

# сообщения объединенного института ядерных исследований 

D99

E1-90-50

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APPLICATION OF A GRADIENT SEARCH METHOD
FOR THE RECONSTRUCTION
OF STRAIGHT TRAJECTORIES

## INTRODUCTION

A method for the reconstruction of straight trajectories based on a gradient search for maximum of the trajectory parameter function has been presented in paper/1/ and developed in ${ }^{\prime 2 /}$. In the second work two-dimensional trajectory of a particle that came through the coordinate detector set-up has been investigated. The gradient search method has been compared with the classical one, based on the minimum $\chi^{2}$ search. It has been shown that both methods give approximately the same results in the case when experimental points are not too much deflected from the straight line. However, starting from some "critical" values of deflection the gradient search method allows to determine parameters of the trajectory the more precisely, the bigger the deviation of one of the experimental points is, providing that the other points lie near to the straight line. This is a result of a corrective property of this method that automatically does not take into account experimental points that are too much deflected from the straight trajectory. In the present paper the gradient search method has been applied for the data obtained in the experiment for fragmentation of oxygen nuclei with the impulse of $4.5 \mathrm{GeV} /$ nuc. Parameters of the trajectory of the oxygen nucleus before the target and it's fragment after the target have been obtained by $x^{2}$ method and then by the gradient method. The point of interaction as a point of a minimal distance between the trajectory of an incident particle and it's fragment has been defined. The histograms of the $z$-coordinate of the interaction points obtained by both methods are compared.

## 1. THE EXPERIMENTAL SET-UP DESCRIPTION

The experimental set-up "Anomalon" is consistent with three sections of three or four coordinate detectors each. Here we are only interested in two sections. The first one: three wire chambers before the target, and the second one: four wire chambers between the target and the magnet. Two car-
bon plates of thickness 8 mm and 20 mm have been used as a target.

## 2. EXPERIMENTAL DATA AND THE "CLASSICAL" METHOD OF THEIR TREATMENT

We start from a set of points, every one described by $\left(x_{i k}, s_{x i k}, y_{i k}, s_{y i k}, z_{k}\right) i=1, \ldots, N_{k}, k=1, \ldots, 7$. Here $z_{k}$ is a coordinate of a $k$-th detector along the axis of the step-up, $\mathrm{x}_{\mathrm{ik}}, \mathrm{y}_{\mathrm{ik}}$ are horizontal and vertical coordinates of $i$-th experimental point at $k$-th detector, $s_{x i k}, s_{y i k}$ stand for standard deviations for determination of respective coordinates of an experimental point. The procedure of obtaining above number from the wire chambers data will be described in one of the next papers. In the ideal case we should have only one point in every chamber (i.e. $N_{k}=1, k=1, \ldots, 7$ ) and the points should lie exactly at the straight line. In practice in many cases there is none, or there are several points at one detector. One has to choose these that are the least deviated from the straight line.

The procedure that is normally used is to look through every combination of points and to find out (using $x^{2}$ test) if they can be regarded as belonging to the straight line. Separate fits have been done for the first section of detectors (before the target) and the second section (after the target).

Fig. la shows the histogram for the coordinate " $z$ " that has been found for a point of interaction for selected in above way cases.


Fig. 1. Histogram of " $z$ " coordinate of the point of interaction. Trajectories of particles have been obtained: a) by $X^{2}$ method, b) by the gradient search method.

## 3. APPLICATION OF A GRADIENT SEARCH METHOD

The gradient search method and formulae for it's application for straight trajectory reconstruction have been presented in paper ${ }^{/ 2 /}$. Here we follow the rules given there with exception of that now we deal with the three-dimensional trajectory instead of the two-dimensional one. The formula of Yu.A.Yatsunenko ${ }^{1 / /}$ for reconstruction of the straight trajectory in the three-dimensional space is given by:
$R\left(A_{x}, B_{x}, A_{y}, B_{y}\right)=\sum_{n=1}^{N}\left(\sigma_{x n} * \sigma_{y n} * F\left(x_{n}-a_{n}, \sigma_{x n}, y_{n}-b_{n}, \sigma_{y n}\right)\right)$.
Here $N$ is the number of detectors. The function $F$ is given by the formula:
$\mathrm{F}\left(\mathrm{x}_{\mathrm{n}}-\mathrm{a}_{\mathrm{n}}, \sigma_{\mathrm{xn}}, \mathrm{y}_{\mathrm{n}}-\mathrm{b}_{\mathrm{n}}, \sigma_{\mathrm{yn}}\right)=1 /\left(2 \pi \sigma_{\mathrm{xn}} \sigma_{\mathrm{yn}}\right) * \exp \left(\frac{\left(\mathrm{x}_{\mathrm{n}}-\mathrm{a}_{\mathrm{n}}\right)^{2}}{2 \sigma_{\mathrm{xn}}^{2}}-\frac{\left(\mathrm{y}_{\mathrm{n}}-\mathrm{b}_{\mathrm{n}}\right)^{2}}{2 \sigma_{\mathrm{yn}}^{2}}\right)$,
where $x_{p_{2}}=A_{x} * Z_{n}+B_{x}, y_{n}=A_{y^{*}} * Z_{n}+B_{y}$. Other remarks given in work ${ }_{2 /}$ that have made the procedure of maximum calculation more time-economical have been applied as well. As an initial point for iteration, values of parameters obtained by $\chi^{2}$ method have been adopted.

The gradient search method is expected to be especially effective in the case when one of the experimental points belongs to the other set of data, e.g. to the background, and does not fit to the others. Let us suppose that, because of an uneffectivity of the chamber, there is no track of a particle in a first chamber (fig. 2) but, instead there is a point from a background. The background point is near enough the trajectory, so the test $\chi^{2}$ would be still positive. However the gradient method would automatically exclude such a point from the treatment.

The histogram of " $z$ " coordinate of interaction point when trajectories of particles have been calculated by the gradient search method is presented
 in fig.lb.

Fig. 2. The set-up of four coordinate detectors with tracks of a charged particle. There is a false track in the first from the left-side detec. tor; the broken line - trajectory obtained by $\chi^{2}$ method, the full line trajectory obtained by the gradient search method.


The histogram for numbers of iteration steps that has been necessary to obtain the maximum of the function (1) is shows in fig.3.

Fig. 3. The number of iteration steps to obtain the maximum of the function/1/by the gradient search method, starting from parameters calculated by $X^{2}$ method.

## 4. COMPARISON OF BOTH THE METHODS

As can be seen from fig. 1, results obtained by the gradient search method correspond almost exactly to these obtained by $x^{2}$. It comes from the fact that, as can be seen in fig. 3 , in most cases only a few numbers of iteration steps have been enough to get a maximum, so the parameters of trajectories obtained by the gradient method are very similar to these obtained by $\chi^{\mathcal{L}}$ method. As has been pointed out in ${ }^{\prime 2 /}$, it takes place when deflections of experimental points from the trajectory are not too large. In our experiment, deflections are usually small, so the corrective properties of the gradient search method cannot be visible.

To see how the gradient search method would work in the case of the presence of big deflections, we have repeated the above calculations assuming that one of detectors has been shifted by about three mean standard deviations into " $x$ " direction. Histograms of " z " coordinate of interaction point obtained by both the methods are shown in fig. 4.

## 5. CONCLUSION

As could be seen, the gradient search method gives exactly the same results as the classical $x^{2}$ method in the case when the deflections of experimental points are small enough, there are no systematic errors and the background is small. In the case when the experimental data are not so "clear" and especi-
ally when any systematic arrors are involved, the gradient search method gives results much closer to the reality than the classical method. Computing time for the gradient search method has been about $30 \%$ bigger than for the classical one. One has to mention that here the simplest version of the classical method has been used. In fact, one should compare the gradient method with such a version of the $x^{2}$, which allows one to correct the trajectory when one of experimental points is too much deflected (as it is automatically done by the gradient method). The time relation should then be more favoured for the gradient method. Such a comparison could be done only if we had more detectors in our set-up. Having only three detectors, the corrections of parameters by excluding one of experimental points are impossible.

The authors are very grateful for Yu.A.Yatsunenko for introducing into the problem, the interest into the work and helpful discussion. The authors are grateful also to D.A.Smolin for fruitful comments and suggestions.

## REFERENCES

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