сообщения
объединенного
института
ядерных
исследований
дубна

E1-91-256

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RELATIONS BETWEEN THE MATTER DENSITY AND LAYER THICKNESS RADIAL DISTRIBUTIONS IN SPHERICALLY SYMMETRIC OBJECTS: FOR HIGH ENERGY NUCLEAR PHYSICS AND ASTROPHYSICS USE

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1991

1. INTRODUCTION

In many problems, met in experimental high energy nuclear physics and in astrophysics, one should obtain the matter density in spherically symmetric objects from a known radial distribution of the matter layer thicknesses at various distances from the diameter of the objects. In other words, from the known distribution of the thicknesses $\lambda(b)$ of an object at various impact parameters b, the radial distribution $\rho(r)$ of the matter density should be obtained. In our works, the problem appeared directly in the studies of the matter density distributions in atomic nuclei by means of high energy hadronic projectiles/1/. Similar problems may be met in high energy astrophysics and cosmic ray physics, and in astronomy.

In nuclear physics and astrophysics the task is being set in a following manner: to find out the radial distribution of the matter density or radiation density inside a spherically symmetric object from a known distribution of the matter layer thicknesses inside this object. The thickness of the matter layers inside the object, covered by a probe, may be determined experimentally; as the probe a projectile particle (in high energy nuclear physics) or a beam of radiation (in high energy astrophysics) may be employed.

The method applied usually so far in such cases is: 1. Some hypothetical density radial distribution is accepted for the object under study, on the basis of a known properties of the object (some exponential or Gaussian, for example); 2. The distribution is fitted to corresponding experimental data. We would like to have some method in which not any assumptions on the density distribution are used.

The method presented here does not use any assumptions on a character of the distribution wanted. The only assumption is that the distribution is in an object of the spherical symmetry, and the initial conditions are: the density ρ at the maximum size of the object $d_{max} = r_0$ is $\rho(r_0) = 0$ and d is given for the object under study.

The subject in this paper is to describe the method and to test it, and to estimate its accuracy.

2. THE METHOD

Let us assume that the thicknesses $\{S_1, S_2, \ldots, S_k, \ldots, S_n\}$ of a radially symmetric object at the distances $\{r_1, r_2, \ldots, \ldots, r_k, \ldots, r_n\}$ from its center are known from an experiment, see, for example, fig.1. The thicknesses may be expressed in mass units per the square of the length unit (g/cm^2, nucleons/cm^2 etc., for example); the radii $\{r_i,\ i = 1,2,\ldots,n\}$ may be in length units. The series of the auxiliary concentric spheres - with the radii r_i, $i = 1,2,\ldots,n$ - may be drawn, as it is shown in fig.1, where the series of the largest circles are presented on the plane with the center of the spheres.

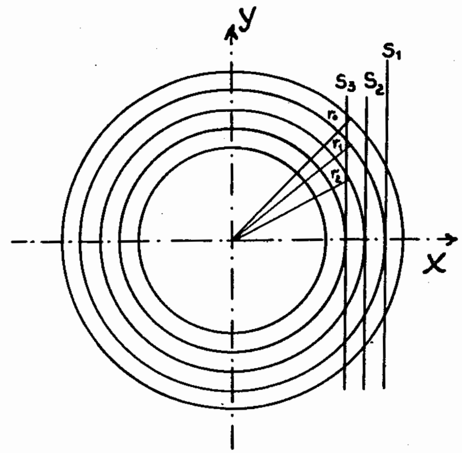

Fig.1. The auxiliary scheme for derivation of formula (1).

It is evident that the length of the section $r_{k-1,k}$ on the thickness S_k is as large as $r^2_{k-1} - r^2_k$, because $\sqrt{r^2_{k-1} - r^2_k} = \sqrt{(r^2_{k-1} - x^2) - (r^2_k - x^2)} = \sqrt{y^2_{k-1} - y^2_k}$. Then, the radial density $\rho(r_i)$, which should be ascribed to a radius r_i is expressed by the recurrent formula:

$$\rho_1 = \frac{0.5\cdot S_1}{\sqrt{r_0^2 - r_1^2}},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

$$\rho_n = \frac{0.5\, S_n - \sum\limits_{k=1}^{k=n-1} \rho_k\left(\sqrt{r^2_{k-1} - r^2_k} - \sqrt{r^2_k - r^2_n}\right)}{\sqrt{r^2_{n-1} - r^2_n}}.$$

If the radius of the spherical object r_0 is large enough, a desired accuracy of the radial distribution $\rho(r)$ estimation may be achieved at large values of n.

3. THE EXAMPLES OF AN APPLICATION

3.1. The Testing of the Method

Let us analyse two given radial distributions: $f_1(r)=e^{-r^2}$ and $f_2(r) = e^{-r}$ inside spherical objects 1 and 2. The radius r_0 of the first object is accepted to be $r_{01} = 5$; and of the second one, $r_{02} = 50$. For both of the two spheres integrals

$$S_{i,n}(r_n) = \int_{-\infty}^{+\infty} f_i(\sqrt{x^2 + r_0^2})dx \qquad (2)$$

$$i = 1,2$$

were calculated numerically, for the radius sections $\Delta r = 0.1$. From the thicknesses $S_{1,n}$ and $S_{2,n}$ distributions inside the objects 1 and 2, corresponding radial distributions $f_1(r)$ and $f_2(r)$, were obtained, using the above described method. Results of the confrontation of the corresponding distributions assumed are presented in fig.2; agreement well enough is seen evidently.

Then, the method described above provides correct transformation of the thicknesses distribution inside a spherically symmetric object into radial density distribution inside this object. The method can be applied for the radial density distribution estimation on the basis of the known thicknesses distribution inside a spherically symmetric object, therefore. For example, an unknown radial distribution $\rho(r)$ of the matter density d in g/cm³ may be obtained from the known distribution $N(\lambda)$ of the thicknesses $\lambda(r)$ in g/cm², in the spherically symmetric object with the radius r_0.

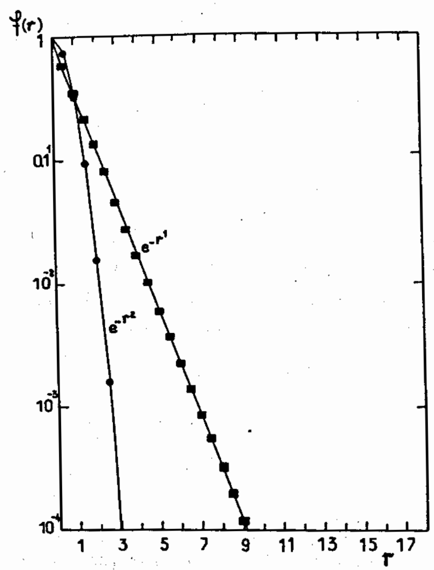

Fig.2. Testing of the method. Solid lines - the known distributions e^{-r^2} and e^{-r}, correspondingly; • and ■ the distributions obtained by the method.

3.2. The Determination of the Intranuclear Matter Density Radial Distribution Inside Atomic Nuclei; a Formulation of the Problem

In the work "Matter Density Distribution in Atomic Nuclei as Illuminated by High Energy Hadrons", important experimental facts are stressed out[2]:

1. The spatially unpolarized atomic nucleus for the hadronic probes falling randomly on it is a spherical object of intranuclear matter, it can be characterized as a target by its maximum thickness, mean thickness, and the thickness at a given impact parameter.
2. The number n_N of the nucleons emitted in any hadron-nucleus collision event, at an incident hadron kinetic energy high enough - higher than a few GeV, provides the information about the thickness λ, in nucleons/S, of the intranuclear matter layer covered by the projectile hadron in the target nucleus; $S \doteq 10.3$ fm².
3. The multiplicity n_N distribution $N(n_N)$ of the nucleons emitted in a numerous sample of hadron-nucleus collisions of a definite hadrons with a definite nucleus is in fact the distribution $W(\lambda)$ of the thicknesses λ in nucleons/S of the spherical target nucleus thicknesses at various distances from its center or, at various impact parameters b.
4. Relations can be written between the multiplicity n_p distribution $N(n_p)$ of the proton emitted from the target nucleus and the intranuclear matter layer thickness λ in protons/S distribution $W(\lambda)$.

In other words, from experiments the data on $W(\lambda)$ may be obtained for any of the atomic nuclei, in hadron-nucleus high energy collisions. But, from the $W(\lambda)$ distributions the intranuclear matter density radial distribution $\rho(r)$ can be evaluated simply, by means of the method proposed and described above.

4. CONCLUSIONS

Method is proposed for a transformation from the known distribution of the thicknesses of a matter layers in a radially symmetric object into an unknown radial distribution of the matter density in this object.

This simple method is worked out adequately for experimental conditions - usually, in probing a structural properties of an object by appropriate energetic probes, one obtains the information about the thicknesses of the object at various distances of the projectile course from the center of the object - at various impact parameters. In a sample of events of the probings, numerous enough, the distribution $N(\lambda)$ of the thicknesses λ of the object under study is obtained; the thicknesses may be expressed in the mass unit per cm^2, for example. The total sample of the events may be treated as the collision of the beam of the projectiles with an object; if the object is spatially unpolarized, the sample may be treated as the collision of the beam of probes with the spherically symmetric object.

The method may be applied in solving many physical and astrophysical problems. We can recognize simply this case as similar to that concerning the hadron-nucleus collisions, we have in section 3.2 written about.

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Receibed by Publishing Department
on June 7, 1991.

Зелинска М. и др. E1-91-256

Соотношения между радиальными распределениями плотности и толщинами слоев материи в сферически симметричных объектах: для нужд ядерной физики высоких энергий и астрофизики

Описан метод перехода от данного радиального распределения толщин материи к неизвестному радиальному распределению плотности материи в сферически симметричных объектах. Даны основные формулы, проведена проверка полученных результатов. Приведен случай применения метода в ядерной физике высоких энергий для получения радиального распределения материи в ядре-мишени, облучаемой адронами.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1991

Zielińska M. et al. E1-91-256

Relations between the Matter Density and Layer Thickness Radial Distributions in Spherically Symmetric Objects: for High Energy Nuclear Physics and Astrophysics Use

It is shown how it is possible to transform known data on radial distribution of the matter layer thicknesses to unknown radial distribution of the matter density inside spherically symmetric objects. Appropriate formulas and testing of them are presented. An application of the method for the radial distribution of the matter density inside a target nucleus is discussed, as an example.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Inst