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OF HIGH ENERGY γ -QUANTA WITH
A TOTAL ABSORPTION SPECTROMETER

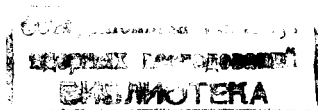
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**THE MEASUREMENT OF THE COORDINATÉS
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Submitted to "Nuclear Instruments and Methods"



Ten years ago when we proposed an idea of spectrometer, afterwards called a "Cherenkov mass-spectrometer", it was difficult to predict the future of this new technique ^{/1-3/}.

The Cherenkov mass-spectrometer was proposed to investigate the resonances in the systems of electromagnetic particles, i.e., electron and positron or two and more γ -quanta ^{/4,5/}.

The operating principle of the Cherenkov mass-spectrometer is based on the possibility of measuring simultaneously with high accuracy both the directions of particles (with spark chambers) and their energy (with total absorption γ -spectrometers).

A mass-spectrometer of this type belongs to those devices which permit the effective resonance mass to be measured in a direct way. The last factor is of decisive importance in investigating the processes in which there is no recoil particle, as in decay reactions of unstable particles, experiments on colliding beams, nucleon-anti-nucleon pair annihilation, etc. The possibility of identifying each event with the mass-spectrometer, similar to that which takes place in bubble chambers is of fundamental importance in investigations of rare processes.

The second generation of the Cherenkov mass-spectrometers, characterized by increasing the number of spectrometric channels more than 100 using filmless readout chambers and application of "on-line" computers, enables the possibilities of the method to be extended considerably ^{6-8/}.

However, these advantages could not get rid of a variety of essential defects.

In fact, in order to detect γ -quanta by spark chambers, it is necessary to use converters whose total thickness is limited by the requirement for high energy resolution^{/9/}. For γ -quanta with energies up to 10 GeV this thickness does not exceed one radiation unit. In this case the conversion efficiency of the mass-spectrometer is $\sim(0.5)^n$, where n is the number of γ -quanta. Large errors in measuring the decay angles of γ -quanta are due to bremsstrahlung of conversion pairs and multiple scattering.

It should be noted that spark chambers in similar devices have large longitudinal dimensions (about 100 m and more) and dead time (of the order of tens ms) which significantly deteriorate the geometric efficiency and fast operation of the mass-spectrometer.

These disadvantages make the problem of construction of a γ -mass-spectrometer without spark chambers of great interest. This problem is complicated because the coordinates of particles should be measured with accuracy comparable to that achieved by means of modern chamber technique.

In this paper we consider one of the possible solutions of this problem - a cell structure γ -spectrometer consisting of a large number of independent spectrometers-modules of small transverse dimensions.

The first question which should be answered can be formulated as follows: what transverse dimensions must the module of the γ -spectrometer have in order to measure the γ -quantum coordinates with an accuracy of ~ 1 mm ?

The experimental data on the transverse electromagnetic shower development for electrons with the energy $E=1$ GeV^{/10/} show that the radius of the shower is weakly dependent on the energy and is equal approximately to $1 X_m^*$, where X_m is a Moliere unit. If the transverse dimensions of the spectrometer module satisfy the condition

$$D_m/D_s < 1, \quad (1)$$

* In the spectrometer $2 X_m$ in diameter about 90% of the shower energy is absorbed.

where D_m and D_s are correspondingly the module and electromagnetic shower diameters, the shower energy will be distributed in the module group. The analysis of the energy distribution between the modules in the group permits, in principle, the γ -quantum coordinate to be localized by measuring the shower center of gravity. However, a detailed consideration of the problem indicates that in addition to condition (1) it is necessary to fulfil the second condition: The energy resolution of the spectrometer and hence of each module must be sufficiently high so that the energy difference in a module may exceed measurement errors due to small shifts of the shower axis. Figure 1 shows the energy distribution of the shower ($E_\gamma = 4$ GeV) in the modules 1.2 r.l. in diameter for the distances from the module center 6 mm and 9 mm , respectively*.

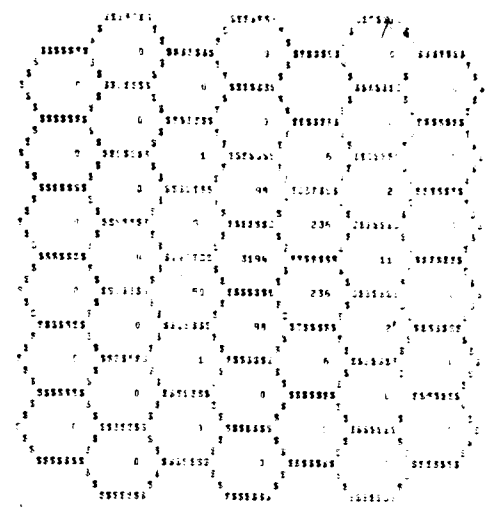
The Monte-Carlo method was used for quantitative estimates. This method made it possible to determine errors in measuring the γ -quantum coordinates ($\Delta X, \Delta Y$) in the spectrometer versus

- 1) the module size " D_m ";
- 2) the coordinates " X, Y " of the γ -quantum entry point on the spectrometer;
- 3) the γ -quantum incident angle " θ " (angle between the γ -quantum direction and the module axis);
- 4) the energy resolution " $\Delta E/E$ " of the γ -spectrometer.

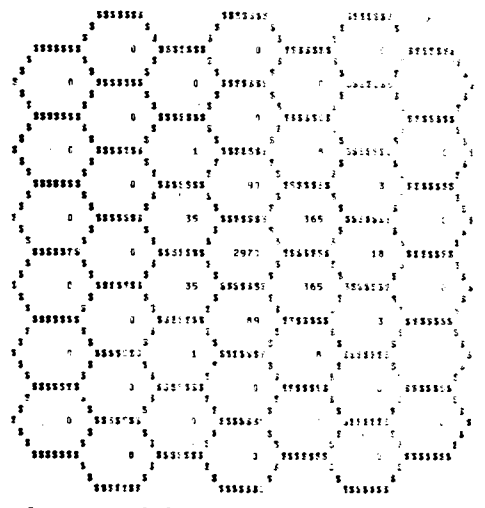
The Monte-Carlo calculations were made for the γ -spectrometer which comprises 45 modules of hexagonal shape (see *fig. 1*).

The experimental data on the longitudinal and transverse energy distribution of the electromagnetic shower obtained in ref.^{/10/} for 1 GeV electrons were used in the Monte-Carlo calculations. The data on the radial and longitudinal distribution of the shower energy for 11 values of t , where t is the matter thickness in radiation lengths along the γ -quantum trajectory, were used in the program. The values of t used in the program are listed in the table.

* Here and then 1 r.l. = 25 mm .



a)



b)

Fig. 1. The scheme of the γ -spectrometer. The energy distribution of γ -quanta (MeV) in the modules ($E_\gamma = 4000$ MeV, $D_m = 1.2$ r.l.) when a) the shower axis is shifted relative to the module axis (X axis) by 6 mm; b) the shower axis is shifted relative to the module axis by 9 mm.

Table

t rad.	0.43	0.87	1.74	2.61	3.47	4.34	5.21	6.95	8.68	10.42	13.90
lengths											

Each radial curve is represented by 20 points with a 0.1 r.l. step in the interval of t_R from 0 to 2 r.l. The energy E_i , absorbed by the shower in the module, is calculated by integration if the longitudinal and radial shower distributions are taken into account. Energy errors are introduced into the program by accidental numbers distributed by the Gaussian law with a mean value of E_i and dispersion $(\Delta E_i)^2 (\Delta E_i = \text{const} \sqrt{E_i})$. The particle coordinates in the spectrometer are calculated by the formula

$$(X, Y) = \sum_i (X_i^c, Y_i^c) \cdot w_i, \quad (2)$$

where X_i^c, Y_i^c are the center coordinates of an i -th module ($i = 1, 2, \dots, 45$).

$$w_i = E_i / E_t \quad \text{or} \quad (2a)$$

$$w_i = \exp[B(E_t - E_i) / E_t] \cdot E_i / E_t \quad (2b)$$

and $E_t = \sum E_i$; "B" is a constant which depends on the module radius ($1.5 \leq B \leq 1.7$).

By means of the described program we calculated errors in measuring the coordinates $\Delta X, \Delta Y$ of the entry points of γ -quanta on the spectrometer versus the energy resolution. Figure 2 gives these results for the modules 1.2, 1.6 and 2.0 r.l. in diameter. As is seen from fig. 2, $\Delta X, \Delta Y$ have a minimum for the module 1.2 r.l. in diameter and are equal to ± 0.4 mm if $\Delta E / E = 5\%$.

The results in fig. 2 concern the case when the incident angle of γ -quanta on the spectrometer surface is equal to zero, and the coordinate corresponds to $X_0 = Y_0 = 0.5 R_m$, where R_m is the module radius.

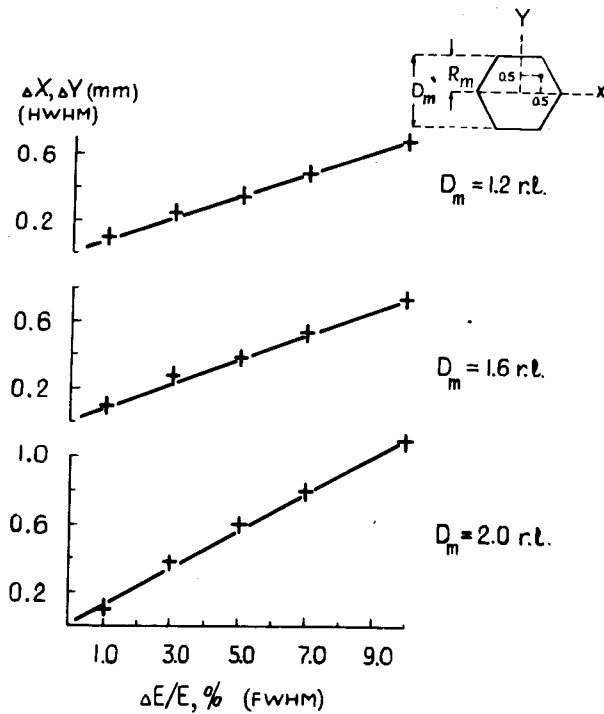


Fig. 2. The errors in measuring the coordinates ΔX , ΔY (HWHM of the distributions of the centers of gravity) calculated by formulae (2) and (2a) of the γ -quantum entry point on the spectrometer vs the energy resolution $\Delta E/E$ for the modules 1.2, 1.6 and 2.0 r.l. in diameter; $X_0 = Y_0 = 0.5 R_m$, where R_m is the module radius.

In fig. 3 the values of $\Delta X, \Delta Y$ calculated for $\Delta E/E = 3\%$ vs the distance to the module center for three directions are given. One can see from fig. 3 that moving away from the module center the absolute values of $\Delta X, \Delta Y$ decrease due to the increase of the fraction of energy distributed between the neighbouring modules. Errors in measuring the coordinates increase with increasing the module diameter.

Figure 4 illustrates the systematic errors, i.e., the shift of the distribution maximum of the shower gravity centers to the real shower center, arising in the calcu-

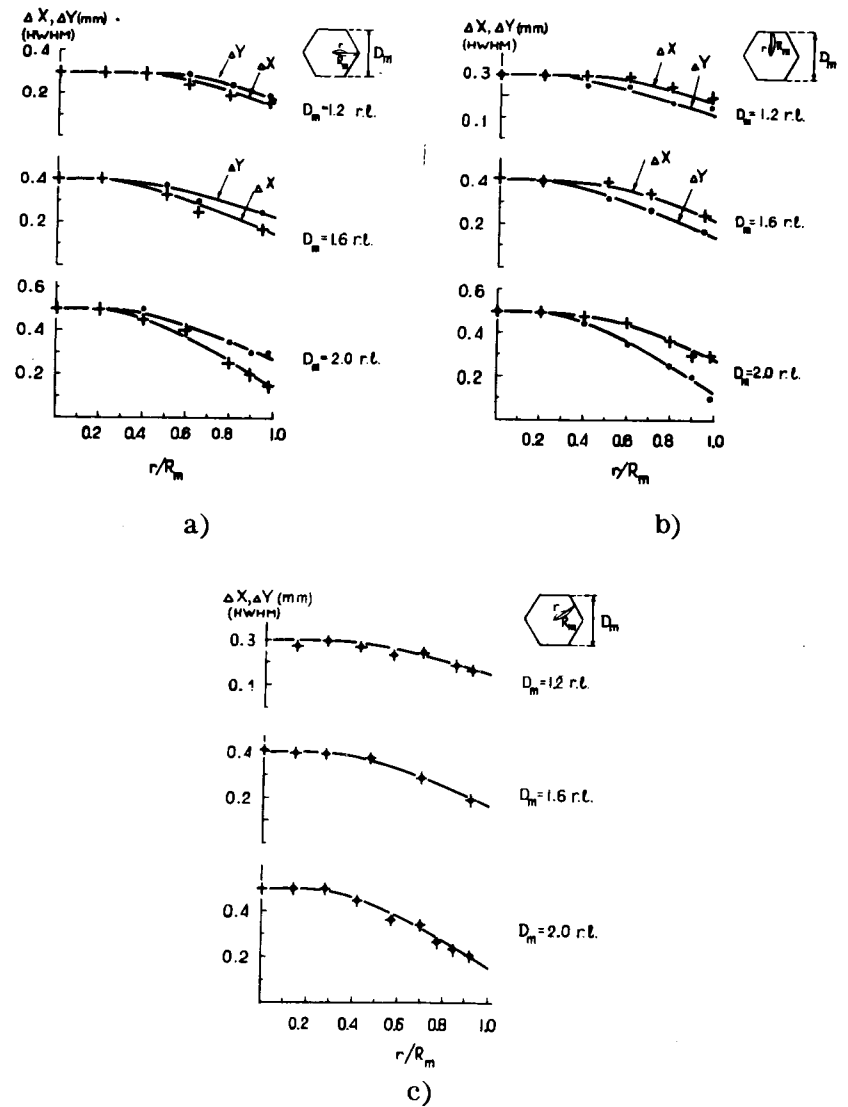


Fig. 3. The errors in measuring the coordinates $\Delta X, \Delta Y$ (HWHM of the distributions of the centers of gravity) calculated by formulae (2) and (2a) vs the γ -quantum entry point on the spectrometer module a) along the "X" axis, b) along the "Y" axis, and c) at an angle of 45° to the "X" axis; $\Delta E/E = 3\%$.

lation by formulae (2), (2a) and (2), (2b) for the module 1.6 r.l. in diameter.

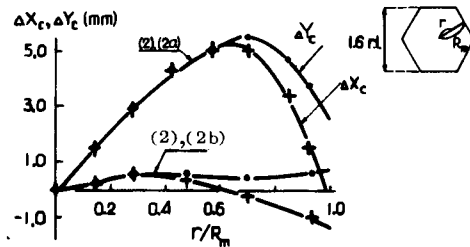


Fig. 4. The systematic errors calculated by formulae (2), (2a) and (2), (2b) versus the distance to the module center ($D_m = 1.6$ r.l., $\Delta E/E = 3\%$).

The obtained distributions show that, if the coordinates are calculated by formula (2), (2a), there appear significant systematic errors. These errors are due to the nonlinear redistribution of the shower energy between the modules when the shower axis is shifted along the module radius. As is seen from fig. 4, formula (2), (2b), which has an exponential part, permits the systematic shift to be reduced essentially. Yet the errors in measuring the coordinates calculated by formula (2), (2b) increase (see figs. 5 and 6). Till now we have considered the errors in measuring the coordinates arising for particles incident perpendicular to the spectrometer surface.

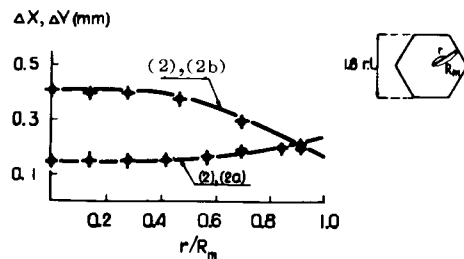


Fig. 5. The errors in measuring the coordinates ΔX , ΔY vs the distance to the module center calculated by formulae (2), (2a) and (2), (2b); $D_m = 1.6$ r.l., $\Delta E/E = 3\%$.

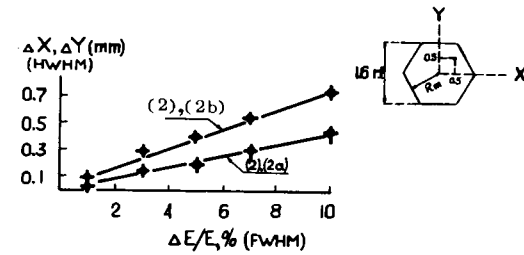


Fig. 6. The dependence of the errors in measuring the coordinates ΔX , ΔY on the energy resolution of the spectrometer for $X_0 = Y_0 = 0.5 R_m$ calculated by formulae (2), (2a) and (2), (2b); $D_m = 1.6$ r.l.

Figure 7 presents the systematic errors of measurement of the γ -quantum coordinate "Y" versus the incident angle in the plane "YOZ" for the point coincident with the module center. The dependence of the systematic error on the distance to the module center (axis "Y") is shown in fig. 8 for $\theta = 3^\circ$.

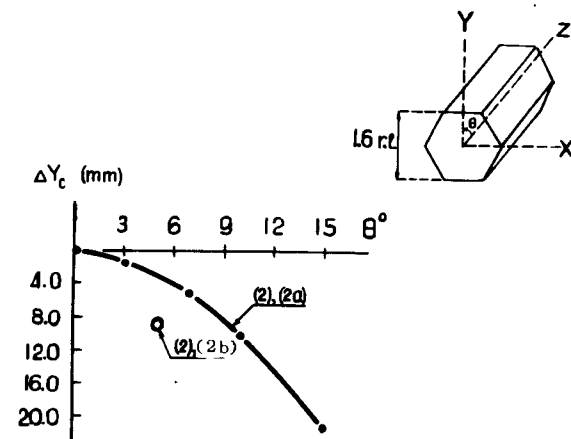


Fig. 7. The systematic errors of measurement of the γ -quantum coordinate "Y" vs the incident angle in the plane YOZ for the point coincident with the module center.

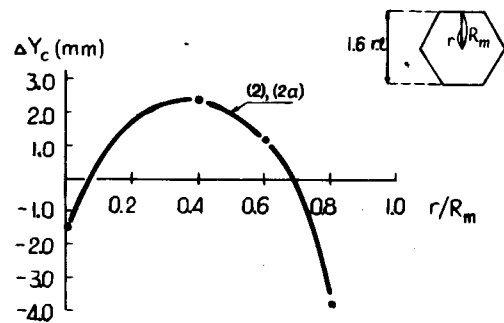


Fig. 8. The dependence of the systematic errors on the distance to the module center (along the axis "Y") when γ -quanta are incident at an angle of $\theta = 3^\circ$; $D_m = 1.6$ r.l., $\Delta E/E = 3\%$.

Figures 7 and 8 show that there exists a strong dependence of the values of systematic errors both on the γ -quantum incident angle and on the radius.

In order to overcome the above difficulties, we have used the "matrix" method of calculating the coordinates. Using this method, the particle coordinates are determined by comparing the "experimental" energy distribution in the modules with those obtained by calibration (for this purpose it is possible to use, e.g., the beam of monoenergetic electrons). In this paper the indicated distributions have been obtained by the program method. In the last case the energy information for the given values of energy, incident angle and particle coordinates, is given as a matrix.

The calibration matrices were generated with a computer over the whole surface (2 mm step) by the method similar to that used above. The calculations were performed for the modules 1.6 r.l. in diameter and $\Delta E/E = 5\%$ in the interval of γ -quantum incident angles from 0° to 10° with a 2° step.

In the program, the coordinates (X, Y) of the γ -quantum entry points on the spectrometer module were calculated by minimizing the function

$$F \left| \frac{E_i^M}{E_t^M} - \frac{E_i^Y}{E_t^Y} \right| \rightarrow \min, \quad (3)$$

where E_i^M/E_t^M and E_i^Y/E_t^Y are correspondingly the matrix and "experimental" ratios of energies in the i -th module to the total γ -quantum energy.

Figure 9 illustrates the errors in measuring the γ -quantum coordinates versus the incident angle and the entry point on the module calculated by the matrix method.

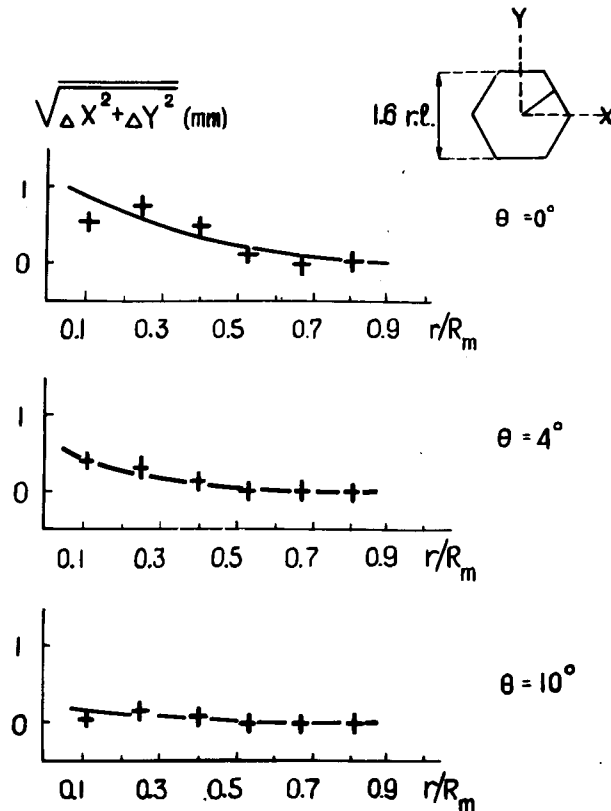


Fig. 9. The errors of measurement of the γ -quantum coordinates versus the incident angle and the entry point on the module spectrometer calculated by the matrix method; $D_m = 1.6$ r.l., $\Delta E/E = 5\%$.

In the figure the systematic errors, which do not exceed 1 mm in a wide range of angles θ and r/R_m , depend only on the matrix step (2 mm).

For experiment it is important to know how the value of systematic error in measuring the coordinates of γ -quanta incident at angle θ_i depends when the experimental matrices are compared with the calibration matrix measured for angle ϕ . These results for $r/R_m=0.5$ and $\phi=10^\circ$ and five values of angle θ_i are presented in fig.10. An evident minimum of the curve points to that, in principle, one can determine not only the coordinates of the entry point on the module but also of the γ -quantum incident angle.

The dependence of the systematic errors on the energy resolution of the spectrometer is shown in fig. 11 for $r/R_m=0.5$ and $\theta=10^\circ$.

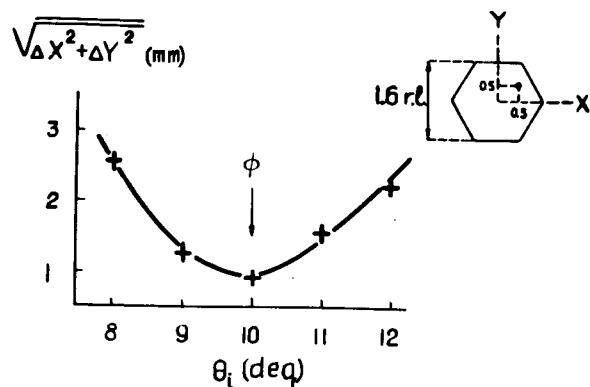


Fig. 10. The dependence of the systematic errors in measuring the coordinates of γ -quanta incident at angle θ_i when the experimental matrices are compared with the calibration matrix measured for $\phi=10^\circ$ at the point $X_0 = Y_0 = 0.5R_m$; $\Delta E/E = 5\%$.

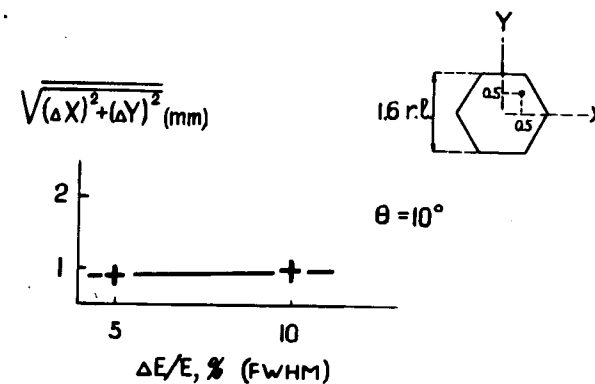


Fig. 11. The dependence of the systematic errors on the energy resolution of the spectrometer for the point $X_0 = Y_0 = 0.5R_m$ and $\theta = 10^\circ$.

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