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THE πHe^3H^3 COUPLING CONSTANT ESTIMATION USING THE CHEW-LOW EQUATION

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Оценка константы связи $\pi He^3 H^3$ при помощи уравнения Чу-Лоу

В работе получена оценка константы связи $\pi He^3 H^3$ при помощи уравнения Чу-Лоу. Проведен анализ дифференциальных сечений упругого рассеяния $\pi^{\pm}$-мезонов на He^3 при энергии 98, 120, 135 и 156 МэВ.

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The $\pi He^3 H^3$ Coupling Constant Estimation Using the Chew-Low Equation

In this paper it is presented an estimation of the $\pi He^3 H^3$ coupling constant using the Chew-Low equation and and a semi-phenomenological analysis of the $\pi^{\pm} He^3$ elastic differential cross sections at 98, 120, 135 and 156 MeV.

The investigation has been performed at the Laboratory of Nuclear Problems.

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1. Introduction

The theoretical estimations of the $\pi He^3 H^3$ coupling constant are between $f^2_{\pi He^3 H^3} = 0.08$ from simple impulse approximation [1] and $f^2_{\pi He^3 H^3} = 0.16$ from the dispersion relation for the pionic formfactor of He^3 and Goldberger-Treiman relations [2].

The experimental values for the π-nucleus coupling constant are now available only for Li^7 and Be^9 and are obtained from forward dispersion relations [3,4]

$$f^2_{Li^7} \cong f^2_{Be^9} \cong 0.06 .$$

The dispersion relations for the forward amplitude provide one of the most accurate phenomenological determinations of the coupling constants. With this method accurate results for the pion-nucleon as well as for the nucleon-nucleus coupling constants [5,6] have been obtained. The well-known difficulties of using this method are connected with the treatment of the unphysical cut and of the asymptotic behaviour of the amplitude. But for the pion-nucleus scattering another important problem in using the dispersion relations method arises from the lack of good information on $\pi^{\pm}$ total cross sections especially in the low energy region.

On the other hand, the hypothesis about the analyticity of the scattering amplitude in the $\cos\theta$ plane can be used to obtain information on the coupling constants /7,8/.

But for the $\pi He^3 H^3$ coupling constant determinations neither dispersion relations nor analytical continuation in the $\cos\theta$ plane can help because of the lack of the $\pi^{\pm} He^3$ total cross sections and because of the lack of measurements of the elastic differential cross sections at sufficiently large energies /9/*.

However information on the $\pi He^3 H^3$ coupling constant value can be obtained by using the Chew-Low equation for π-nucleus scattering /10/.

In this paper we present a semi-phenomenological analysis of the $\pi^{\pm} He^3$ elastic scattring (at 98, 120, 135 and 156 MeV /14/) in order to obtain information on the $f^2_{\pi He^3 H^3}$ coupling constant using the Chew-Low plot for the P_{33} partial wave.

*At low energies (up to 400 MeV) the analytical continuation method in $\cos\theta$ plane (with the optimal conformal mapping) doesn'n work even for $\pi^+ p$ scattering (the analog of our $\pi^+ He^3$ scattering) but this failure seems to be connected with the existence of the Δ_{33} resonance in the s channel.

2. The Chew-Low Equation for πHe^3 Scattering

The spin and isospin of the He^3 nucleus are $I = S = 1/2$ and He^3 therefore together with H^3 forms an isodoublet like the proton and neutron, and as a consequence of the charge independent interaction, as in the pion-nucleon case, absorption or emission of a pion by He^3 is allowed.

The conservation of the angular momentum and the parity implies that the pion can be absorbed by He^3 only in the $\ell = 1$, $J = 1/2$ state (P_{11} state) - the direct process, - and, in the limit of $m/M \to 0$ in all four P waves - the exchange process (the nucleus first emits the final pion and then absorbs the initial pion). The strongest exchange process can occur in the P_{33} state.

To obtain the Chew-Low equation for the P wave we follow the simplified treatment from the πp case according to Hamilton /11/. Using the same basic theoretical ideas as in the πp scattering, the singularity structure in the s variable (s is the total CMS energy) of the πHe partial waves turns out to be the same, except the right hand cut: $s_0 = (M_d + M_n)^2 \leq s \leq \infty$ (corresponding to the unphysical cut from forward dispersion relations).

We will use the peripheral method in order to reduce as much as possible the contribution of the short range parts of the interaction.

The partial wave amplitude

$$f^I_{\ell\pm}(s) = \frac{1}{2ik}\left(\eta^I_{\ell\pm}\, e^{2i\delta^I_{\ell\pm}} - 1\right) \qquad (1)$$

is replaced by the reduced partial wave amplitude

$$F^{I}_{\ell\pm}(s) = f^{I}_{\ell\pm}/k^{2\ell} \qquad (2)$$

for which for $\ell \geq 1$ is no pole at $s=0$ and the contribution of the cut $-\infty < s < 0$ is strongly suppressed and can be ignored.

The contribution of the short cut:

$(M-m^2/M)^2 \leq s \leq M^2 + 2m^2$ to partial waves for physical s is given by the angular integration of the exchange term (partial wave projection of the exchange term) which in the static approximation (small m/M) is given by:

$$F^{3/2}_{\ell\pm} = \frac{(-1)^{\ell}}{M^2-s} \cdot \frac{2Mf^2}{m^{2\ell}} \int_{-1}^{+1} (2xP_\ell(x) - P_{\ell\pm1}(x))dx \qquad (3a)$$

$$F^{1/2}_{\ell\pm} = \frac{(-1)^{\ell+1}}{M^2-s} \cdot \frac{Mf^2}{m^{2\ell}} \int_{-1}^{+1} (2xP_\ell(x) - P_{\ell\pm1}(x))dx. \qquad (3b)$$

Here f is called the equivalent pseudo-vector coupling constant

$$f^2 = \frac{G^2}{4\pi}\left(\frac{m}{2M}\right)^2 \qquad (4)$$

and is 0.08 in πp scattering.

The polar contributions to P partial waves in the physical region are (in the static approximation the above integrals vanish unless $\ell = 1$):

$$F^{1/2}_{1-} = \frac{\frac{2}{3}Mf^2}{m^2(s-M^2)}; \quad F^{3/2}_{1+} = \frac{\frac{8}{3}Mf^2}{m^2(s-M^2)}; \qquad (5)$$

$$F^{1/2}_{1+} = F^{3/2}_{1-} = \frac{-\frac{4}{3}Mf^2}{m^2(s-M^2)},$$

and adding the contribution of the direct Born term to P_{11}:

$$F^{1/2}_{1-} = \frac{-\frac{16}{3}Mf^2}{m^2(s-M^2)}.$$

Now we can construct a new analytic functions:

$$h^{I}_{\ell\pm}(s) = \frac{1}{s-M^2} \cdot \frac{1}{F^{I}_{\ell\pm}(s)}, \qquad (6)$$

which has a cut along $s_0 = (M_d + M_n)^2 \leq s \leq \infty$ and no pole at $s = M^2$. . Using Cauchy theorem we can write one substracted dispersion relation for $h(s)$::

$$\mathrm{Re}\, h^{I}_{\ell\pm}(s) = \frac{1}{A^{I}_{\ell\pm}} - \frac{s-M^2}{\pi} \mathcal{P} \int_{s_0}^{\infty} \frac{K'^3 ds'}{x^{I}_{\ell\pm}(s')(s'-M^2)^2(s'-s)}, \qquad (7)$$

where $x(s)$ is the elasticity factor:

$$x^{I}_{\ell\pm}(s) = \frac{\sigma^{I}_{\ell\pm}(s)_{\text{elastic}}}{\sigma^{I}_{\ell\pm}(s)_{\text{total}}}, \qquad (8)$$

with $1 \geq x \geq 0$ ($x = 1$ for the pure elastic case - for example πp up to ≈ 200 MeV) and $A^{I}_{1\pm}$ are constants related to f^2 :

$$A^{1/2}_{1-} = -\frac{16}{3}\frac{M}{m^2}f^2;\ A^{1/2}_{1+} = A^{3/2}_{1-} = -\frac{4}{3}\frac{M}{m^2}f^2;\ A^{3/2}_{1+} = \frac{8}{3}\frac{M}{m^2}f^2.$$

And now we can use the hypothesis that the integral

$$I(s) = \frac{1}{\pi}\mathcal{P}\int_{s_0}^{\infty}\frac{K'^3 ds'}{x(s')(s'-M^2)^2(s'-s)} \qquad (10)$$

as in the πp case, is a slowly decreasing function on s.

A simple approximation to the low energy behaviour of P waves is $I(s) \approx \text{const.} = \beta$ even if in the πHe^3 scattering the integral has a contribution from unphysical region $s_0 = (M_d + M_n)^2$ to $s_1 = (M+m)^2$.

The equation which is obtained can be used for continuation outside the physical region (Chew-Low plot) in order to obtain information on the $\pi He^3 H^3$ coupling constant (for P_{33}):

$$\frac{8}{3}\cdot\frac{M}{m^2}\cdot\frac{k^3}{x^{3/2}_{1+}(s)(s-M^2)}\cdot\frac{\eta^{3/2}_{1} + \sin(2\delta^{3/2}_{1+})}{1-\eta^{3/2}_{1+}\cos(2\delta^{3/2}_{1+})} = \frac{1}{f^2_{\pi He^3 H^3}} - (s-M^2)\beta. \qquad (11)$$

This equation represents a straight line $y = \alpha + \beta z$ (with $z = s - M^2$) .) .

In the πp case there is a good agreement between experiment and the straight line up to the resonance, but a disagreement appears for larger energies (Fig. 5, curve a). This is due to the approximation which assumes the cut-off function $v(k)$ to be constant. The cut-off function is introduced in the interaction Hamiltonian and it can be thought of some kind of Fourier transform of the particle density and actually eq. (11) should contain $(v(k))^2 k^3$ in place of k^3.

For example Layson /15/ has used a Yukawa type source distribution:

$$\rho(r) = \frac{a^2 e^{-ar}}{4\pi r} \qquad (12)$$

to obtain a smooth cut-off function $v(k)$: :

$$v(k) = \int e^{i\vec{k}\vec{x}}\rho(x)\,dx = \frac{1}{1+k^2 a^{-2}}\ (a^{-1} = 0.38\ \text{fm}) \qquad (13)$$

for which he was able to extend the validity of the Chew-Low formula.

This correction due to the non-point interaction between particles is more important in our scattering case. Using Ingraham's expression (eq. 4 from /10/) we can obtain the following equation for the Born approximation of the reduced partial wave:

$$F^{I}_{\ell\pm}(s) = \frac{2f^2_{\pi He^3 H^3}}{s - M^2} \; \frac{M}{m} \; \frac{\lambda^{\ell}_{2J,2I}(k)}{2\ell + 1} , \qquad (14)$$

where $\lambda^{\ell}_{2J,\,2I}(k)$ is given by:

$$\lambda^{\ell}_{2J,2I} = \{ a_I \,[\ell V_{\ell-1} + (\ell + 1) V_{\ell+1}] + \\ + [\tfrac{3}{4} + \ell(\ell + 1) - J(J + 1)]\,[V_{\ell+1} - V_{\ell-1}] \} \qquad (15)$$

with $a_{3/2} = 1$ and $a_{1/2} = -2$.

Here we assumed equal spin and isospin distributions $\rho_I(r) = \rho_J(r)$ and we defined

$$V_\ell(k) \equiv V_\ell = 4\pi \int J^2_\ell(kr)\,\rho(r)\, r^2\, dr , \qquad (16)$$

with normalization $V_\ell(0) = \delta_{\ell,0}$

$$V_\ell(k) = (-i)^\ell e^{-\frac{a^2 k^2}{2}} J_\ell\left(i\,\frac{a^2 k^2}{2}\right) , \qquad (17)$$

for $\rho(r) = \frac{1}{(\pi a^2)^{3/2}} e^{-r^2/a^2}$ with $a = 1.38$ fm.

For P_{33} the partial wave amplitude from πHe^3 scattering we obtain $\lambda^1_{33} = 2V_0(k) + V_2(k)$ instead of $\lambda^1_{33} = 2$ as in the π-nucleon. Figure shows the Layson's cut-off function and our $\lambda^1_{33}(k)$ as function of the pion kinetic energy.

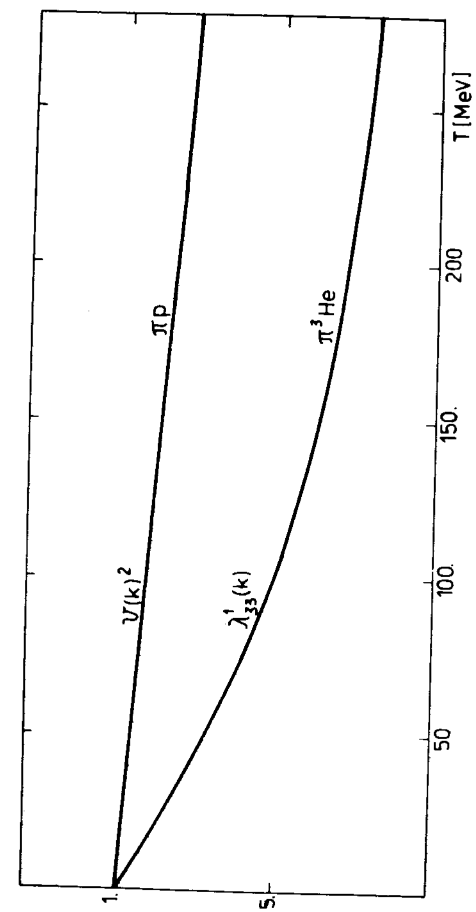

Fig. 1. The cut-off function πp kinetic energy for scattering (Layson /15/) and for πHe^3 scattering (eq. (15)).

3. Semi-Phenomenological Analysis of $\pi^{\pm}He^3$ Elastic Differential Cross Section

We have analysed experimental data on $\pi^{\pm}He^3$ elastic differential cross section at four energies: 98, 120, 135 and 156 MeV /14/.

At those energies the last significant partial waves seem to be the D and F, which means that for a normal phase shift analysis it is necessary to determine 20 or 28 phase shifts.

On the other hand up to now only elastic differential cross sections are available and for a rigorous phase shift analysis total cross section and polarization measurements are also necessary.

For these reasons we have chosen a theoretical model of πHe^3 scattering to support our analysis.

We have performed an analysis of experimental data using the computed phase shifts from optical model and keeping as free parameters only few partial waves (for $\ell \geq 1$).

The Kisslinger type of an optical model seems to give a qualitative agreement with exoerimental data /12/: : the position for the minima in the differential cross section and the fact that the minima are less deep for $\pi^{-}He^3$ reaction than for $\pi^{+}He^3$ (Fig.2,3, curve (a)).

But there remain large discrepancies at small scattering angles (similar to those observed for $\pi^{\pm}He^4$ elastic scattering /13/). In a preliminary fit of the differential cross sections we have found for our energy interval a common normalization factor (2/3) to the scattering amplitude obtained from

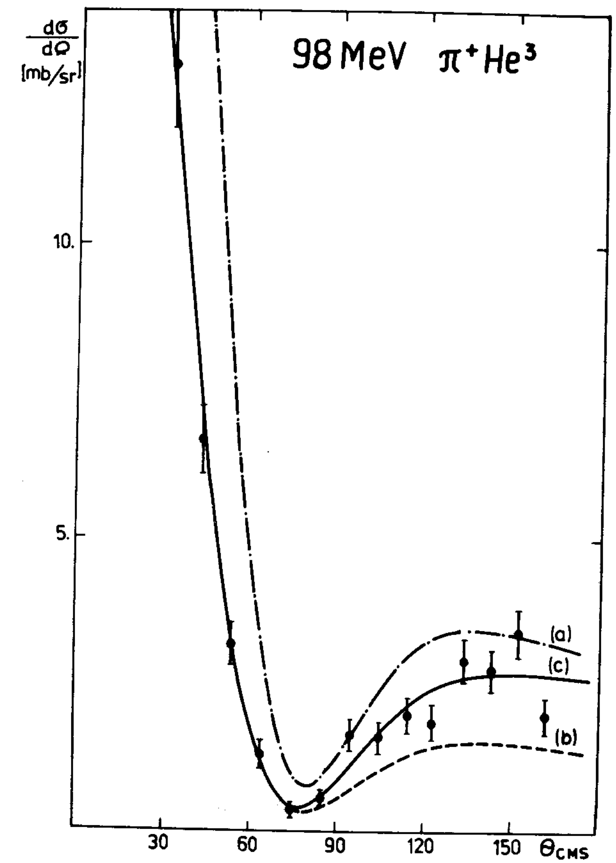

Fig. 2. The differential cross section for $\pi^{+}He^3$ elastic scattering. curve (a) Optical model calculations /12/ ; curve (b) 2/3 normalization of scattering amplitude from the optical model; curve (c) the S_3 and P_{33} as free parameters.

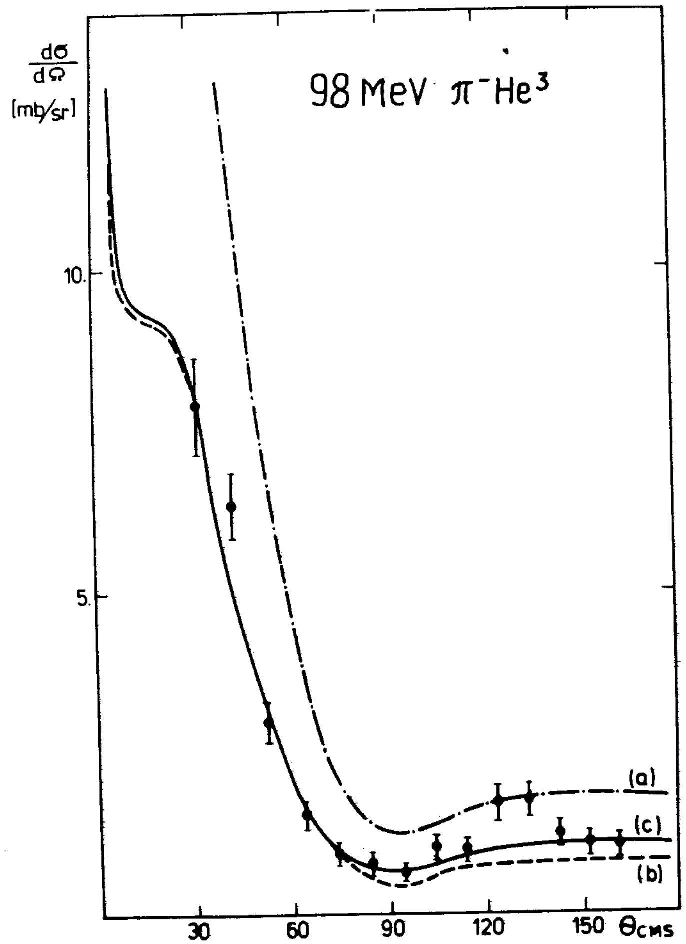

Fig. 3. The same as in fig. 2 - for $\pi^- He^3$ elastic scattering.

the optical model (Fig. 2,3, curve (b)) and we have tested an optimal combination (from the χ^2 point of view for free parameters.

The least χ^2 is obtained if the S_3 and P_{33} partial waves are free parameters (we have fitted the π^+ and π^- differential cross sections together), all other phases being fixed at the theoretical model values (and the common normalization factor of the scattering amplitude being fixed at 2/3) - fig.2, 3, curve (c).

The χ^2/N_{points} obtained for these four energies is ≈ 1.37.

The errors on phase shifts are determined in a standard way: the change in parameter that changes χ^2 by 1 when all other parameters are searched.

In fig. 4 both $Re\,\delta^1_{33}$ and $Im\,\delta^1_{33}$ obtained from the fit to the differential cross sections and also computed from optical model are presented. In this figure we can observe that there is a resonant behaviour of the δ^1_{33} at around 140 MeV where $Re\,\delta^1_{33}$ is going through zero and $Im\,\delta^1_{33}$ has a maximum, but for a definitive conclusion more experimental information is necessary.

Using the values of δ^1_{33} obtained from our fit in eq. (11) we have found the following value of the $\pi He^3 H^3$ coupling constant (see the Chew-Low plot - fig. 5, curve (b))

$$f^2_{\pi He^3 H^3} = 0.048 \pm 0.009.$$

Taking into account the cut-off function (which in πHe^3 scattering is more important than in the πp case) - eq. (15), we have found the following value (fig. 5, curve (c))

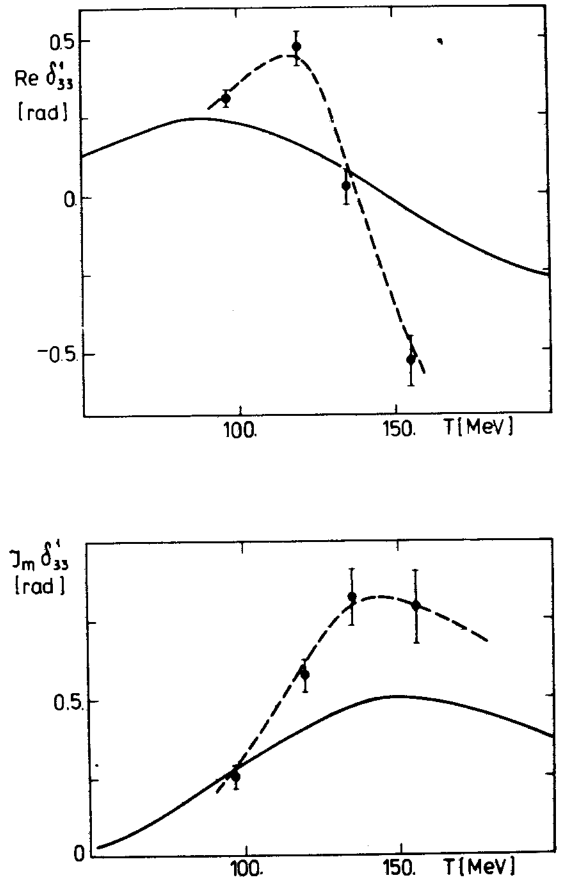

Fig. 4. The $Re\,\delta^{1}_{33}$ and $Im\,\delta^{1}_{33}$ from the fit and also computed from optical model (full line).

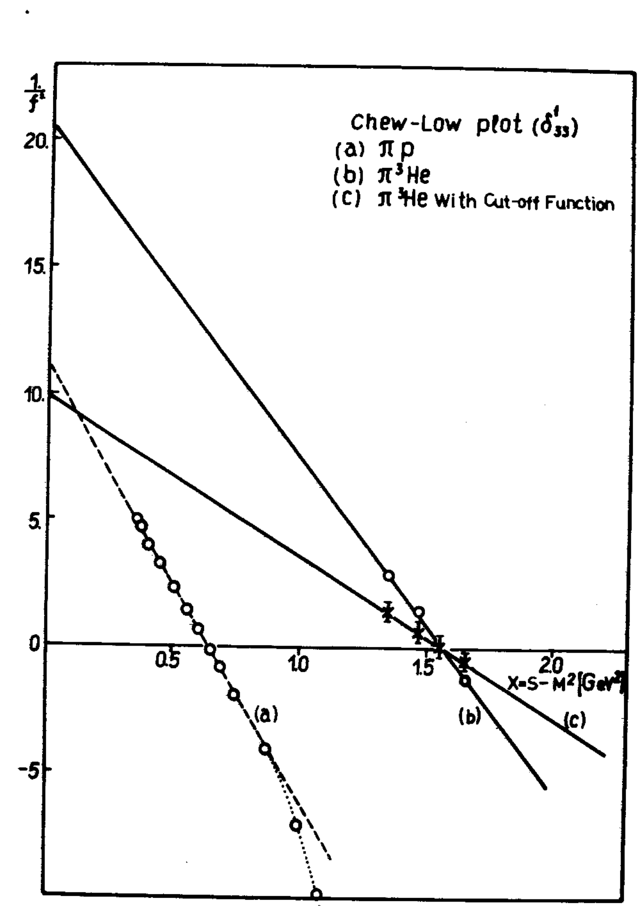

Fig. 5. The Chew-Low plot for P_{33} partial wave. (a) for πp scattering (from phase shifts analysis); (b) for πHe^3 without cut-off function; (c) for πHe^3 with cut-off function (eq. (15)).

$$f^2_{\pi He^3 H^3} = 0.101 \pm 0.018.$$

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References

1. T.E.O.Ericson, M.P.Locher. Nucl.Phys., B148, 1 (1970).
2. B.Z.Kopeliovich. Yadern.Phys., 18, 1157 (1973).
3. P.Osland. Nucl.Phys., B56, 308 (1973); C.Wilkin, C.R.Cox, J.J.Domingo, K.Gabathuler, E.Pedroni, J.Rohlin, P.Schwaller, N.W.Tanner. Nucl.Phys., B62, 61 (1973).
4. G.T.A.Squier, M.E.Cage, G.J.Pyle, A.S.Clough, G.K.Turner, B.W.Allardyce, C.J.Batty, D.J.Baugh, W.J.McDonald, R.A.J.Riddle, L.H.Watson. Phys.Rev.Lett., 31, 389 (1973); M.Ericson, M.Krell. Preprint LYCEN 7457 (1974).
5. V.K.Samaranayake, W.S.Woolcock. Nucl. Phys., B48, 205 (1972); N.Sznajder. Hald. Nucl.Phys., B48, 549 (1972).
6. M.Locher. Nucl.Phys., B23, 116 (1970); B36, 634 (1972).
7. G.F.Chew. Phys.Rev., 112, 1380 (1958); R.E.Cutkosky, B.B.Deo. Phys.Rev.Lett., 20, 1272 (1968).
8. G. von Holtey, D.Schwela. Nucl.Phys., B41, 438 (1972); S.Dubnicka, O.V.Dumbrais, F.Nichitiu. Nucl.Phys., A217, 535 (1974).

9. O.V.Dumbrais, F.Nichitiu, Yu.A Shcherbakov.Yadernaya Fiz., 16, 327 (1974).
10. R.L.Ingraham. Lett. Nuovo Cimento, 9, 331 (1974).
11. J.Hamilton. High Energy Physics, v. 1 (Ed. E.Burhop), New York, Academic Press, 1967.
12. R.Mach, F.Nichitiu, Yu.A.Schcherbakov. Phys.Lett., B53, 133 (1974).
13. R.Mach. Nucl.Phys., A154, 56 (1973).
14. Yu.A.Shcherbakov et al. JINR preprint, Dubna, 1975.
15. W.M.Layson. Nuovo Cim., 20, 1207 (1961).

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