СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E1-95-402

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THE SYSTEMATIC ERRORS IN Γ_1^d DUE TO UNCERTAINTIES IN THE STRUCTURE FUNCTION $R(x,Q^2)$

1995

1 Introduction

According to the quark - parton model hadron are composed of quarks, spin 1/2 fermions with fractional charge carrying the quantum numbers of the parent particle, and gluons, neuteral spin 1 gauge bosons. Deep inelastic polarization experiments complete our understanding of the internal dynamics of the nucleon, as they probe the spin distribution of the constituent quarks. The spin dependent structure function $g_1(x, Q^2)$ is extracted from the measurement of the spin-dependent asymmetry according to the expression:

$$g_1(x, Q^2) = \frac{A(x, Q^2)}{D} \cdot \frac{F_2(x, Q^2)}{2x \cdot (1 + R(x, Q^2))}, \quad (1)$$

where $A(x, Q^2)$ is the measured lepton-nucleon asymmetry, D is the kinematical factor, $F_2(x, Q^2)$ is the spin independent structure function and $R = \sigma_L/\sigma_T$ is the ratio of longitudinal and transverse double differential photo-absorbtion cross sections.

The quark-parton model interpretation of the nucleon structure functions allows to formulate a considerable numbers of so-called sum rules based on quite intuitive physical grounds. These sum rules are expressed in the form of integral relations between the structure functions and their verification constitutes an important test for the theory.

The most famous spin dependent sum rule was obtained by Bjorken [1] and relates the integral of the spin dependent structure function $g_1(x)$ to the ratio of axial to vector weak coupling constants in nucleon beta decay g_A/g_V. In the scaling form it reads :

$$\int_0^1 [g_1^p(x) - g_1^n(x)]dx = \Gamma_1^p - \Gamma_1^n = \frac{1}{6}|g_A/g_V|,$$

where Γ_1 was called *the first moment value.* In this paper we estimate the error in the first moment value caused by the uncertainty in the knowledge of the function R. To apply our calculation we use SMC 1992 deuteron data.

SMC has measured asymmetry $A_1^d(x, Q^2) = A^d(x, Q^2)/D$ in kinematical range $0.006 \leq x \leq 0.6$ and $1.2 \leq Q^2 \leq 15.5 GeV^2$ [2]. To calculate the $g_1^d(x)$ and the first moment value SMC has used NMC parametrization of the structure function F_2^d [3] and the SLAC global fit result for function R [4]. The calculation is done at average $Q^2 = 4.6\ GeV^2$ of the SMC experiment [2].

The total systematic error of Γ_1^d value is combined from the uncertainties of the measurement and errors of the used parametrizations ($R(x, Q^2)$ and $F_2(x, Q^2)$). To calculate the first moment value we have to know these function in the whole

x range. The error caused by the parametrization of $R(x, Q^2)$ is not dominate in the total systematic error of the SMC result (see [2]) but, as it is shown below, the influence of R is most important at small x region, that is the reason we study this source of systematic.

Table 1. *Experimental data on the function R.*

Experiment	x-range	Q^2-range GeV^2
SLAC	> 0.1	$0.5 < Q^2 < 20$
CDHSW	$0.015 < x < 0.65$	$0.19 < Q^2 < 194$
EMC	same region	same region
BCDMS	> 0.07	> 8.0

Data on the function R have been collected by SLAC, CDHSW, EMC and BCDMS collaborations [4,5,6,7], see Table 1 and Figure 1.
Table 1 and Figure 1 allow us to conclude that there are no experimental data on the structure function R(x,Q^2) in a small x region (which is covered by the SMC measurements) and existing measurements have a large errors. So, for the calculation of g_1^d at x<0.1 and x>0.8 it is necessary to rely on the extrapolations of existing measurements, using parametrizations. The most popular parametrization of R is the SLAC global fit [4] which is also shown in Fig.1 together with QCD predictions (R^{QCD} and R^{QCD+TM}). This plot shows the SLAC parametrization reasonably well represents experimental data but has a different x and Q^2 dependencies than QCD prediction.
Experimental data on R obtained at $Q^2 \approx 5GeV^2$, which is average value of the SMC measurements, are presented in Figure 2 as a function of x together with R^{1990}, which is our notation for the SLAC parametrization [4]. The SLAC recommended procedure for calculation of R has uncertainties ΔR(x,Q^2) which are shown in Fig. 2 by grey area. The comparison of ΔR with uncertainties of the experimental data and other parametrizations shows that ΔR is probably underestimated, especially in the small and large x regions.
From this we can conclude that:

1. Other than R^{1990} parametrisations can not been excluded by the existing data,
2. The uncertainties of the R^{1990} are probably underestimated,

3. Parametrisation R^{1990} at $x > 0.2$ has a flat x-dependence, presumably due to higher twist effects, presence of which was not confirmed by the EMC and BCDMS data at higher Q^2 [6,7].

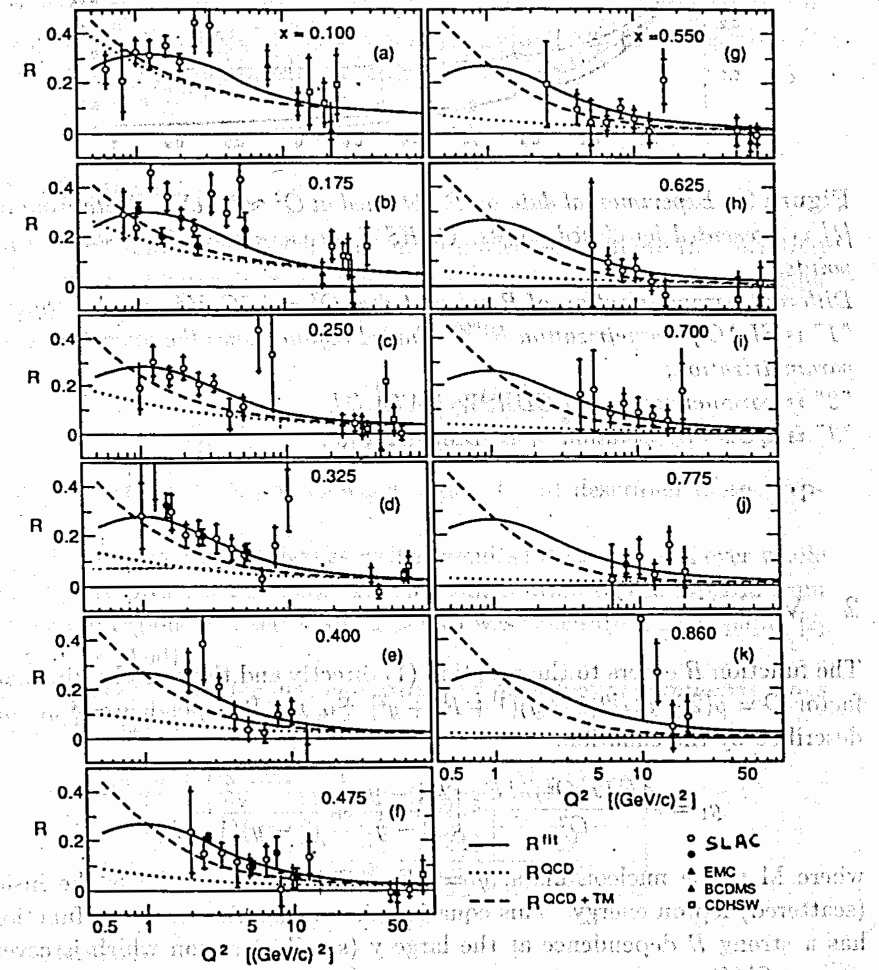

Figure 1. *Experimental data on R(x, Q^2) with statistical (inner part of the error bars) and systematic errors. The errors shown on all SLAC data do not include the ±0.025 systematic uncertainty due to radiative corrections. The solid curve represents SLAC global fit, the dotted and dashed curves represent calculation of R^{QCD} without [9] and with target mass correction [10], respectively.*

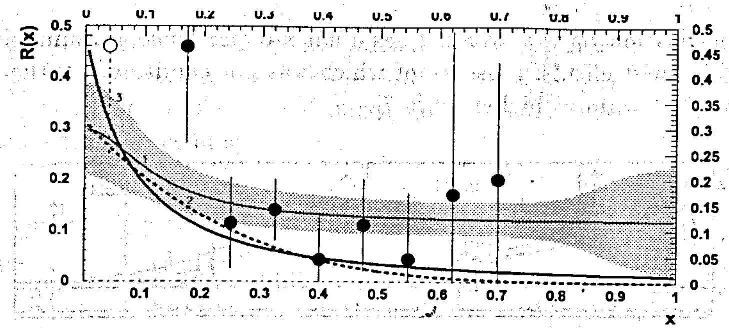

Figure 2. *Experimental data on R obtained at $Q^2 \approx 5GeV^2$. Data from SLAC [4] is presented by closed points, CDHSW measurement [5] is shown as open points.*
Different parametrization of R calculated at $Q^2 = 4.6GeV^2$ are also shown:
"1" is SLAC parametrization R^{1990}, shaded region shows the uncertainty of this parametrization;
"2" is parametrization of CDHSW DATA [5];
"3" is QCD extrapolation of BCDMS data [8].

2 Variation of g_1.

The function R enters to the equation (1) directly and through the kinematetic factor $D = y(2-y)/[2(1-y)(1+R)+y^2]$. So, the R-dependence of g_1 can be described by the equation:

$$g_1 = \frac{AF_2(x,Q^2)ME}{Q^2} \cdot \Big[\frac{2(1-y)}{2-y} + \frac{y^2}{(2-y)(1+R)}\Big], \qquad (2)$$

where M is the nucleon mass, $y = (E-E')/E$ and E (E') is the insident (scattered) lepton energy. This equation shows that the structure function g_1 has a strong R dependence at the large y (small x) region which is covering most of SMC measurements .
The structure function F_2 also dependes on R:

$$F_2(R) = \phi(\sigma, E, x, y) \cdot \Big[\frac{1}{1-y+y^2/2(1+R)}\Big], \qquad (3)$$

where function ϕ includes all kinematical factors. To examine R-dependence of F_2 we have calculated the ratio $F_2(R)/F_2(R \pm \Delta R)$ at different **x**. We found that :

- in the small x region $(0.01 < x < 0.1)$ variation of the function R to 20% leads to variation $\Delta F_2 < 1\%$;
- at moderate x $(0.1 < x < .4)$ variation $\Delta R = \pm 30\%$ leads to $\Delta F_2 < 0.5\%$ and at large x $(x > 0.4)$ variation $\Delta R = 100\%$ leads to $\Delta F_2 < 0.2\%$.

Therefore we assume in our analysis that the function F_2 is R-independent in the x range covered by SMC measurements [2] and eq. (2) fully describes R-dependence of the spin-dependent structure function g_1 [1].

3 Variation of Γ_1^d.

To be free of the small x extrapolation uncertainties we estimate the variation of Γ_1 value due to different parametrization of R evolving it over the x–region of SMC measurements 0.006<x<0.6 [2]

$$\Gamma_1' = \int_{0.006}^{0.6} g_1(x)dx = \int_{0.006}^{0.6} \frac{A_1(x)F_2(x)}{2 \cdot x(1+R(x))}dx.$$

Used parametrisations of R are shown in figure 1 and described in the **Appendix.**
Results for Γ_1' (see equation above) as well as results for the integral over whole x–region, Γ_1^d are presented in Table 2. The contribution to the integral from the unmeasured regions($x < 0.006$ and $x > 0.6$) was taken from SMC paper [2] and amounts to -0.001.

Table 2. *Values of Γ_1' and the first momentum Γ_1^d for different parametrisations of the function $R(x)$ at Q^2=4.6 GeV^2*

parametrisation	Γ_1'	Γ_1^d	$\Delta\Gamma'/\Gamma'_{SLAC}$
SLAC	0.0245	0.0235	—
CDHSW	0.0254	0.0244	4%
BCDMS	0.0265	0.0255	8%

4 Conclusion

The uncertainy of SLAC global fit [4] leads to the systematic error in the first moment value $\Delta\Gamma_1^d = 0.0005$ or about 2% (see [2]). According to our calculations the systematic uncertainties caused by another than SLAC parametriza-

[1] we do not discuss here the region $x < 0.01$, is studing at HERA , where the function F_2 has a strong x dependence and requires of the special consideration

tion of R could be about $\Delta\Gamma_1^d \approx 0.001 \div 0.002$.

5 Appendix

Here we describe parametrisations of R(x,Q^2) used in our calculations.

1. SLAC global fit [4]

$$R = \frac{R_a + R_b + R_c}{3}$$

$$R_a = \frac{a_1 \cdot \Theta(x, Q^2)}{ln(Q^2/.04)} + \frac{a_2}{\sqrt[4]{Q^8 + a_3^4}}$$

$$R_b = \frac{b_1 \cdot \Theta(x, Q^2)}{ln(Q^2/.04)} + \frac{b_2}{Q^2} + \frac{b_3}{q^4 + 0.3^2}$$

$$R_c = \frac{c_1 \cdot \Theta(x, Q^2)}{ln(Q^2/.04)} + \frac{c_2}{\sqrt{(Q^2 - Q^2_{thr})^2 + c_3^2}}$$

$$Q^2_{thr}=5\cdot(1-x)^5$$

$$\Theta = 1 + 12 \cdot \frac{Q^2}{Q^2+1} \cdot \frac{.125^2}{.125^2 + x^2}$$

Parameters of the fit a_i, b_i and c_i, where i=1,2,3 , see in Ref. [4]

2. Global fit of CDHSW data [5]

$$R = 1.5 \cdot \frac{(1-x)^4}{log(Q^2/\Lambda^2)}$$

where Λ=200 MeV.

3. QCD fit of BCDMS data [8] at Q^2=5 GeV^2

$$R = \frac{0.015}{(x + 0.149)^{1.85}}$$

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Received by Publishing Department
on September 18, 1995.