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# INTEGRAL ANALYSIS OF TRACKS DATA IN SOLENOID

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## I. THE MATHEMATICAL FORMALISM

a) The system of coordinates and designations: the (XYZ) or ($R\phi Z$) system will be used, magnetic field has the Z-component (H), there are "N" detectors along the Z-axis, each detector ($Z_n$;n=1,2,..,N) registers "$M_n$" space points $\{(R_{mn},\phi_{mn},Z_n),\ m=1,2,\dots,M_n\}$ with the accuracy "$\sigma_{mn}$".

b) Analytical view of the trajectory:

$$x(t)=X_o + r_o\sin\alpha + r_o\sin(\omega t-\alpha)(\equiv R\cos\phi\ ) \tag{I.1a}$$

$$y(t)=Y_o - r_o\cos\alpha + r_o\cos(\omega t-\alpha)(\equiv R\sin\phi\ ) \tag{I.1b}$$

$$z(t)=Z_o + tv_o\cos\beta\ , \tag{I.1c}$$

where $X_o,Y_o,Z_o$ are the vertex position, "$\beta$" is the (pZ) angle and "$\alpha$" is the (XY) angle of momentum "p",

$$\omega = \frac{ecH}{E},\qquad E=\frac{pc^2}{v_o}\ ,\qquad r_o=\frac{v_o\sin\beta}{\omega} \tag{I.2}$$

$$A = \frac{1}{2}\,\frac{\omega}{v_o\cos\beta} = \frac{eH}{2cp\ \cos\beta}\ . \tag{I.3}$$

c) The vertex function (abbr.VF). In accordance to the rule of the VF creation [1,p.424] it is necessary to redefine parameters "$r_o,\alpha,A$" through $X_o,Y_o,Z_o$ and two formal detector counts at $Z_\nu$ and $Z_k$. Simple transformations of expressions (I.a-c) allow one to get follows relations

$$\alpha + A(2Z_o-Z_k-Z_\nu)= \mathrm{Arctg}\,\frac{y_{1k}-y_{\mu\nu}}{X_{1k}-X_{\mu\nu}}\ (\equiv P_{1k\mu\nu}) \tag{I.4}$$

$$\alpha - A(Z_k - Z_o) = \mathrm{Arctg}\,\frac{y_{1k}-Y_o}{X_{1k}-X_o}\ \ (\equiv F_{1k}(X_o,Y_o)) \tag{I.5}$$

$$\alpha - A(Z_\nu - Z_o) = \mathrm{Arctg}\,\frac{y_{\mu\nu}-Y_o}{X_{\mu\nu}-X_o}\ \ (\equiv F_{\mu\nu}(X_o,Y_o))\ . \tag{I.6}$$

These expressions redefine parameters "A" and "α"

$$A = -\frac{F_{1k} - F_{\mu\nu}}{Z_k - Z_\nu} \qquad (I.7)$$

$$\alpha = P_{1k\mu\nu} - A(2Z_o - Z_k - Z_\nu)\,. \qquad (I.8)$$

The expression (I.6) can be used for the definition of the vertex function:

$$V(X_o,Y_o,Z_o) = \sum_{1=1}^{M_k}\sum_{\mu=1}^{M_\nu}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\alpha - A(Z_n - Z_o) - F_{mn};\sigma_{mn}\}\sigma_{mn}\,, \qquad (I.9)$$

where parameters "$\alpha, A$" are used as (I.7,8).

$G(x;s)$ is the coordinates precision (s) function that can be used in several forms (views):

$$G(x;s) = \exp\left(-\frac{x^2}{2s}\right)/s\sqrt{2\pi} \qquad (I.10a)$$

or

$$G(x;s) = \begin{cases} (2s)^{-1} & \text{at } |x| \le s \\ 0, & |x| > s \end{cases} \qquad (I.10b)$$

or

$$G(x;s) = \begin{cases} \dfrac{3}{s4\sqrt{5}}\,(1 - x^2/5s^2) & \text{at } |x| \le s\sqrt{5} \\ 0, & |x| > s\sqrt{5}\,. \end{cases} \qquad (I.10c)$$

These interpretations of the $G(x;s)$ reflect various types of detectors (for instance, (I.10b) is typical for proportional chambers), but simplify also the analysis of VF.

The function (I.9) requires about $M^3$ number of operations (the abbreviation NOP will be used in the following) and reflects itself the track reconstruction by the exhaustive search.

However one can exclude the ($\mu$) summation out of this definition or to change "$\nu$ ->n" and to get the new expression of this VF



$$V(X_o,Y_o,Z_o) = \sum_{1=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\frac{F_{1k}-F_{mn}}{Z_k - Z_n}(Z_o - Z_k) - F_{mn} + P_{1kmn};\sigma_{mn}\}\sigma_{mn}\,. \qquad (I.11)$$

It looks more attractive in the order of NOP ($\simeq M^2$). It can be mentioned that this definition of VF does not contain a possibility to do a simple step to the reconstruction of trajectories (as it was for straight tracks) and this VF(I.11) is "independent" description of the track pattern.

## II. SEARCH FOR THE PRIMARY VERTEX

To solve this task one can neglect the transversal sizes of colliding beams which are small enough (15 μm at LHC, <1mm for SSC) in the comparison to counts of detectors (it means that minimal coordinate of a detector is larger than the beam radius). It can be mentioned also that the problem of the z-separation of primary vertices will be very severe [2].

So, the $V(Z_o)$ gets the view:

$$V(Z_o) = \sum_{1=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{\frac{\phi_{1k}-\phi_{mn}+(i-j)\pi}{Z_k - Z_n}(Z_o - Z_k) - \phi_{mn} + P_{1kmn} - j\pi;\sigma_{mn}\}\,\sigma_{mn} \qquad (II.1)$$

$i,j = \ldots -2,-1,0,1,2,\ldots$ these parameters should be determined for particular experiment.

In the following the notation of the $V(Z_o)$ will be used in other form (for the simplicity):

$$V(Z_o) = \sum_{1=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{A(Z_o - Z_k) - C;\sigma_{mn}\}\sigma_{mn}\,. \qquad (II.1a)$$

This function has several extremes, especially for the multivertices events and the determination of the vertex position $Z_o$ foresees two stages:

1) The localization of the subregion of global maximum of $V(Z_o)$.

In the same way it was done for the straight tracks, the economic method [1,p.426] can be used. The main task at this step is the analytical integration:

$$I_\mu = \sum_{l=1}^{M_k} \sum_{n=1}^{N} \sum_{m=1}^{M_n} J^\mu_{lkmn} \qquad (\mu=0,1,\ldots,4) \qquad \text{(II.2)}$$

where

$$J^\mu_{lkmn} = \int_{Z_L}^{Z_U} dZ(Z_o-\bar{Z}_o)^\mu \, G\{A(Z_o-Z_k)-C;\sigma_{mn}\}\sigma_{mn} \,. \qquad \text{(II.2a)}$$

This problem is very simple if the G(x;s) is used in discrete form (I.10b). It is necessary to solve simple system of unequalities:

$$\begin{cases} |A(Z_o-Z_k) - C| \le \sigma_{mn} \\ Z_L \le Z_o \le Z_U \,. \end{cases} \qquad \text{(II.3)}$$

If this system has no solutions, it means that for the "base" point $(R_{lk},\phi_{lk},Z_k)$ there are no "track extention" into the point $(R_{mn},\phi_{mn},Z_n)$, or $J^\mu_{lkmn} \equiv 0$. In opposite case

$$J^\mu \equiv \int_{Z_1}^{Z_2} dZ_o(Z_o-\bar{Z}_o)^\mu \equiv \frac{1}{\mu+1}(Z_o-\bar{Z}_o)^{\mu+1}\Bigg|_{Z_1}^{Z_2} \qquad \text{(II.4)}$$

where:

$$Z_1 = \max\,(Z_L, Z_k + \frac{C}{A} - \frac{\sigma}{|A|}) \qquad \text{(II.5)}$$

$$Z_2 = \min\,(Z_L, Z_k + \frac{C}{A} + \frac{\sigma}{|A|}) \,,$$



$Z_U, Z_L$ are upper and lower limits of the searching region (it can be comparable with sizes of whole experimental installation). Approximate position of vertex is determined by the solution of cubic equation that has coefficients determined by integrals $I_\mu$.

2) Precise determination of the vertex. On this stage the parabolic fit [1,p.427] also can be used. The parabolic interpretation (I.12c) of the accuracy function is useful too. Final result has exact value of $Z_o$ and one can get the dispersion $S(Z_o)$ of the $V(Z_o)$.

Results of very simple test of this vertex determination are shown on Fig.1-3.

## III. THE REMOVAL OF THE VERTEX AND PREPARATION OF ARRAYS FOR THE TRACK RECONSTRUCTION

This possibility is obviously enough if one looks back to the integration (II.2-5). The width of the $V(Z_o)$ peak is narrow in the comparison with initial big limits. Coordinates of the trajectories emitted from finding vertex correspond to this peak and while the integration is performed over the peak width, one can prepare the array $\{R_{mn},\phi_{mn},Z_n\}_k$ of the "track extentions" for the point $(R_{lk}, \phi_{lk},Z_k)$. This array, of course, will have false points too (a noise, counts of other vertices, trajectories), this "overloading" is determined mainly by the detector accuracy. However, the task of the reconstruction of single trajectory over this limited array looks more simple than the analysis of all track data.

Similarly, one can exclude all coordinates that belong to the vertex out of original data or, in other words, to do the "cleaning" (removal) of current vertex for the following search for other vertices.

With these methods (ch.II+III) one can exclude all points of all primary intensive vertices and thereby get more simple track data in the remainder for the following analysis (let it be the search of rare decays).

Fig.4 illustrates how this method works for simple simulated 3-vertices event.

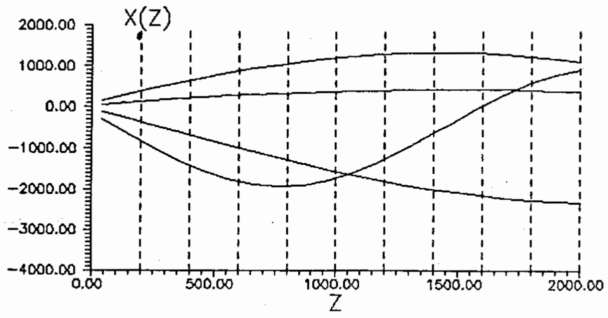



Fig.1 The simplest simulation of 4−tracks pattern. The X(Z) plane. Detectors are placed at z=200,400,600,... mm .

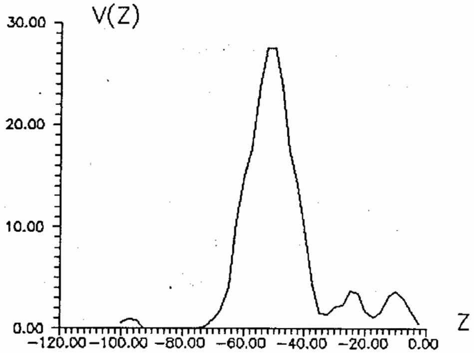



Fig.2 The view of vertex function V(Z) for track pattern shown in Fig.1.

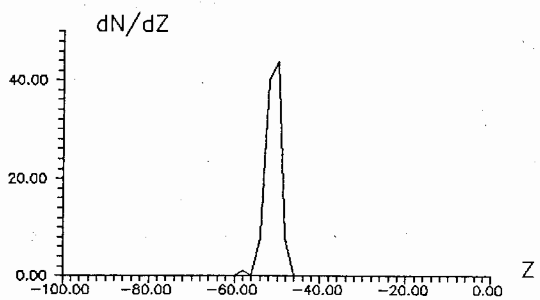
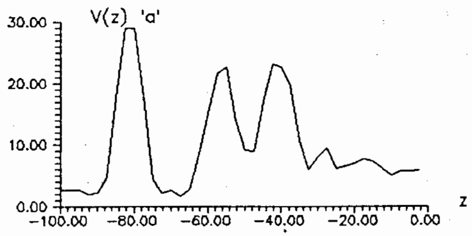
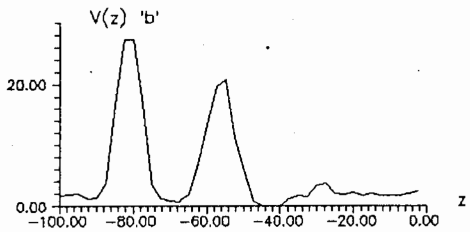



Fig.3 Statistical distribution of the vertex determination for 100 simulated events like shown in Fig.1.





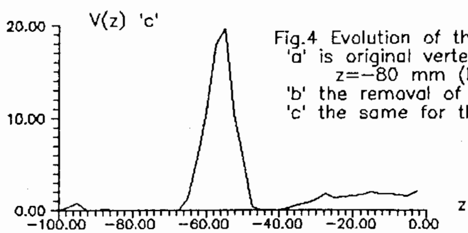




Fig.4 Evolution of the V(z) at the 'cleaning':
'a' is original vertex function for 3 vertices at z=−80 mm (Mult.=4),−55mm(M=3),−40mm(M=3);
'b' the removal of the vertex (z=−40) was done;
'c' the same for the vertex at z=−80.

## IV. SEPARATION OF THE TRACKS DATA INTO "KINEMATICAL" SUBREGIONS

The "integral model" allows one to get not only the vertex functions, but also to construct the functions for the other parameters. For the solenoid one can create the function F(A), that has "kinematical" parameter "A"(I.3) as the argument. This parameter connects the particles charge and the value of longitudinal momentum. Looking back again to equations (I.5,6) one can exclude parameters $\alpha$, $Z_o$ and get:

$$F(A)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{A - A^*_{lkmn};\sigma_{mn}\}\sigma_{mn}, \qquad (IV.1)$$

where

$$A^*_{lkmn} \equiv - \frac{\phi_{lk}-\phi_{mn}+(i-j)\pi}{Z_k-Z_n} . \qquad (IV.1a)$$

Real trajectories manifest themselves in this functions as high enough peaks (amplitude is about of N) beyond the combinatorial background. Simple track pattern (fig.5) was simulated for the illustration of the F(A). The view of the F(A) is shown in fig.6-6b.

For the high multiplicity event this F(A) will have a very complicated view and the analytical search for trajectories does not seem expedient. However, this function is simple for the integration like (II.2-5) and, respectively, for the extraction of the single track arrays, as it was done in ch.III. This F(A) can be used by the following method.

One can determine boundaries of the parameter "A" for the particular experiment. This region is divided onto "Q" subregions, each of them contains "K=M/Q" tracks (here "M" is total quantity of tracks). And then, for various methods of the track reconstruction with typical value of NOP bigger than "K" (for instance, equals "$K^2$"), one will have considerable profit in the number of operations ($N_t$) in comparison with NOP (about of $M^2$) to reconstruct over whole array:



a) for the sequential reconstruction of all subregions

$$N_t \simeq Q*(M/Q)^2 = M^2/Q , \qquad (IV.2)$$

b) for the parallel reconstruction of all subregions

$$N_t \simeq (M/Q)^2 = M^2/Q^2 . \qquad (IV.3)$$

It should be noted, that such separation is not a "geometrical division", which can lead to the distortion of the track picture.

The separation of the whole track pattern into "kinematical" subregions is a simple way for concurrent processing for the track reconstruction problem. This idea is based on the full independence the data arrays for different subregions. One can perform procedures of the track reconstruction in several computers of either high and low performance using various methods.

The considerable decrease of the overall processing time will be achieved if the time needed just for data separation is short enough (the same order as the data transfer time).

Let us estimate the value of the data transfer time ($T_t$) for the hypothetical detector unit with the number of position sensitive planes N=10, the multiplicity in each plane M=1000, the data range word length of the track information L=16 bit. For the transfer rate R=20 MB/s this time is $T_t \simeq \frac{2*N*M}{R} \simeq 1$ ms.

The time estimation for the separation of the full track pattern into Q=10 "kinematical" subregions, if one uses the computer with performance P=40 MIPS, is $T_s \simeq \frac{N*M^2*Q}{R} \simeq 2.5$ s.

This time is too large with respect to transfer time $T_t$. It is necessary to notice that the time to reconstruct the track will not be smaller but of the same order or even more.

The decrease of the time $T_s$ may be achieved only with the processing of several pieces (K) of the whole track picture simultaneously. In this case the time estimation is $T_s \simeq \frac{N*M^2*Q}{K}*t_p$, where $t_p$ is the time to process one coordinate point.

Note again the simple form of the equation for the F(A) (IV.1). It allows to build a very regular architecture of the specialized processor unit for hardware implementation. The algorithm to separate the coordinate samples consists of several simple operations for every points:

- calculate simple arithmetic expression for $A^*$;
- determine if the point belongs to the subregion "$A_q$";
- load the current point to the corresponding array.

It seems that it is correct to estimate time $t_p$ to be no more then 100 ns (especially for interger arithmetics). If K is equal to N*Q ( it corresponds to concurrent separation into Q subregions for all coordinate planes) then $T_s \simeq \frac{M^2}{K} * t_p \simeq 100$ ms.

Moreover, if to process J points in every plane at the same time the result will be $T_s \simeq \frac{100}{J}$ ms.

The last equation is the way to achieve very high performance in separating tracks data into "kinematical" subregions and inspires the further work.

The sketch of one specialized processor unit architecture for hardware implementation is presented at Fig.6c.

In the case of hardware implementation the scale of integration may be very high if some of the modules will be implemented as ASIC chip.

Of course, the suggestions of hardware implementation is only preliminary and requires further discussion.

It should be noted that the authors do not consider any particular method of data transfer between the parts of the data acquisition system because there are many implementations of those in various experimental installations.

## V. SEARCH FOR JETS

As it follows out of the results of the paper[4], devoted, in particular, to the Monte-Carlo simulation of jets at Tev region of the energy, hard component of the jet (the momentum value > 1 Tev) is concentrated in a narrow cone with typical angle about of 50 mrad. In the following the 'hard component of the jet' will be implied as the jet.

Looking back at kinematical function (V.1), one can suppose that peaks, which are corresponding to the jet, will be concentrated in nar-



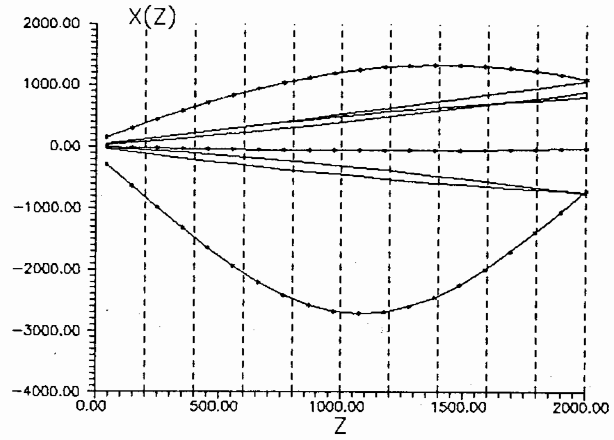



Fig.5 The view of X(Z) for 3 'usual' particles (marked by dots) and 5 'jet' particles.

row region of the "A". The positive-charged part of the jet is concentrated close to the point "A"(for instance, it is positive), and the negative-charged part of the jet is near at the symmetrical point "-A". Perhaps, it is most significant manifestation of the jet in terms of kinematical variable "A". This manifestation can be increased by a very simple method: in the argument of the "G" function (IV.1a) one have to replace $A^*_{lkmn} \Rightarrow |A^*_{lkmn}|$ (see definition (IV.1a) and get the "jet function":

$$J(A)=\sum_{l=1}^{M_k}\sum_{n=1}^{N}\sum_{m=1}^{M_n} G\{A-|A^*_{lkmn}|;\sigma_{mn}\}\sigma_{mn}. \qquad (V.1)$$

Joined this way, total peak of the jet will look as the prominent one at the background of "usual" particles. "Mathematical simplicity" of the J(A) allows one to use analytical method for the search for these 'prominent' peaks (jets), as it is used in the vertex determination. Moreover, for the most amplification of the jet peak, one can engage the coordinate-amplitude information of calorimeters to construct the equation (V.1).

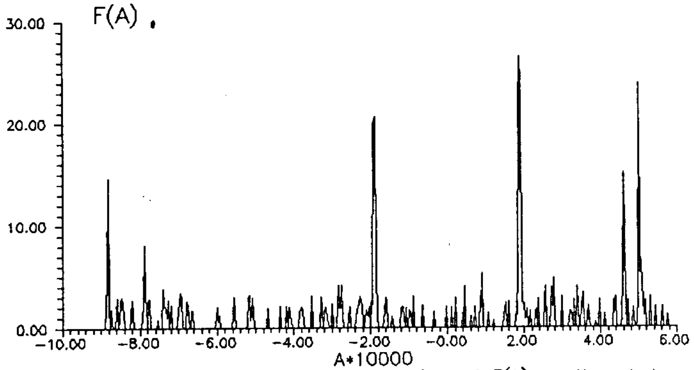



Fig.6 An attempt to get the view of F(A) on the whole. The resolution of graph does not separate single peaks (particles) well.

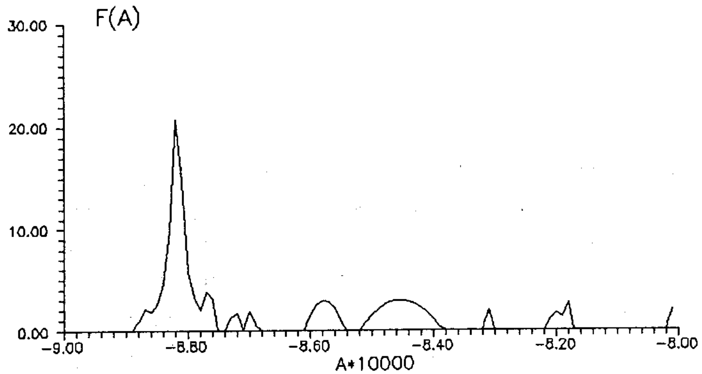



Fig.6a Same view of the F(A) for one particle at A=−8.8E−4.

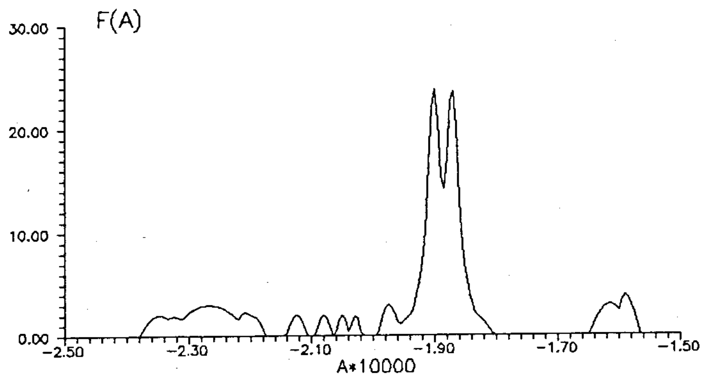



Fig.6b The piec of F(A) for 2 negative 'jet' particles.



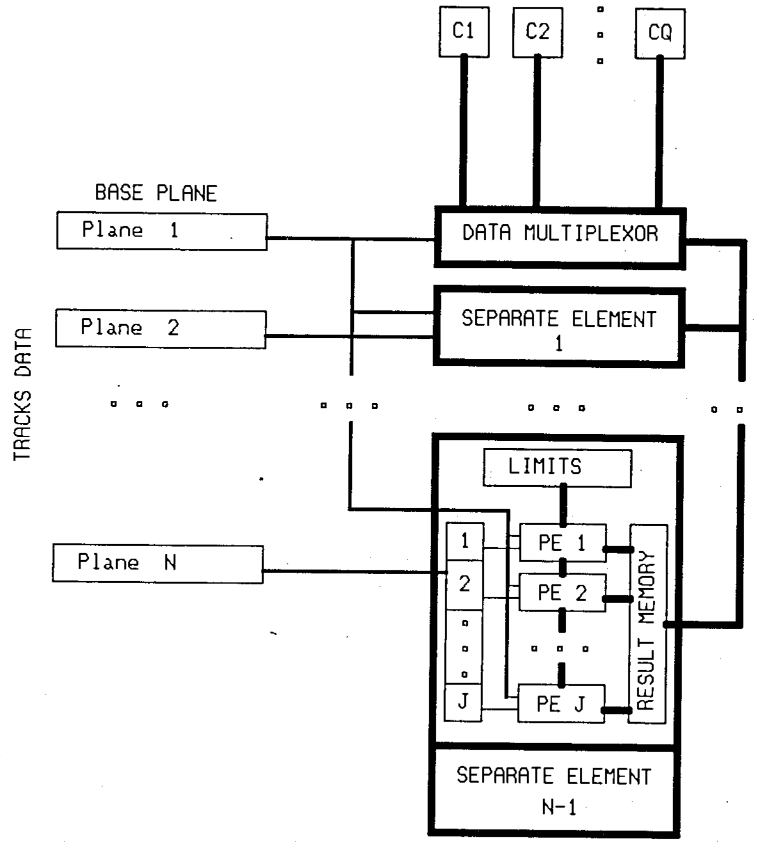



FIG. 6c. THE SKETCH OF THE SPECIALIZED PROCESSOR UNIT ARCHITECTURE TO SEPARATE WHOLE ARRAY OF TRACKS DATA INTO Q "KINEMATICAL" SUBREGIONS.
C1...CQ - COMPUTERS FOR TRACKS RECONSTRUCTION;
PE1...PEJ -SIMPLE PROCESSOR ELEMENTS (ASIC);
LIMITS - MEMORY FOR SUBREGIONS BOUNDARIES.

The jets funotion J(A) for tracks picture on the fig.5 is shown at fig.7.

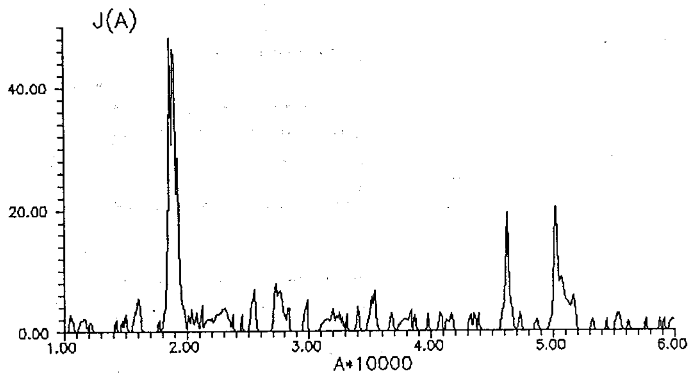



Fig.7 The 'jet' function J(A). The 'jet' looks here as prominent peak at A=2.E−4 .

CONCLUSION

The analysis of vertex function allows one to separate vertices and to prepare arrays for the reconstruction of single track. Moreover, the preliminary vertex determination simplifies considerably the following reconstruction of kinematical parameters.

The using of kinematical separation of tracks data substantiates the simple way for the parallel reconstruction of trajectories and has the profit in the number of operations.

Analytical search for jets can be used as 'software trigger' in some experiments.

This ideology can be used at LHC, SSC, at nuclear accelerators at Los-Alamos, Darmstadt etc.



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Интегральный анализ трековой информации в соленоиде


Рассматривается возможность предварительного анализа трековой информации в экспериментах с высокой множественностью для трековых детекторов, помещенных в соленоидальное магнитное поле. Подобный анализ предусматривает выполнение следующих функций:

1) определение первичных (интенсивных) вершин;
2) подготовка массивов, содержащих координаты для одного трека, для последующего восстановления траекторий;
3) "очистка" (удаление) первичной вершины для последующего поиска других вершин;
4) разбиение трековой информации по "кинематическим" диапазонам, позволяющее сократить время реконструкции события;
5) выделение "струй" (жесткая компонента).

Характерное число операций пропорционально $M^2$, где M — множественность или число треков в событии. Идеология анализа основана на "интегральной математической модели трековых изображений" и не содержит в себе процедуру восстановления траекторий.


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Integral Analysis of Tracks Data in Solenoid


The preliminary analysis of the tracks data for the particles in the solenoidal magnetic field in the high multiplicity experiments is considered. This analysis provides the realization of some functions:

1) the determination of the primary (intensive) vertices;
2) the creation of the measurement coordinates arrays for the reconstruction of single track;
3) the cleaning (removal) of the primary vertex for the following search the next vertices;
4) the separation of the whole tracks data into "kinematical" subregions that allows one to reduce the overall CPU time of the events reconstruction;
5) the determination of "jets" (hard components).

The typical number of operations is proportional to $M^2$ ("M" is the multiplicity or the tracks number). This idea is based on the "integral mathematical model of the track pattern" and does not contain a procedure of the trajectories reconstruction.


The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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