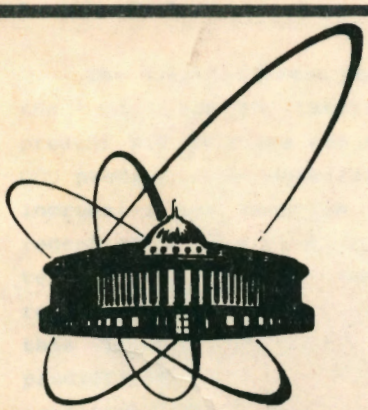

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# NEXT-TO-LEADING-ORDER QCD ANALYSIS OF STRUCTURE FUNCTIONS WITH THE HELP OF JACOBI POLYNOMIALS

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## 1. Introduction

The deep-inelastic scattering (DIS) is commonly considered as the best place for testing quantum chromodynamics (QCD). Though present DIS data are not accurate enough to check the fundamental QCD prediction on decreasing the strong coupling constant $\alpha_s$ with increasing the momentum transfer squared $Q^2$, the precision of recent BCDMS data [1-8] appears to be sufficient to quantitatively test the specific QCD predictions for scaling violations and to reliably determine the QCD mass-scale parameter $\Lambda$. Clearly, for this purpose a precise method of calculation of the QCD predictions is required. Since the QCD analysis of the data represents an extensive fit of the predictions to a large number of experimental points, it is desirable that this method would be fast.

It is well known that these requirements are fulfilled by the method based on Jacobi polynomial reconstruction of structure functions (SF) suggested in ref. [9] and further studied, developed and applied to the analysis of experimental data in refs. [10-13,4,5,7]. In particular, the method was used for the QCD analysis of nonsinglet SFs in refs. [10-12] and extended to the singlet case in our previous paper [13].

In ref. [13], we have limited our analysis of the singlet SF to the leading order (LO) of perturbation theory. Here we describe further development of the method for a complete singlet + nonsinglet QCD analysis of SFs, including next-to-leading order (NLO) QCD corrections. Due to a substantial contribution of the longitudinal SF at low values of the Bjorken $x$ and a strong correlation between the gluon density in a nucleon and $\Lambda$, the complete QCD analysis requires a careful study of the reconstruction accuracy and adjustment of the fit parameters. The corresponding computer code based on the standard MINUIT program [14] has been already applied for QCD fits of the BCDMS carbon and hydrogen data in refs. [4] and [5,7]. Also included into the code are procedures allowing to take into account the preasymptotic corrections to the leading-twist massless theory: flavour threshold corrections (similar to refs. [15,16]), target mass- and

higher twist-corrections (according to refs. [17-19]). Detailed description of these procedures, as well as the results of calculations of NLO-corrections to the longitudinal structure function (according to ref. [20]) and estimates of the uncertainties of the QCD fits due to the preasymptotic corrections (including the higher-order ones), may be found in ref. [21].

The rest of this paper is organized as follows. In Section 2 we briefly review the perturbative QCD predictions for DIS. The Jacobi polynomial method for calculation of the QCD predictions for SFs is discussed in Section 3. The method is tested with the help of the BCDMS hydrogen data in Section 4. The conclusions are summarized in Section 5.

## 2. Perturbative QCD predictions for structure functions

According to the QCD factorization theorem, the SFs are given as the convolution of quark, $q_i$, and gluon, $G$, (partons) densities with the coefficient functions $C_k$ (which are proportional to the corresponding cross sections of the hard process of the absorption of virtual photon or intermediate bosons by a parton):

$$f_k(x,Q^2) = \int_x^1 \frac{dy}{y} \left[C_k^{NS}\left(\frac{x}{y},Q^2\right)A^{NS}(y,Q^2) + C_k^{SI}\left(\frac{x}{y},Q^2\right)A^{SI}(y,Q^2) + C_k^{G}\left(\frac{x}{y},Q^2\right)A^{G}(y,Q^2)\right], \qquad (1)$$

where $f_k$, $k = 1, 2, 3$, are related to the usual SFs by:

$$f_1 = \frac{1}{2}F_1, \quad f_2 = F_2/x, \quad f_3 = F_3.$$

The functions $A^G \propto G$, $A^{SI}$ and $A^{NS}$ are certain flavour singlet and nonsinglet combinations of the parton densities. In the case of charged lepton-proton scattering, assuming $m$ doublets of zero mass quarks with standard charge assignments, these combinations are the following:



$$A^G = \frac{5}{18}G(x,Q^2), \quad A^{SI} = \frac{5}{18}\Sigma(x,Q^2), \quad A^{NS} = \frac{1}{6}\Delta(x,Q^2),$$

$$\Sigma = \sum_{i=1}^{2m}(q_i+\bar{q}_i), \qquad \Delta = (u-\bar{u}+d-\bar{d}+c-\bar{c}+s-\bar{s}+\ldots). \qquad (2)$$

The coefficient functions $C_k$ can be expanded in powers of the running coupling constant $\alpha_s(Q^2)$, which obeys the QCD beta function renormalization group equation, and, in the next-to-leading order, it is given by the implicit equation [22]:

$$\ln\frac{Q^2}{\Lambda^2} = \frac{4\pi}{\beta_0\alpha_s} - \frac{\beta_1}{\beta_0^2}\ln\left(\frac{4\pi}{\beta_0\alpha_s} + \frac{\beta_1}{\beta_0^2}\right), \qquad (3)$$

where $\beta_0 = 11 - \frac{2}{3}f$, $\beta_1 = 102 - \frac{38}{3}f$; $f$ is the number of active flavours. The $\Lambda$ is an unknown integration constant to be determined from experiment. Often the following NLO formula is used [23]:

$$\alpha_s^{(1)}(Q^2) = \frac{4\pi}{\beta_0\ln\frac{Q^2}{\Lambda^2}}\cdot\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\ln\frac{Q^2}{\Lambda^2}}{\ln\frac{Q^2}{\Lambda^2}}\right]. \qquad (3')$$

In the BCDMS $Q^2$-range and at $\Lambda \approx 200$ MeV eq. (3'), as compared with the equally valid NLO expression (3), gives the $\alpha_s$-value lower by about 2%, and, leads to the $\Lambda$-value higher by ~16 MeV. Eq. (3') is used in this paper.

The $Q^2$ dependence of the functions $A^i(x,Q^2)$ is governed by solutions of the generalized Altarelli-Parisi-Lipatov integro-differential evolution equations, the integration kernels of which or the splitting functions $P_{ij}(x,Q^2)$ are the probabilities of the partonic transitions $j \to i$.

The coefficient functions, splitting functions and parton densities are not physical quantities and depend on the renormalization (factorization) scheme. In the following, we use the perturbative QCD results obtained in the modified minimal subtraction ($\overline{MS}$) renormalization scheme [23]. In particular, the parton densities defined in this scheme are universal quantities [24] and satisfy the usual momentum sum rule:

$$\langle x_q \rangle + \langle x_G \rangle = \int_0^1 dx \cdot x \cdot [\Sigma(x,Q^2) + G(x,Q^2)] = 1. \qquad (4)$$

The convolution integrals can be transformed into multiplications of the Mellin moments

$$f(n) = \int_0^1 dx \cdot x^{n-1} f(x).$$

E.g., istead of eq. (1) we have:

$$f_k(n,Q^2) = C_k^{NS}(n,Q^2)A^{NS}(n,Q^2) + C_k^{SI}(n,Q^2)A^{SI}(n,Q^2) + C_k^{G}(n,Q^2)A^{G}(n,Q^2), \qquad (1')$$

where the $Q^2$-dependence of $C_k(n,Q^2)$ is given by the series (the indices denoting the NS-, SI-quark and gluon contributions are omitted):

$$C_k(n,Q^2) = 1 + \frac{\alpha_s(Q^2)}{4\pi} B_{k,n}^{(1)} + [\frac{\alpha_s(Q^2)}{4\pi}]^2 B_{k,n}^{(2)} + \ldots . \qquad (5)$$

The expansion coefficients $B_{k,n}^{(1)}$ can be found in ref. [25].

Given the initial moments $A^i(n,Q_0^2)$ at certain reference point $Q_0^2$ (which are not predicted by perturbative QCD), the solution of the Mellin transformed evolution equations is straightforward:

$$A_\pm^{NS}(n,Q^2) = \phi_\pm^{NS}(n,Q^2,Q_0^2)A_\pm^{NS}(n,Q_0^2),$$
$$A^{SI}(n,Q^2) = \phi_{qq}(n,Q^2,Q_0^2)A^{SI}(n,Q_0^2) + \phi_{qG}(n,Q^2,Q_0^2)A^{G}(n,Q_0^2),$$
$$A^{G}(n,Q^2) = \phi_{Gq}(n,Q^2,Q_0^2)A^{SI}(n,Q_0^2) + \phi_{GG}(n,Q^2,Q_0^2)A^{G}(n,Q_0^2). \qquad (6)$$

The index + (-) in the NS case denotes the evolution of a crossing even (odd) combination of parton densities, i.e. of the one containing $q_i+\bar{q}_i$ ($q_i-\bar{q}_i$). The $\phi$-functions are determined by the Mellin transformation of the splittind function. For example, the nonsinglet $\phi$ - function has a form:



$$\phi_\pm^{NS}(n,Q^2,Q_0^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\cdot[\gamma_\pm^{(0)}(n) / 2\beta_0]} \cdot H_\pm^{NS}(n,Q^2,Q_0^2), \qquad (7)$$

with

$$H_\pm^{NS}(n,Q^2,Q_0^2) = 1 + \frac{1}{4\pi}[\alpha_s(Q^2) - \alpha_s(Q_0^2)] \cdot Z_\pm^{NS}(n),$$

$$Z_\pm^{NS}(n) = [\gamma_\pm^{(1)}(n) - \gamma_\pm^{(0)}(n)\beta_1/\beta_0]/2\beta_0$$

and

$$\gamma(n,Q^2) = -2P(n,Q^2) = \frac{\alpha_s}{4\pi}\gamma^{(0)}(n) + (\frac{\alpha_s}{4\pi})^2\gamma^{(1)}(n) + \ldots; \qquad (8)$$

parton indices are omitted here and below. Similar expressions for the singlet $\phi$-functions are given in eqs. (2.138) - (2.143) of the review [25]. Note, that in the case of charged lepton-proton scattering, the moments $A^1(n,Q^2)$ are related to the ones introduced in [25] by:

$$A^G(n,Q^2)=\frac{5}{18}\langle G(Q^2)\rangle_n, \quad A^{SI}(n,Q^2)=\frac{5}{18}\langle \Sigma(Q^2)\rangle_n, \quad A^{NS}(n,Q^2)=\frac{1}{6}\langle \Delta(Q^2)\rangle_n.$$

In fact, the QCD predictions in terms of the Mellin moments have been originally obtained with the help of the Wilson operator product expansion (OPE); $\gamma_\pm(n,Q^2)$ at even (odd) $n$ are just the anomalous dimensions of the spin-$n$ nonsinglet operators. The anomalous dimensions have been calculated up to the next-to-leading order in [26] and represented in a simple analytical form in [27,28]. For the gluon-gluon anomalous dimension we use a generally accepted result of refs. [28,29] which slightly differs from the one of refs. [26,27].

It should be noted that in the crossing even (odd) case the anomalous dimensions beyond the leading order coincide with the moments in eq. (8) only at even (odd) $n$ [30]. To find out these moments at any $n$, an analytical continuation should be performed for even and odd $n$ separately. As a result, the moments in eq. (7) are related to the OPE anomalous dimensions $\gamma_n^{NS}$ and $\gamma_n^{SI}$ by the following expressions:

$$\gamma_{\pm}^{(1)}(n) = \gamma_n^{NS(1)} + \eta_{\pm}(n)\Delta\gamma_n^{NS(1)},$$

$$\gamma^{(1)}(n) = \gamma_n^{SI(1)} + \eta_{+}(n)\Delta\gamma_n^{SI(1)},$$

$$\eta_{\pm}(n) = \pm 1 - (-1)^n,$$

where $\Delta\gamma_n^{(0)} = 0$, and, the corrections $\Delta\gamma_n^{(1)}$ are known to be quite small and vanishing very fast with $n$ ($\Delta\gamma_n^{NS} \propto 1/n^6$) [30]. For this reason the corrections are often neglected in the literature. However, they appear to be quite important for the Jacobi polynomial SF-reconstruction due to factorially large coefficients weighing the contributions of the moments. The corrections in the crossing even (odd) case can be simply taken into account by the following replacements in the OPE anomalous dimensions [31,32]:

$$\begin{aligned}
&(-1)^n \quad \to \pm 1,\\
&S_2'(\tfrac{1}{2}n) \to (-1)^n\{\pm S_2'(\tfrac{1}{2}n) + \eta_{\pm}(n)[-2S_2(n) + \zeta(2)]\},\\
&S_3'(\tfrac{1}{2}n) \to (-1)^n\{\pm S_3'(\tfrac{1}{2}n) + \eta_{\pm}(n)[-4S_3(n) + 3\,\zeta(3)]\},\\
&\tilde{S}(n) \to (-1)^n[\pm\tilde{S}(n) + \eta_{\pm}(n)\,\tfrac{5}{8}\,\zeta(3)],
\end{aligned} \qquad (9)$$

where the series $S_m(n)$ and the alternating series $S_m(\frac{1}{2}n)$, $\tilde{S}(n)$ are defined in [27] and $\zeta(z)$ is the Riemann zeta function, $\zeta(2) = \pi^2/6$, $\zeta(3) \cong 1.20205\ 69031\ 59594$.

To conclude this brief theoretical summary, the QCD predictions for the moments of the nucleon SFs are given by eq. (1') together with eqs. (5), (6), (8) and (9).

### 3. Jacobi polynomial reconstruction method

The evolution equations allow one to calculate the $Q^2$-dependence of the parton densities $A^i$ provided they are given at a certain reference point $Q_0^2$. The densities $A^i(x,Q_0^2)$ are usually parametrized on the basis of plausible theoretical assumptions concerning their behaviour near the end points $x = 0$, 1, e.g.:



$$\begin{aligned}
xA^{NS}(x,Q_0^2) &= a_{NS}x^{\mu_{NS}}(1-x)^{\nu_{NS}}(1+\gamma_{NS}x),\\
xA^{SI}(x,Q_0^2) &= a_{SI}[x^{\mu_{SI}}(1-x)^{\nu_{SI}} + a_{SEA}x^{\mu_{SEA}}(1-x)^{\nu_{SEA}}],\\
xA^{G}(x,Q_0^2) &= a_G x^{\mu_G}(1-x)^{\nu_G}.
\end{aligned} \qquad (10)$$

The evolution equations can be solved and QCD predictions for SFs obtained with the help of various numerical algorithms [33-35]. Although straightforward, these methods are not cheap in terms of computer time and meet a problem of accumulation of the rounding errors. Therefore a number of analytic methods has been developed to solve these equations with a lower price. One of the simplest and fastest possibilities is the SF reconstruction from the QCD predictions for its Mellin moments as given in an analytical form in eq. (1'). The Jacobi polynomials are especially suited for this purpose since they allow one to factor out an essential part of the $x$-dependence of the SF into the weight function [9]. Thus, given the Jacobi moments $a_m(Q^2)$, a structure function $f(x,Q^2)$ may be reconstructed in a form of the series

$$xf(x,Q^2) = \lim_{M\to\infty} x^{\beta}(1-x)^{\alpha} \sum_{m=0}^{M} a_m(Q^2)\Theta_m^{\alpha\beta}(x), \qquad (11)$$

where the Jacobi polynomials

$$\Theta_m^{\alpha\beta}(x) = \sum_{j=0}^{m} c_j^m(\alpha,\beta)x^j$$

satisfy the orthogonality relation with the weight $x^{\beta}(1-x)^{\alpha}$ (see ref. [10] for details). The Jacobi moments are just linear combinations of the Mellin ones:

$$a_m(Q^2) = \sum_{j=0}^{m} c_j^m(\alpha,\beta)f(j+2,Q^2). \qquad (12)$$

Their $Q^2$-dependence thus simply follows from the QCD eq. (1').

It was shown that a fast convergence of the series (11) can be achieved with an appropriate weight function. Originally, a $Q^2$-dependent weight function (with $\alpha = \alpha(Q^2)$) was proposed [9].

Later on, it was recognized [10,12,13] that a good reconstruction accuracy (better than 1%) can be obtained with constant values of $\alpha$ and $\beta$, and with a reasonable number $N_{max}$ of the terms retained in the series. As expected, the choice $\alpha \approx 3$ and $\beta \approx 0.5$ for the weight function parameters appears to be optimal in the nonsinglet case. For a singlet structure function parametrized at $Q_0^2 = 5\ GeV^2$ as the singlet density in eq. (10) with $\mu_{SI} = 0.25$, $\nu_{SI} = 3$, $\mu_{SEA} = 0$ and $\nu_{SEA} = 8$, two sets of optimal $\alpha$, $\beta$ values have been found [13]: $\alpha_1 \approx 3$, $\beta_1 \approx 0.2$ and $\alpha_2 \in (-0.8,\ 3.3)$, $\beta_2 \approx -0.8$. The relation $\beta_1 \approx \beta_2 + 1$ between the two $\beta$-values is merely a consequence of the polynomial expansion.

The analysis of the SF reconstruction accuracy in ref. [13] does not take into account a rapid $Q^2$-evolution of the sea quarks and gluons. Since the evolution effectively leads to the appearance of a negative power of $x$ in their $x$-distributions [36], we may expect decreasing the optimal $\beta$-value with $Q^2$. This is indeed confirmed (fig. 1) by the analysis of the relative reconstruction accuracy

$$\Delta_k^M(Q^2) = \{\frac{1}{N}\sum_{i=1}^{N}[\frac{F_k^M(x_i,Q^2) - F_k^{test}(x_i,Q^2)}{F_k^{test}(x_i,Q^2)}]^2\}^{1/2}, \quad N = 11, \qquad (13)$$

of a singlet-like test function $F_k^{test}$, $k = 2, L$. The index $M$ indicates that the function $F_k^{test}$ was reconstructed from its first $M+1$ moments. We have approximated the singlet-like structure function by

$$F_k^{test}(x,Q^2) = \sum_{j=1}^{3} c_j x^{\beta_j}(1-x)^{\alpha_j}, \qquad (14)$$

The parameters $c_j$, $\alpha_j$, $\beta_j$, $j = NS, SI$ and $G$ at various values of $Q^2$ are given in Table 1. They have been calculated with the help of the results of a QCD fit to the BCDMS hydrogen data. The approximation (14) is sufficient for studying the reconstruction accuracy, and, as the moments of its rhs are exactly known, it avoids the necessity of the "exact" solution of the evolution



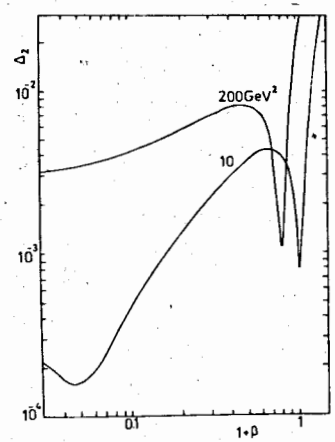


Fig. 1. The relative reconstruction accuracy (13) of a singlet-like SF $F_2(x,Q^2)$ (defined in eq. (14) and table 1) as a function of the weight function exponent $\beta$; calculations are performed with $\alpha = 3$, $N_{max} = 12$ and $Q^2 = 10,\ 200\ GeV^2$.

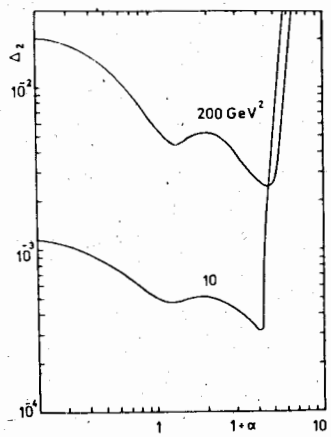


Fig. 2. The same dependence as in Fig. 1 but as a function of the weight function exponent $\alpha$ with fixed $\beta + 1 = 10^{-5}$.

Table 1. The parameters $c_j$, $\alpha_j$, $\beta_j$ in eq. (14) are given in the 1-st, 2-nd and 3-rd column for each $j$. The two values at each $Q^2$ correspond to the structure functions $F_2$ and $F_L$, respectively.

| $Q^2$ $GeV^2$ | NS | | | SI | | | G | | |
|---|---|---|---|---|---|---|---|---|---|
| 10 | .763 | 2.74 | .901 | .618 | 3.22 | .052 | -.553 | 12.6 | .757 |
| | .034 | 4.44 | .734 | .023 | 4.99 | -.001 | .025 | 13.9 | -.259 |
| 25 | .723 | 2.88 | .860 | .513 | 3.26 | -.037 | -.195 | 11.7 | .436 |
| | .027 | 4.56 | .694 | .016 | 4.99 | -.083 | .019 | 14.4 | -.340 |
| 200 | .650 | 3.12 | .790 | .378 | 3.37 | -.173 | -.049 | 10.9 | .054 |
| | .017 | 4.76 | .626 | .009 | 5.03 | -.210 | .012 | 14.3 | -.454 |

equations for this purpose.

It may be seen from figures 1 and 2 that an optimal choice of the weight function parameters $\alpha$ and $\beta$ in the case of a singlet-like structure function $F_2$ would be $\alpha \in (0, 4)$ and $\beta$ close to $-1$ or $\beta \approx -0.15$. In the BCDMS kinematic range this choice guarantees $\Delta_2^{11} < 0.3\%$. This result is more than one order of magnitude better as compared with the case of the constant weight function (Legendre polynomial expansion). For the longitudinal structure function, due to a substantial gluon contribution, the reconstruction accuracy appears to be worse by an order of magnitude (see fig. 3; the optimal values are $\alpha \approx 6$ and $\beta$ close to $-1$). Such an inaccuracy is still acceptable since it is compensated by a small $F_L$-contribution to the cross section (up to several % in a few high-$y$ BCDMS points). Figures 1-3 also indicate, in contrast with the nonsinglet case [12], the sensitivity of the reconstruction accuracy to the analysed $Q^2$-interval. Clearly, this is a consequence of a fast singlet evolution in the low-$x$ region.

The dependence of the reconstruction accuracy on the number $N_{max}$ of the terms in the series and on the length of the IBM computer word is displayed in fig. 4. It may be seen that the single (double) precision is sufficient up to $N_{max} = 8$ (22). The reconstruction accuracy blows up at $N_{max} = 44$ even if the maximal word length of REAL*16 has been used. It also follows from fig. 4



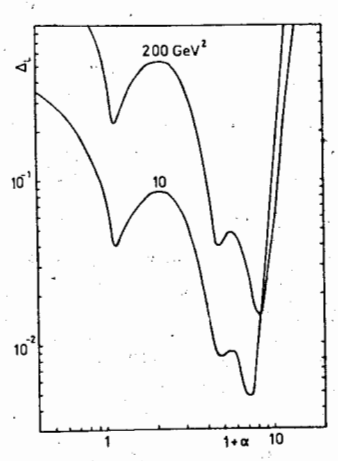


**Fig. 3.** The same dependence as in Fig. 2 but for a singlet-like longitudinal SF $F_L(x,Q^2)$.

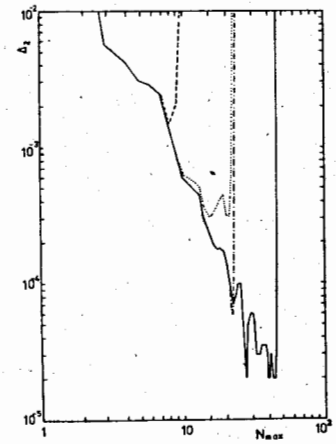


**Fig. 4.** The same dependence as in Fig. 1 but as a function of the number $N_{max} = M + 1$ of the terms retained in the reconstruction series at various lengths of the IBM computer word: REAL*4 (dashed curve), REAL*8 (dashed-dotted curve) and REAL*16 (full curve); the parameters are : $\alpha = 3$, $\beta + 1 = 10^{-5}$ and $Q^2 = 25\ GeV^2$. The dotted curve corresponds to REAL*8 and a numerical integration of the $x$-parametrizations in eqs. (10).

that the IBM double precision allows one to achieve the reconstruction accuracy by about one order better than in the case of the single one, while further doubling of the word length is less effective. We may conclude that the computer precision practically limits the number of the retained terms to $N_{max} < 20$. Due to rapidly increasing computer time with $N_{max}$, an optimum seems to be $N_{max} = 10\text{-}15$. In this case, as may be seen from fig. 4, there is only a minor difference in reconstruction accuracies corresponding to the exact and numerical calculations of the initial moments from eqs. (10).

It should be noted [32] that the convergence of the reconstruction series breaks at $N_{max} > 10$ if the QCD moments have been calculated neglecting the corrections to the OPE next-to-leading anomalous dimensions arising from the substitutions (9).

## 4. QCD fits to BCDMS proton data

The method has been used for QCD analysis of the BCDMS hydrogen data [5,6]. The initial parton densities at $Q_0^2 = 5\ GeV^2$ have been parametrized according to eqs. (10). The corresponding parameters are determined (except $\mu_{SEA}$ and $\mu_G$ assumed to be zero), together with the QCD parameter $\Lambda$ defined in eq. (3'), by fitting the QCD predictions to the cross section data. Note that these points are often given in a form of the function $F_2^0(x,Q^2;E)$ which coincides with the structure function $F_2$ calculated under the assumption $R = \sigma_L/\sigma_T = 0$ (see, e.g. [1,6,37]). We compare the cross section data with the complete singlet + nonsinglet NLO QCD prediction containing both the SFs $F_2$ and $F_L$ calculated in the $\overline{MS}$ renormalization scheme. Different weight functions are used to optimize the Jacobi reconstruction of these SFs: for $F_2$, the corresponding exponents $\alpha$, $\beta$ are treated as free parameters of the fit, and, for $F_L$, they are fixed at $\alpha = 6$ and $\beta$ close to $-1$.

The main results have already been communicated [5,7]. An



excellent agreement of the QCD predictions for $F_2(x,Q^2)$ with the data is demonstrated in fig. 5 (see also figs. 2, 3 of ref. [7]). The values of the fit parameters are given in Table 2 for the SI+NS analysis in a full kinematic range ($x > 0.06$) and for the NS analysis in a restricted domain ($x > 0.25$). The kinematic cuts of ref. [7] are applied. The momentum sum rule (4) is assumed.

Note that rather large errors of the parameters of the quark densities are due to substantial correlations among them. However, these parameters, being determined essentially by the $x$-dependence of the SFs, are practically decorrelated from the QCD mass-scale parameter $\Lambda$, which measures the size of the scaling violations.

Table 2. Averaged results of the NLO QCD fits ($N_{max} = 10\text{-}17$) to the BCDMS hydrogen data [6]. Only statistical errors are given.

| Fit | $\mu_{NS}$ | $\nu_{NS}$ | $\gamma_{NS}$ | $a_{NS}$ | $\mu_{SI}$ | $\nu_{SI}$ | $\langle x_q \rangle$ | $\nu_{SEA}$ | $a_{SEA}$ | $\nu_G$ | $\Lambda_{\overline{MS}}$ | $\frac{\chi^2}{DOF}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SI+NS | 0.5 ±0.2 | 3.5 ±0.2 | 10 ±2 | 1.1 ±0.2 | 0.8 ±0.1 | 4.5 ±0.6 | 0.45 ±0.08 | 13 ±4 | 0.17 ±0.05 | 9.0 ±1.5 | 207 ±21 | 258/270 |
| NS | 0.6 ±0.2 | 3.5 ±0.3 | 0.1 ±0.8 | 2.2 ±0.7 | - | - | - | - | - | - | 198 ±20 | 178/198 |

A ratio $\chi^2/DOF \approx 1$ indicates not only the consistency of the data with the QCD predictions but also a sufficient flexibility of the quark parametrizations in eqs. (10). We have confirmed this with the help of polynomial modifications of these parametrizations and found that the subsequent change of $\Lambda$ is negligible ($< 2\ MeV$).

In the fits we have constrained the gluon density with the help of the momentum sum rule (4). This may be questionable as it requires an interpolation of the singlet quark and gluon densities into the unmeasured region of $x < 0.06$. It appears, however, when treating both $\langle x_q \rangle$ and $\langle x_G \rangle$ as free parameters and assuming $\mu_{SEA} = \mu_G = 0$, that the results of Table 1 remain practically unchanged (except for 50% increase of the error in $\nu_G$), and, that the sum rule is well satisfied: $1 = 1.05 \pm 0.13$.

The softness of the gluon distribution permits to neglect its contribution in the evolution equations at sufficiently large

$SI + \frac{1}{6} NS$

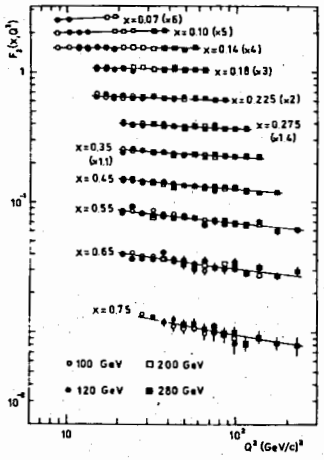


**Fig. 5.** Comparison of the BCDMS proton SF $F_2(x,Q^2)$ [5] with the result of a complete SI+NS QCD fit (full curves); the corresponding parameters are given in Table 2.

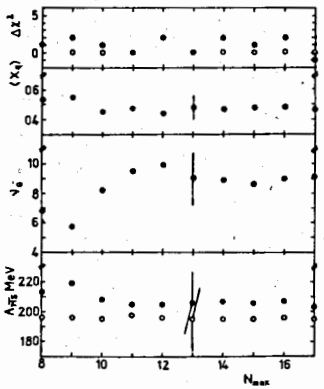


**Fig. 6.** The $N_{max}$-dependence of the results of the NS (open circles) and the complete SI+NS (closed circles) NLO QCD fits to the BCDMS hydrogen data [5]; $\Delta\chi^2 = \chi^2(N_{max}) - \chi^2(13)$. Typical statistical errors are shown at $N_{max} = 13$.



$x$-values ($x \gtrsim 0.3$) and to determine $\Lambda$ with the help of a more constrained nonsinglet analysis. In an interval of $x \geq 0.25$ the NLO nonsinglet approximation, as compared with the complete SI+NS treatment, yields practically the same $\Lambda$ (see Table 1 and ref. [7]).

The $N_{max}$-dependence of the results of NLO fits is displayed in fig. 6. In agreement with the analysis of the reconstruction accuracy, the parameters of NS and SI+NS fits show stable behaviour for $N_{max} \geq 8$ and $N_{max} \geq 10$, respectively. Note that a good $\chi^2$-stability is achieved by treating the weight function exponents $\alpha$ and $\beta$ in the $F_2$-reconstruction as free parameters of the fit. Without such a tuning of the weight function the $\chi^2$ oscillates with $N_{max}$ by ~ 5 units while the fitted parameters remain practically unchanged. Small fluctuations of $\Lambda$ with $N_{max}$ may be considered as a measure of the systematic error of the method. As is seen from fig. 6, the fluctuations are less than 2 MeV which is negligible as compared with the statistical and systematic errors in $\Lambda$.

The results of the fits well agree with the ones [7,8] obtained by a different method (based on a numerical solution of the evolution equations [35]), except for a small systematic difference of ~ 10 MeV in the $\Lambda$-values which cannot be considered as a significant one as compared with the errors. A part of this difference (3-5 MeV) is due to a different treatment of the $R$-function (see discussion in Section 2.4 of ref. [21]). The results obtained by the two methods would be fully identical provided [12] the exact solution of the NLO equation (3) for $\alpha_s(Q^2)$ is used instead of the equally valid NLO approximation in eq. (3').

## 5. Conclusions

We may conclude that the simple procedure suggested for the Jacobi polynomial reconstruction of both the transverse and longitudinal SFs is possible to make the reconstruction uncertainties in the predicted cross section less than a fraction

of %, i.e. negligible as compared with the errors of present data.

The corresponding computer code for QCD fits was successfully tested with the help of BCDMS hydrogen data. The results of the SI+NS (NS) fits show a stable behaviour of the physical parameters provided the number of the terms, $N_{max}$, retained in the reconstruction series is larger than ~ 10 (8). The typical time for one full iteration in the SI+NS case at $N_{max} = 13$ is about 30 CP seconds at the CDC 6500 computer.

The BCDMS data [6] show a perfect agreement with the QCD predictions on scaling violations in the SFs: $\chi^2/DOF \approx 1$. The QCD mass scale parameter $\Lambda_{\overline{MS}}$, determined for the first time from the full SI+NS analysis of the proton SFs, is equal to 207 ± 21 MeV. This value is in a good agreement with the one (198 ± 20 MeV) obtained from the NS fit (neglecting the gluon contribution) in the restricted kinematic range $x \geq 0.25$. This result as well as the large exponent $\nu_{SEA} = 13 \pm 4$ at $Q_0^2 = 5\ GeV^2$ confirm the earlier observations from muon-nuclear and neutrino-nuclear experiments that the dominant contribution to the SF $F_2$ at $x >_{\sim} 0.3$ comes from valence quarks. The valence-quark exponents $\mu_{NS} = 0.5 \pm 0.2$ and $\nu_{NS} = 3.5 \pm 0.2$ fitted at $Q_0^2 = 5\ GeV^2$ agree with the predictions based on the Regge theory and on the quark counting rule, respectively.

Together with similar results obtained by the BCDMS Collaboration using another method of the analysis [7,35], also showing an excellent agreement of the data with the QCD predictions, the best value of $\Lambda_{\overline{MS}}$ is 205 ± 22(stat) + 60(syst) MeV, where the quoted systematical error is due to experimental uncertainties [7]; the theoretical uncertainties are expected to be of a similar size [21]. This is the most precise measurement of the $\Lambda_{\overline{MS}}$ from deep inelastic lepton-proton scattering experiments.

A soft gluon distribution has been obtained at $Q_0^2 = 5\ GeV^2$: $xG(x,5\ GeV^2) \propto (1-x)^{9 \pm 1.5}$; the exponent $\nu_G$ is twice the one predicted by the quark counting rule at low $Q^2$ and found in the LO analysis. This result indicates the importance of the NLO corrections for the SF analysis. The above gluon parametrization should be however considered only as an effective one in the range $0.06 < x < 0.30$, where an essentially nonzero gluon contribution



is required by the measured slopes $\partial \ln F_2/\partial \ln Q^2$ characterizing the scaling violations (see fig. 4 of ref. [7]). On the other hand, direct photon production data are sensitive to the gluon density in the region $0.35 < x < 0.6$ [38]; they require $\nu_G = 4.0 \pm 0.8$ at $Q_0^2 = 2\ GeV^2$ in agreement with $\nu_G = 6.4 \pm {}^{2}_{3}$ determined at the same $Q_0^2$ from the BCDMS hydrogen data.

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КХД-анализ структурных функций
в следующем порядке теории с помощью
полиномов Якоби

Исходя из КХД-предсказаний для меллиновских моментов, даем описание метода КХД-анализа синглетных и несинглетных структурных функций, основанного на их реконструкции с помощью полиномов Якоби. Точность и стабильность метода продемонстрированы с помощью водородных данных, полученных сотрудничеством БЦДМС.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

**Препринт Объединенного института ядерных исследований. Дубна 1990**

Krivikhizhin V.G. et al. E1-90-330

Next-to-Leading-Order QCD Analysis of
Structure Functions with the Help of
Jacobi Polynomials

The method of QCD analysis of singlet and nonsinglet structure functions, based on their Jacobi polynomial reconstruction from perturbative QCD predictions for the Mellin moments, is described. The accuracy and stability of the method are demonstrated with the help of BCDMS hydrogen data.

The investigation has been performed at the Laboratoty of Superhigh Energies, JINR.

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