at proton energies $E_{\rm p} = 24, 70, 450$, 1000 GeV and $A_{2\pi}$ emission angles $\theta_{lab} = 1°, 2°, 3°, 4°, 5°, 6°$ as a function of the momentum of one of the π–mesons forming the atom ($p_\pi = \frac{1}{2}p_A$).

The probability of $A_{2\pi}$ production in the momentum interval Δp_π and the solid angle $\Delta\Omega$ at the emission angle θ can be estimated by multiplying the mean value of the yields in the given momentum interval by the values of this interval in GeV/c and the solid angle $\Delta\Omega$ in sr.

The $\pi^+\pi^-$–atom yields for the same conditions integrated over the momentum are shown in Fig.2 versus the emission angle (solid lines). For comparison, the dashed lines denote unreal $A_{2\pi}$ yields when the contribution of the pions arising from long-lived sources is not subtracted from the single particle production cross section of π^+ and π^-–mesons. As can be seen the contribution of π–mesons emitted from long-lived sources is actually significant for calculation of the $\pi^+\pi^-$–atom yields and has to be subtracted.

Similar yield distributions were obtained for the nuclei Be, Cu, Ag and Ta. The ratios of $A_{2\pi}$ yields integrated over the momentum for the reactions p$A \rightarrow A_{2\pi}X$ to the same for pAl $\rightarrow A_{2\pi}X$ as a function of the emission angle are presented in Fig.3.. To give representation of the $A_{2\pi}$ yields for other nuclei from this paper these ratios may be used. The approximate momentum distributions of $A_{2\pi}$ for pA can be obtained by multiplying yields for pAl presented in Fig.1 and the corresponding coefficients obtained from Fig.3. This estimation is aproximate because the A-dependence varies with p_π.

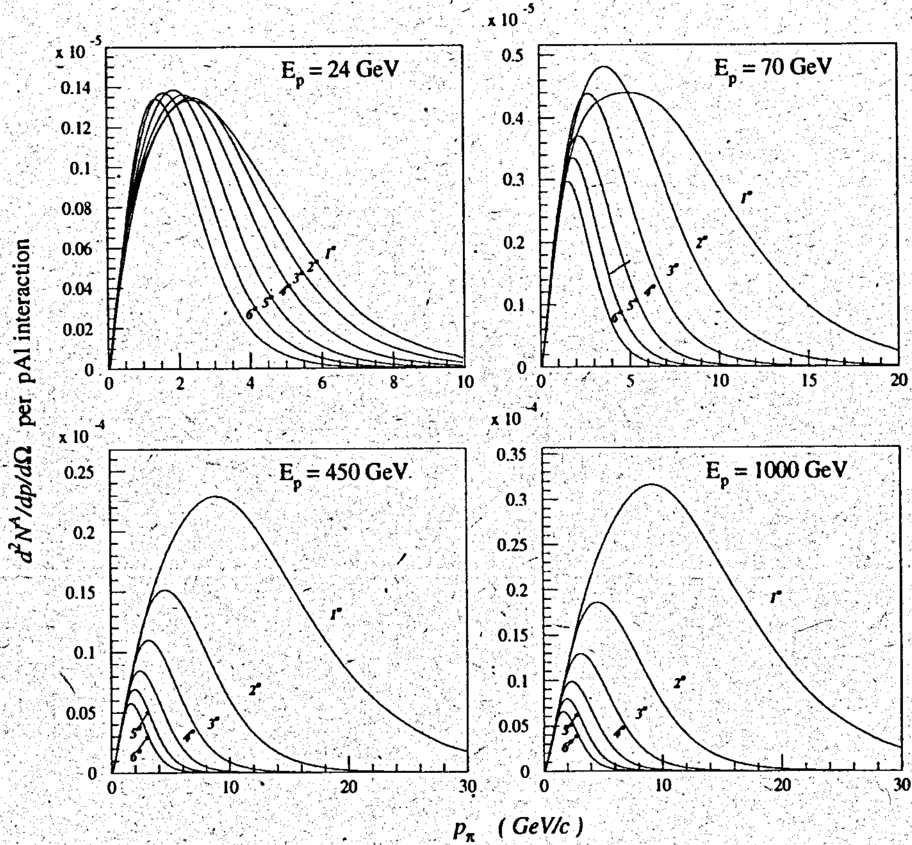

Figure 1: The yields of $A_{2\pi}$ for the reaction pAl $\rightarrow A_{2\pi}X$ (per pAl interaction) at energies $E_{\rm p} = 24, 70, 450, 1000$ GeV and emission angles $\theta_{lab} = 1°$, 2°, 3°, 4°, 5°, 6° as a function of the momentum of one of the π–mesons forming the atom ($p_\pi = \frac{1}{2}p_A$).

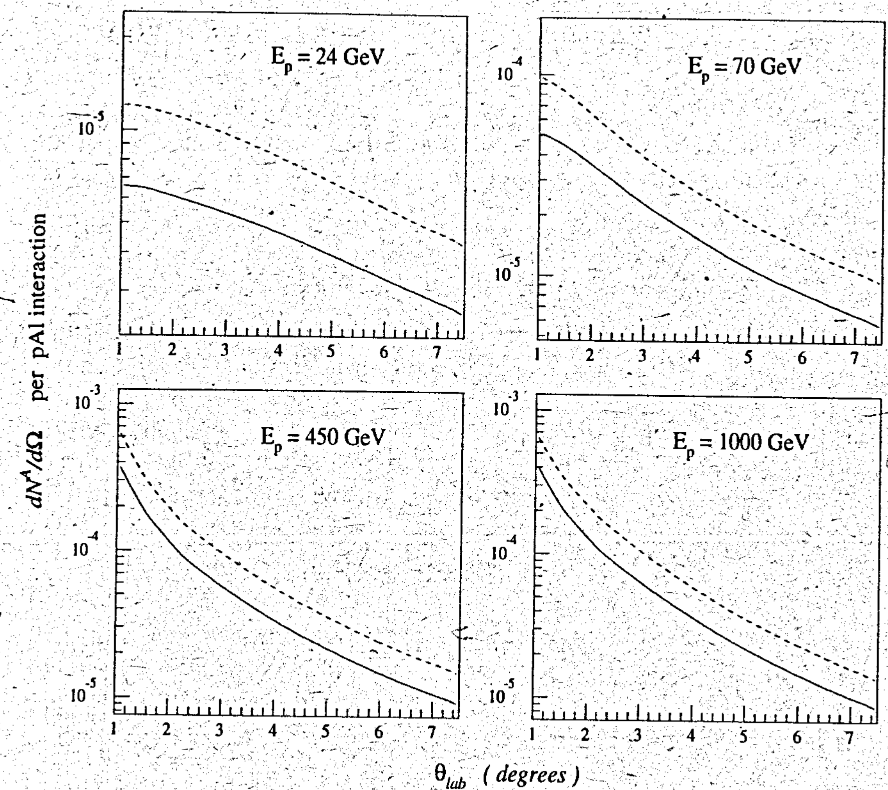

Figure 2: The $A_{2\pi}$ yields for the reaction $\mathrm{pAl} \to A_{2\pi}X$ at energies $E_{\mathrm{p}} =$ 24, 70, 450, 1000 GeV integrated over the momentum versus the emission angle are shown by solid lines. The $A_{2\pi}$ yields, when the yields of π^+ and π^--mesons from all sources are taken in (4) instead of those from short-lived sources alone, are shown by dashed lines.

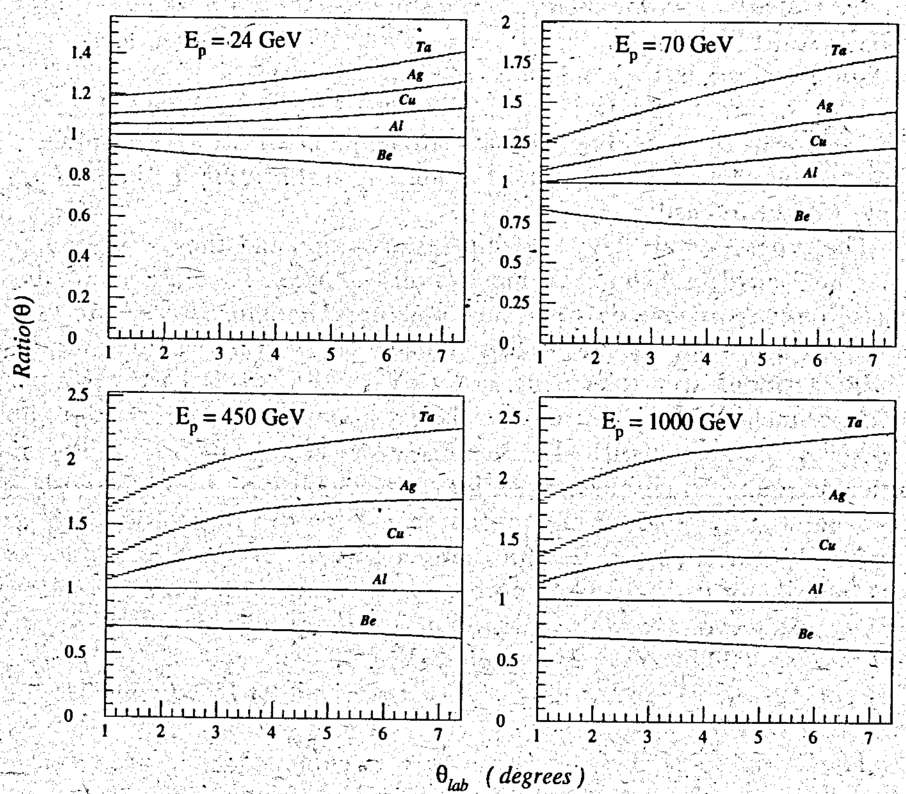

Figure 3: The ratio of $A_{2\pi}$ yields for the reactions $\mathrm{p}A \to A_{2\pi}X$ (A – Be, Cu, Ag, Ta) to $A_{2\pi}$ yield for $\mathrm{pAl} \to A_{2\pi}X$ as a function of the emission angle.

4. Testing of the inclusive single particle $\pi^{\pm}$ yield description

Quality of the Lund description of single particle yields was studied by its comparison with experimental results [16] for pA interactions (A – Be, Al, Cu, Pb) at proton energy of 24 GeV and for π meson laboratory emission angles θ in the interval from 17 to 127 mrad.

The experimental data [16] were obtained with an accuracy of $\pm 15\%$. The errors in the experimental data arose mainly from uncertainties in the spectrometer acceptance and the absolute calibration of the primary proton beam intensity.

The experimental $\pi^{\pm}$ yields for pAl interaction and the corresponding calculated yields are shown in Fig.4 as an example. The discrepancy between the experimental data and the Lund model does not exceed $15 \div 20\%$, as a rule. The same conclusion can be made for Be, Cu and Pb.

In the momentum region $p_{\pi} < 4$ GeV/c at $E_{\rm p} = 24$ GeV there are no experimental points. So for the whole π–momentum interval the Lund model was compared with the phenomenological formula [17]. This empirical formula was obtained by fitting all accelerator data on pp collisions available at that time for the proton energy interval $6.6 \div 1500$ GeV in the lab system and $p_{\perp} = 0.2$, 0.4 and 0.8 GeV/c. To get an accuracy of this formula for $p_{\perp} < 0.2$ GeV/c we used the experimental data on differential cross sections of $\pi^{\pm}$ meson production in pp-collisions: [18] — data compilation, [19] — $\sqrt{s} = 23$ and 45 GeV, [20] — $p_{\rm p} = 12.45$ GeV/c, [21] — $p_{\rm p} = 19$ and 24 GeV/c, [22] — $p_{\rm p} = 18.8$ and 23 GeV/c and [23] — $p_{\rm p} = 12.2$ GeV/c.

The formula describes the data with an accuracy of 20% and occasionally 40% except the data [22] that are described worse at 0°.

To apply the formula to a proton-nuclear interaction the $\pi^{\pm}$ yields calculated by the formula for pp collisions were multiplied by normalizing coefficients $K(\theta)$ which were obtained from $\pi^{\pm}$ yields calculated by Lund for each of pBe, pAl, pCu, pPb reactions. Then these yields were compared with the Lund yields. This comparison shows that, depending on p_{π}, the difference between the shapes of the $\pi^{\pm}$ yields calculated by the formula and by Lund for the pA interaction is not larger than 10%.

So the Lund model describes the $\pi^{\pm}$ yields for pA interactions at $E_{\rm p}$=24 GeV with an accuracy of $\sim 15\%$ in the θ region $17 \div 87$ mrad and $p_{\pi} < 10$ GeV/c. The Lund predictions in the region $\theta < 1°$ have not been tested

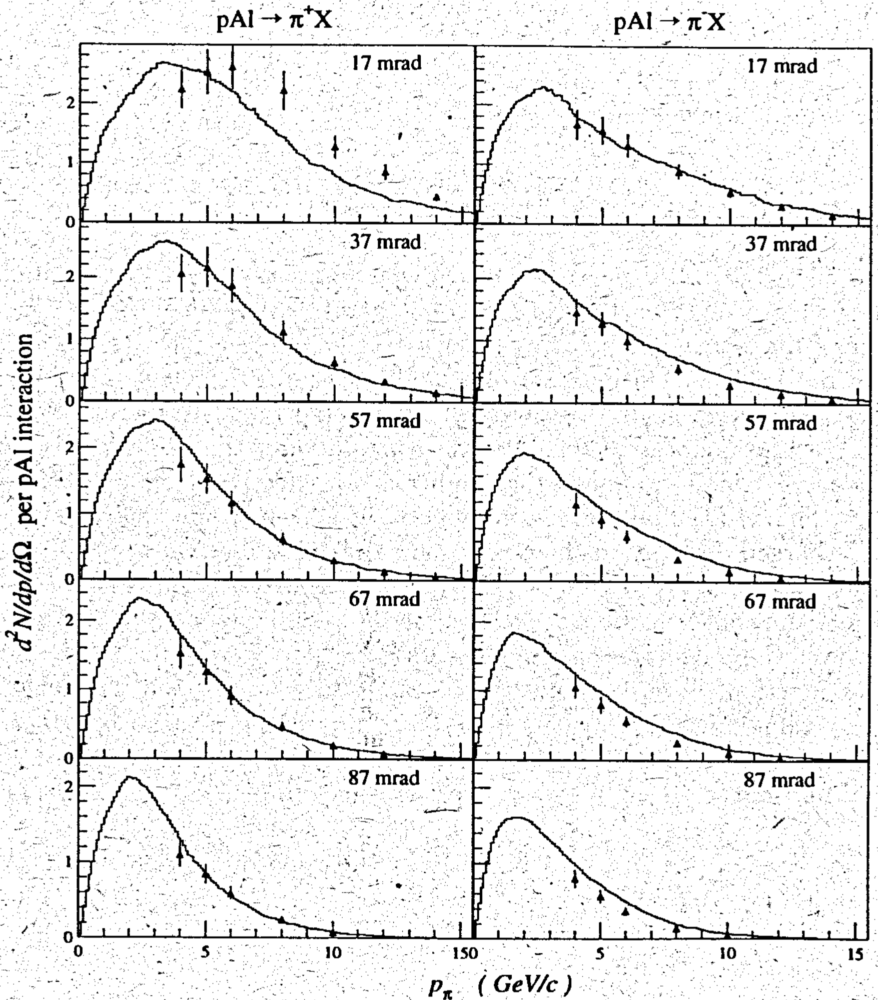

Figure 4: The inclusive single π^{+} and π^{-} yields for pAl interactions obtained by Lund (line) in comparison with the experimental yields [16] for the lab emission angles 17, 37, 57, 67, 87 mrad.

in this paper.

The Lund string model should work more properly at higher energies, so the accuracy under those conditions should be better.

Since uncertainties in single particle cross sections lead to squared errors in the double particle cross sections, the $A_{2\pi}$ yield accuracy obtained in the present paper is about 35%.

5. Conditions of $A_{2\pi}$ detecting

To select the proton energy and the dynamic range of the experimental setup it is nessesary to take into account many factors affecting performance of the setup.

To reach high counting rate of the atomic pairs the yields of pions from $A_{2\pi}$ break-up should be large in the selected momentum and angle ranges. Atomic pairs are detected on the background of the free pairs of the secondary particles. So one of the conditions for effective $A_{2\pi}$ registration is a maximum of the ratio of the number of pions from atomic pairs to the flux of charged particles in the momentum and angle intervals.

The ratio of yields of $A_{2\pi}$ to the sum of inclusive π^+ and p is shown in Fig.5. It would be more acceptable if the measurement range of the setup over the angle and the momentum corresponded to greater values of this function. However this range should also correspond to higher absolute values of the pion yields from $A_{2\pi}$. Also, from consideration of the conditions we should exclude the region with low momenta $p_\pi < 1$ GeV/c where multiple scattering, which adversely affects detector resolutions, is larger.

One can see from Fig.1 and Fig.5 that more appropriate ratios for $A_{2\pi}$ registration are at E_p=450 and 1000 GeV.

We also note that for a heavy target nucleus the ratio increases up to 25% with a small change in the form.

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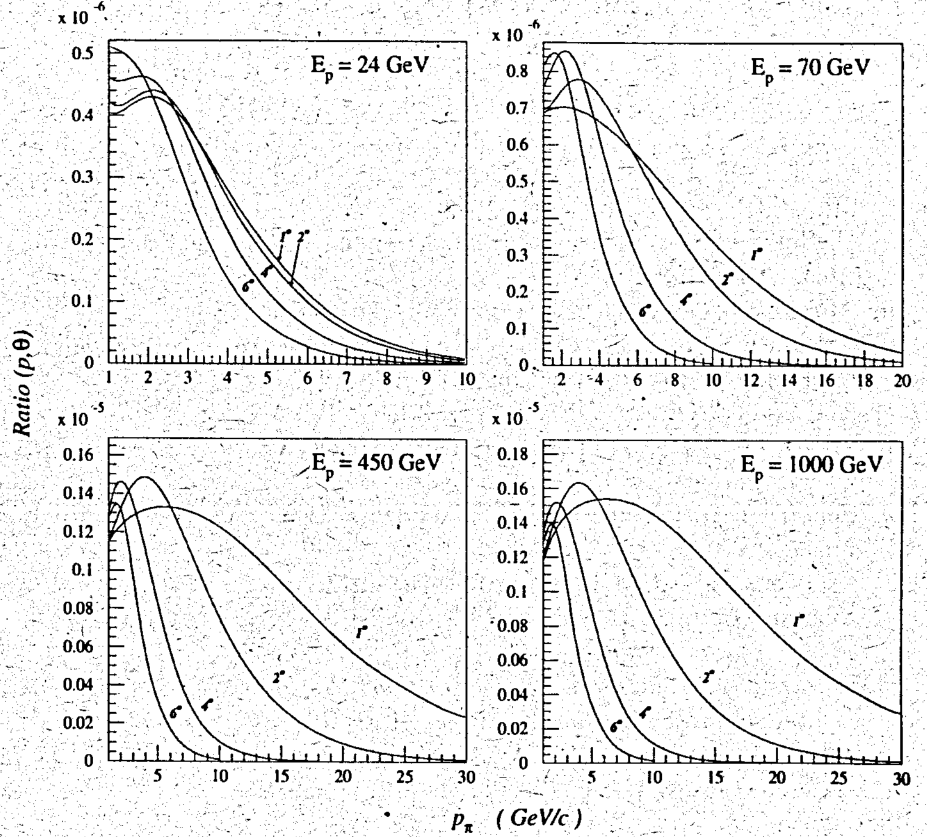

Figure 5: The ratio of yields of $A_{2\pi}$ to the sum of π^+ and p for the lab emission angles 1°, 2°, 4° and 6°.

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