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THE EXPERIMENTAL TEST OF THE ADEQUATENESS OF RELATIVISTIC IMPULSE APPROXIMATION WHEN DESCRIBING THE LIGHTEST NUCLEI BREAK-UP

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Экспериментальный тест адекватности релятивистского импульсного приближения при описании фрагментации легких ядер

Проанализировано поведение сечений реакций фрагментации легких ядер под нулевым углом вблизи максимума. Показано, что имеет место асимметрия сечений относительно максимума, которая конфликтует с нерелятивистским импульсным приближением, но хорошо согласуется с одним из подходов релятивистского описания процесса.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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The Experimental Test of the Adequateness of Relativistic Impulse Approximation when Describing the Lightest Nuclei Break-up

The behaviour of the lightest nuclei break-up cross sections at zero angle have been analyzed in vicinity of the maximum. It is shown that asymmetry of cross sections relatively maximum is in conflict with nonrelativistic impulse approximation, but agrees well with one of relativistic approaches to describe this process.

The investigation has been performed at the Laboratory of High Energies, JINR.

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1 Introduction

Intensive attempts to find an adequate relativistic description of bound states have been undertook during last two decades by many authors. The majority of these investigations is based on classical paper of P.A.M. Dirac[1].

Study of the lightest nuclei disintegration in the wide momentum region of fragments needs to have a reliable theoretical base to clarify the structure peculiarities of investigated objects. Meanwhile, the developed relativistic approaches is not commonly accepted up to now. So, even in recent papers[2] the observed effects are analyzed assuming that fragment momentum in the deuteron rest frame is the argument of the wave function. The C.F.Perdrisat's and V.Punjabi's report at Dubna symposium[3] witnesses that connection between observed and internal momenta of fragments in nuclei are a subject of discussion up to now.

In this paper we try to clarify this question analyzing the published experimental data on disintegration of the deuteron[4], ^3He[5] and ^4He[6].

2 The Relativistic Impulse Approximation

Here we review briefly one of the version of relativistic impulse approximation (RIP)[7] that is used to describe lightest nuclei break-up at zero angle. We restrict ourself by consideration of strictly collinear particular case.

Let us define the fragmenting nuclear, the fragment-spectator and the active fragment as A, s and f respectively. The projectile, disintegrating A, let be P. The momentum q of s in the A rest frame, opposite to P one, we define as positive.

Let us consider the disintegration process as

$$P + A \to X + sf \quad , \quad sf \to s + f. \tag{1}$$

We have

$$M_{sf} = \varepsilon_s + \varepsilon_f, \tag{2}$$

where ε_s and ε_f are energies of s and f in the $s+f$ rest frame, M_{sf} is effective $s+f$ mass.

We can introduce a Lorentz-invariant variable α, defined as

$$\alpha = \frac{E_s + p_s}{E_{sf} + p_{sf}}, \tag{3}$$

where E_i and p_i are energies and 3-momenta of s and $s+f$. In the center of mass $s+f$ we have

$$\alpha = \frac{\varepsilon_s + k}{M_{sf}}, \tag{4}$$

where k and $-k$ are 3-momenta of s and f in this system. At $k \to \pm\infty$ we have

$$\varepsilon_s \simeq \varepsilon_f \simeq |k|,$$

and, substituting (2) into (4), one can see that α changes from 0 to 1, when k changes from $-\infty$ to $+\infty$. Inverting (4) one can easily obtain

$$M_{sf} = \frac{m_s}{\alpha} + \frac{m_f}{1-\alpha}. \tag{5}$$

We also introduce α' by

$$\alpha' = \frac{E_s + p_s}{E_A + p_A}, \tag{6}$$

which in the A rest frame have a form

$$\alpha' = \frac{E_q + q}{m_A}, \tag{7}$$

where $E_q = \sqrt{m_s^2 + q^2}$.

Consideration of a relativistic bound state on the light cone $p_z + t = 0$ leads to equation $\alpha = \alpha'$. In this approach the variable k is implied as "internal momentum" of the relativistic bound state. The relation between observed momentum q and "internal" one k has a form

$$\frac{E_q + q}{m_A} = \frac{\varepsilon_s + k}{M_{sf}}. \tag{8}$$

Using introduced variables one can obtain

$$k = \left(\alpha - \frac{1}{2}\right) M_{sf} - \frac{m_s^2 - m_f^2}{2M_{sf}}, \tag{9}$$

where α is defined in (6) or (7), and M_{sf} in (5). We also mention here well known expression for the fragment momentum in the compound system (which, however, does not remember the momentum direction):

$$k^2 = \frac{\lambda(M_{sf}^2, m_s^2, m_f^2)}{4M_{sf}^2}, \tag{10}$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc.$$

The Jacobian between q and k has a form:

$$\frac{dk}{dq} = \frac{\varepsilon_s \varepsilon_f}{E_q M_{sf}(1-\alpha)}. \qquad (11)$$

In addition to the reasoning in favor of k as more preferable candidate to be an argument of the wave function, which are discussed in refs.[7]), we mention the following one, that is out of field of vision in case one considers the deuteron fragmentation only.

Substituting the extreme value of $\alpha = 1$ into (7) one can immediately obtain the absolute restriction of q:

$$q_{max} = \frac{m_A^2 - m_s^2}{2m_A}. \qquad (12)$$

Overcoming the strange circumstance that the wave function argument can be restricted we just meet another one. If we consider the dissociation of A into two fragments with different masses we reveal that kinematical limit is different for each of them. These different limits take place at any collision energy including infinite one. For example, if we study the

$$^4He \rightarrow t + p$$

reaction, we find that the absolute momentum restriction for protons is 1.74 GeV/c, but for tritons is only 0.81 GeV/c. Using k, we escape not only the problem of restricted argument of the wave function, but in this case the extreme value of k at any collision energy is the same for each from two fragments with different masses. Indeed, at an arbitrary collision energy of the reaction (1) we have the restriction for M_{sf}, defined by

$$M_{sf} + m_P \leq \sqrt{s}.$$

Then we find k through expression (10) which is symmetrical relatively an order of its arguments.

But not all problems we escape using this approach. If we consider a kinematic of a real break-up process we find that only at $t = 0$, where $t = (P_{sf} - P_A)^2 = (P_X - P_P)^2$, we have condition (in the A rest frame)

$$E_{sf} + p_{sf} = m_A \qquad (13)$$

and, hence, the relation (8) is valid.

When $s \gg m_X$, M_{sf} in the process (1) the squared 4-momentum transfer t can be expressed as

$$t \simeq -\frac{(M_{sf}^2 - m_A^2)(m_X^2 - m_P^2)}{s} - \frac{m_P^2(M_{sf}^2 - m_A^2)^2}{s^2} \qquad (14)$$

where $s = (P_N + P_A)^2$. Only second member (t_{min}) in (14) survives at $m_x = m_P$. When M_{sf}, m_X are arbitrary, but fixed, we have $t \rightarrow 0$ at $s \rightarrow \infty$.

In a real inclusive experiment one observes only s parallel (or antiparallel) to P (in the A rest frame), and so, X and f can have perpendicular part of their momenta. In this case we have $-t' > -t_{min}$. We, hence, must suppose that effective $|t_{eff}|$ does not exceed too much $|t_{min}|$ when considering equation (8) as more or less adequate. At fixed collision energy t_{min} increases when q (or k) increases. But the larger the collision energy, the wider the region of q, k, when t_{min} is negligible.

In this paper we deal with the region of q where $|t_{min}| < 3 \cdot 10^{-4}\ GeV^2$. We analyze the asymmetry of cross sections in vicinity of maximum that is different whether old or modified description more adequate. If we neglect by double scattering effects that are negligible in vicinity of maximum, than relation between the cross section and wave function (for the deuteron) in the framework of nonrelativistic Impulse Approximation (NIA) has a following form[8]:

$$\frac{d^3\sigma}{dq^3} = C \cdot |\Psi(q)|^2. \qquad (15)$$

By analogy, in the framework of the RIA we write

$$\frac{d^3\sigma}{dk^3} = C \cdot |\Psi(k)|^2,$$

and, using the standard representation of cross sections, we have[9]

$$E_q\frac{d^3\sigma}{dq^3} = E_q\frac{d^3\sigma}{dk^3}\,\frac{d^3k}{dq^3} = C \cdot \frac{\varepsilon_s\varepsilon_f}{(1-\alpha)M_{sf}}|\Psi(k)|^2. \qquad (16)$$

Here $E_q \cdot d^3\sigma/dq^3$ is the invariant cross section, $dk_\perp = dq_\perp$, $dk_\parallel/dq_\parallel$ is defined in (11). For the description of ^3He and ^4He break-up we use denotation $n_s(k)$ instead of $|\Psi(k)|^2$ (overlapping integral between nuclear and fragment wave functions).

3 The experimental test

To investigate the difference between these two approaches at small q, k let us derive approximate but clear relations between q and k, when they are small. Taking into account only first order members on q in(11) we have

$$\frac{dk}{dq} \simeq \frac{m_s m_f m_A}{m_s(m_s + m_f)(m_A - m_s - q)}$$

$$\simeq \frac{m_f m_A}{(m_A+\varepsilon)(m_f-\varepsilon-q)} \quad (17)$$
$$\simeq \left(1+\frac{m_s}{m_A m_f}\varepsilon+\frac{1}{m_f}q\right),$$

where $\varepsilon = m_s + m_f - m_A$ is a binding energy. Neglecting also by the binding energy we have

$$\frac{dk}{dq} \simeq 1+\frac{1}{m_f}q. \quad (18)$$

Finding the zero order member by substitution of $q = 0$ into (8) and integrating (17) we obtain

$$k \simeq \frac{m_s}{m_A}\varepsilon + \left(1+\frac{m_s}{m_A m_f}\varepsilon\right)q + \frac{1}{2m_f}q^2 \quad (19)$$

The accuracy of finding of absolute value of q during measurements was within several MeV. It is not sufficient to evaluate the first member in (19). So, the position of maximum was assumed to be equal 0 in both approaches. To evaluate difference from 1 of the second member we need to know exactly the wave function, but that is a subject of investigation. And, finally, the third member in (19) is a source of asymmetry between q and k. When q is positive, k is growing faster than q, but in the region of negative q we have the controversial situation. Proceeding from $|\Psi(k)|^2$ is unconditionally symmetrical and have a close to gaussian-like behaviour, we rewrite (15) in form

$$E_q\frac{d^3\sigma}{dq^3} \simeq C\cdot E_q\cdot exp(-Bq^2)$$
$$\simeq C\cdot E_q(1-Bq^2+o(q^4)) \quad (20)$$

and (16) in form

$$E_q\frac{d^3\sigma}{dq^3} \simeq C\cdot E_q\left(1+\frac{1}{m_f}q\right)exp\left[-B\left(q+\frac{q^2}{2m_f}\right)^2\right]$$
$$\simeq C\cdot E_q\left(1+\frac{q}{m_f}-Bq^2-\frac{B}{m_f}q^3+o(q^4)\right) \quad (21)$$

Comparing (20) and (21) one can see, that at $q > 0$, if the RIA is better, the cross section, due to member, proportional to q in (21), goes upper at smallest q and later, due to proportional to q^3 member, lower than in case the NIA is preferable. And we have the controversial picture at $q < 0$. Such a difference in the behaviour of cross sections is not negligible and can be tested experimentally.

One can see from (21) the lesser m_f the asymmetry between k and q is stronger. Therefore we present here, apart from the $d \to p+X$, $^3He \to d+X$ and $^4He \to t+X$ reactions as more sensitive in comparison with the $^3He \to p+X$ and $^4He \to p+X$ ones. The latter ones indeed do not show such an asymmetry as those we have selected to demonstrate. In figs. 1,3,5 it is shown $n_s(q)$ extracted from data using the relation (15). The points with different signs of argument are presented by different symbols. It is seen that in all three reactions the points with negative arguments are appreciably higher at largest values of $|q|$. Due to a number of reasons it is naturally to expect increasing of background in the region of negative q, that could explain marked discrepancy of cross sections at different signs of q. But it is difficult to imagine, that this increasing have exactly such a character to provide symmetrical behaviour of $n_s(k)$ (Figs. 2,4,6), extracted using RIA formula (16). This evaluation is free of any model of a wave function.

To have more contrast picture let us present data in form

$$\frac{d^3\sigma/dq^3}{f^2(q)},$$

where parameters A_i, B_i of

$$f(x) = A_1 exp(-B_1x^2) + A_2 exp(-B_2x^2) \quad (22)$$

is fitted. In this representation we expect the points distribution along the y=1 line if the NIA is a good approximation, and near more complicated curve if the RIA is more adequate. This representation (Figs. 7,8,9) allows one to distinguish not only the asymmetry at $|q| > 0.1$ GeV/c, but also one of opposite sign at $|q| < 0.05$ GeV/c. As was mentioned above, both of these asymmetries are evident when one compares approximate formulas (20) and (21).

The approximation of data by function (22) with free parameters using RIA or NIA approach gives, respectively, the following χ^2: 16/14 and 54/14 for the $d \to p+X$ reaction; 16/27 and 35/27 for the $3He \to d+X$ reaction; 26/38 and 50/38 for the $4He \to t+X$ reaction.

4 Conclusion

It is shown that precise measurements of lightest nuclei break-up cross section in vicinity of maximum is a good test to choose more adequate approach.

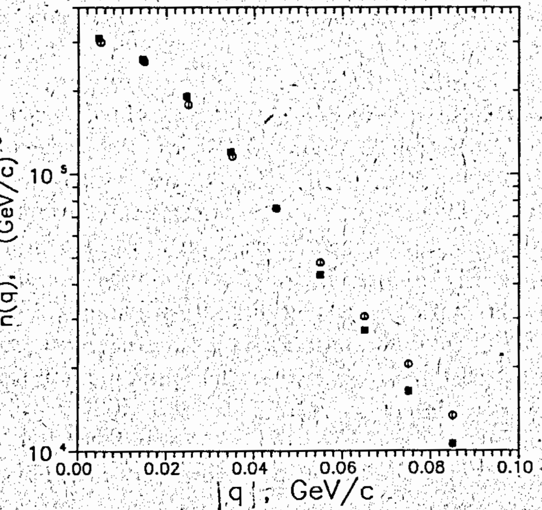

Figure 1: The proton momentum distribution in the deuteron, extracted from cross sections in the framework of NIA. Open points are points with negative values of q.

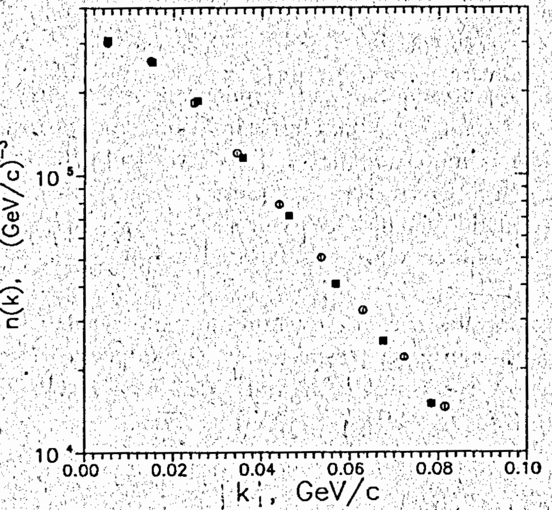

Figure 2: The proton momentum distribution in the deuteron, extracted from cross sections in the framework of RIA. Open points are points with negative values of k.

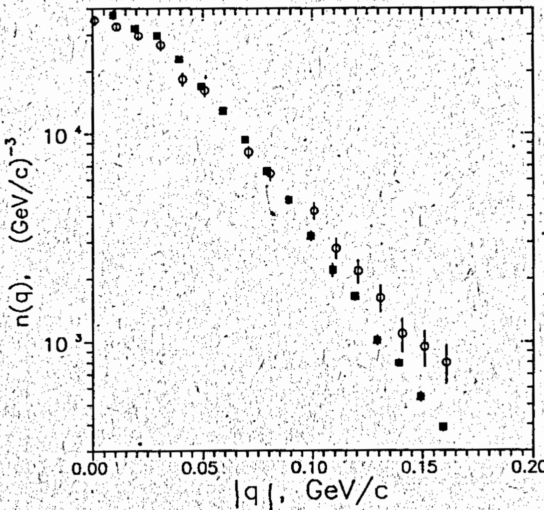

Figure 3: The same as in fig.1, but for $^3He \to d$ reaction.

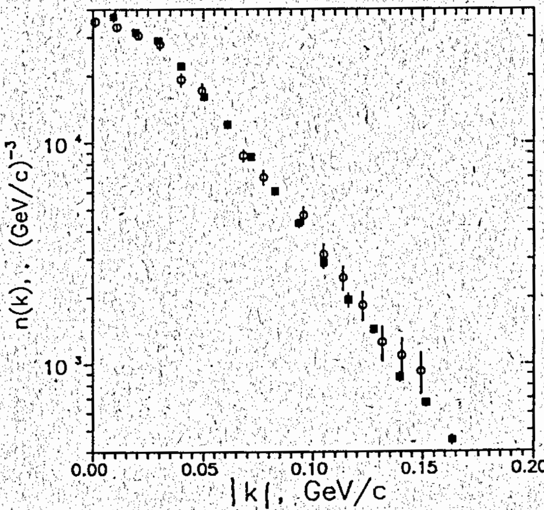

Figure 4: The same as in fig.2, but for $^3He \to d$ reaction.

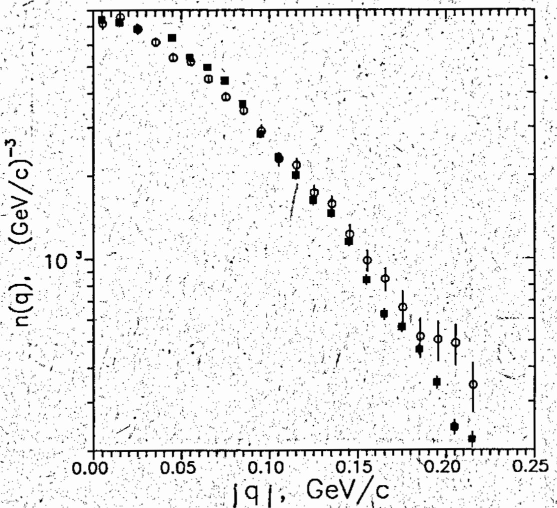

Figure 5: The same as in fig.1, but for $^4He \to t$ reaction.

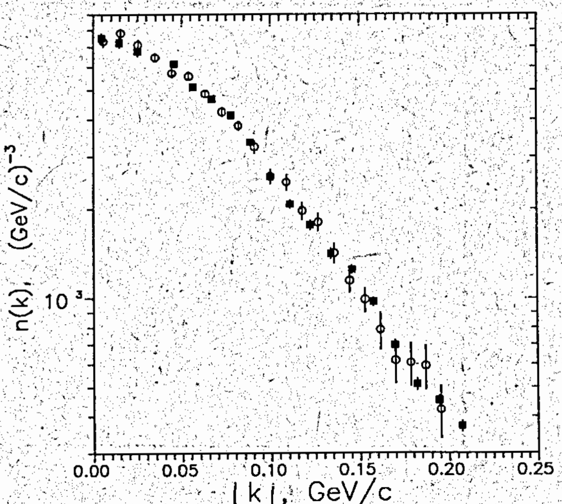

Figure 6: The same as in fig.2, but for $^4He \to t$ reaction.

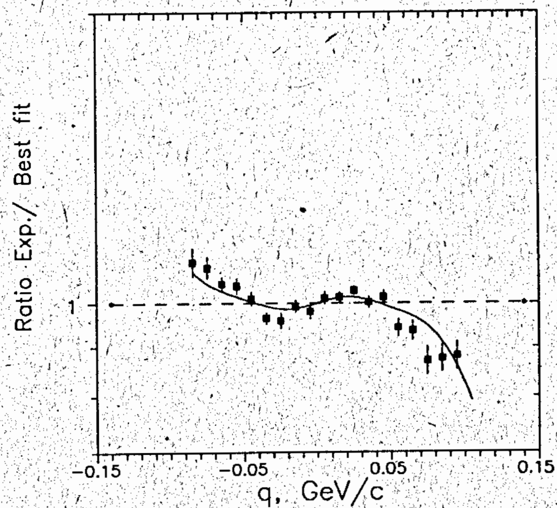

Figure 7: Ratio of the $d \to p + X$ cross sections to fitted function. The solid line is the expected behaviour in the framework of RIA. The dashed line is the expected behaviour in the framework of NIA.

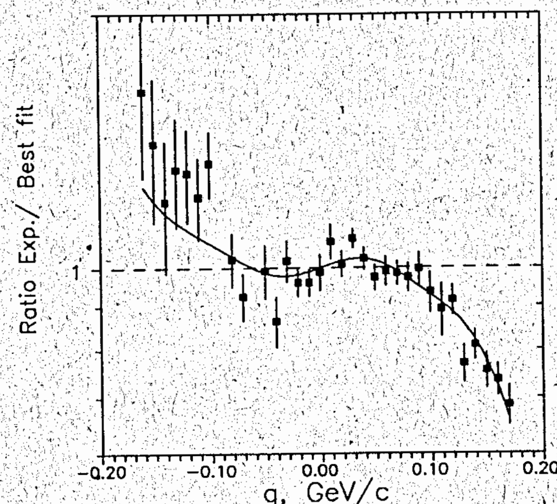

Figure 8: The same as in fig.7, but for $^3He \to d$ reaction.

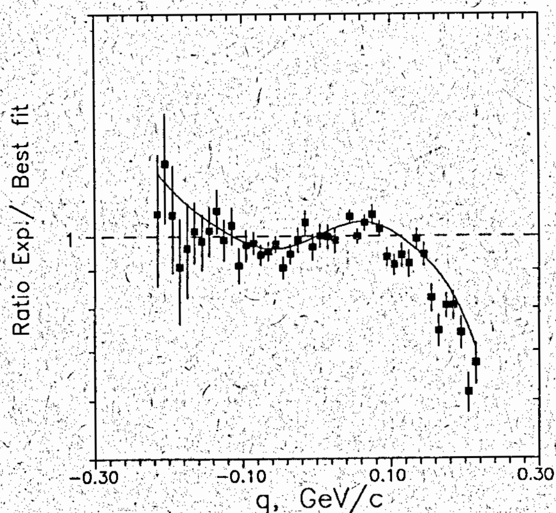

Figure 9: The same as in fig.7, but for $^4He \to t$ reaction.

The observed forward-backward asymmetry points out the considered version of relativistic impulse approximation is more adequate than nonrelativistic one. Such an asymmetry was also observed in ref.[10]. Unfortunately, the importance of precise measurements of cross sections in the region of negative values of q was not realized during carrying out of experiments. Therefore, the data at $q < -0.1$ GeV/c whether are absent (in case of the $d \to p + X$ reaction) or not sufficiently statistically accurate. The considered asymmetry is the more strong the more sharp behavior of cross section in vicinity of maximum. Taking into account this circumstance, the $d \to p + X$ reaction is most suitable to carry out the more reliable experimental test. Such kind of investigation is also possible when measuring polarization observables of this process.

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