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ELECTROMAGNETIC CASCADES PRODUCED BY GAMMA-QUANTA WITH THE ENERGY E_{γ} = 100 - 3500 MeV. II. Longitudinal Development

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I. INTRODUCTION

The longitudinal development of an electromagnetic cascade process (ECP or e.m.shower) produced by high energy gamma quanta (GQ) is traditionally considered to be a basic characteristic of the ECP mainly for reason of relative simplicity of its theoretical reconstruction (at least on the qualitative level) as well as of its great importance from practical point of view. Consequently, the problem of the longitudinal development of the e.m.shower (LDS) is studied more frequently than another one. Recently relevant experimental data /1,2/ are used often to check the accuracy of different formulas approximating the LDS and computer modeling programs, particularly EGS-4 code system /3/.

In the present work we study the longitudinal distribution of deposition of energy released as ionization loss of electrons and positrons (later: electrons) in e.m.showers produced by GQ with the energy E_{γ}= 100-3500 MeV in liquid xenon. The investigation has been performed with the help of pictures of the 180 l xenon bubble chamber (XeBC) of ITEP (Moscow) exposed to the beam of π^{-} mesons at 3.5 GeV/c.

The advantage of using for this purpose the pictures of the XeBC with its dimensions 25.7*11*10 expressed in units of radiation length (RL) is such that, firstly, it is possible to ensure maximum potential length (L_{pot}) for LDS as large as 25 RL and, secondly, on a XeBC picture one can distinguish many details of the e.m.shower structure (fig.1) and to measure all ranges of cascade electrons in a picture plane (PP)/4/. Here L_{pot} is counted out along the shower axis (SA) from a primary GQ conversion point up to the edge of the registering volume of the XeBC.

Fig.1. Electromagnetic shower initiated by a high energy photon in the 180 l xenon bubble chamber of ITEP (Moscow).

On the whole 908 events of e.m.showers recorded on XeBC pictures have been selected during the scanning and after preliminary analysis they were grouped into 22 intervals of primary GQ energy E_γ. Selection criteria assume, in particular, that the SA is in the PP. For further analysis such events were accepted only in which $L_{pot}(E_\gamma)$ is not lower than the maximum length $t_{max}(E_\gamma)$ on a sample of showers belonging to a given interval centred at E_γ (the length $t(E_\gamma)$ is measured, as $L_{pot}(E_\gamma)$, from the conversion point of a primary GQ up to the remotest visible shower electron tracks along the SA).

In case of each event of e.m.shower satisfying the aforesaid criteria partial shower electron ranges in the PP, $\Delta\Sigma r(E_\gamma,t)/\Delta t$, within a layer of liquid xenon of the thickness Δt=0.6 RL at the depth t were measured. As has been pointed out earlier /4,5/, multiplied by some constant value η=3.63 MeV/mm they correspond, on the average and in sufficient good accuracy ($\sim$3%), to the ionization loss of relevant shower electrons, $\Delta\Sigma E(E_\gamma,t)/\Delta t$, inside a layer of the thickness Δt. Minimum range of cascade electrons in the PP which can be discerned reliably enough is equivalent to their cut-off energy of 0.5-1.5 MeV. In more detail methodical problems and experimental material used in the work have been described in /4/ and references quoted herein.

II. DIFFERENTIAL DISTRIBUTION

Figure 2 shows the differential distribution of ionization loss (DD) in e.m.showers produced in liquid xenon by GQ within six energy intervals centred at E_γ (bins of 0.6 RL). Smooth lines correspond to the approximating functions:

$$\frac{\Delta\Sigma E(E_\gamma,t)}{\Delta t} = A_0\frac{A_1}{A_2}\left(\frac{t}{A_2}\right)^{A_1-1}\exp\left(-\left(\frac{t}{A_2}\right)^{A_1}\right) \qquad (1)$$

(solid lines) and

$$\frac{\Delta\Sigma E(E_\gamma,t)}{\Delta t} = A_0\, t^{A_1}\exp(-A_2\, t) \qquad (2)$$

(dashed lines), where the parameters A_i (i=0,1,2) have been determined using the common fitting and goodness-of-fit testing procedure /6/. The relevant numerical values of these parameters are given in the table. Two parameters, viz. A_1 and A_2, of the Weibull distribution (1) are also plotted in fig.3.

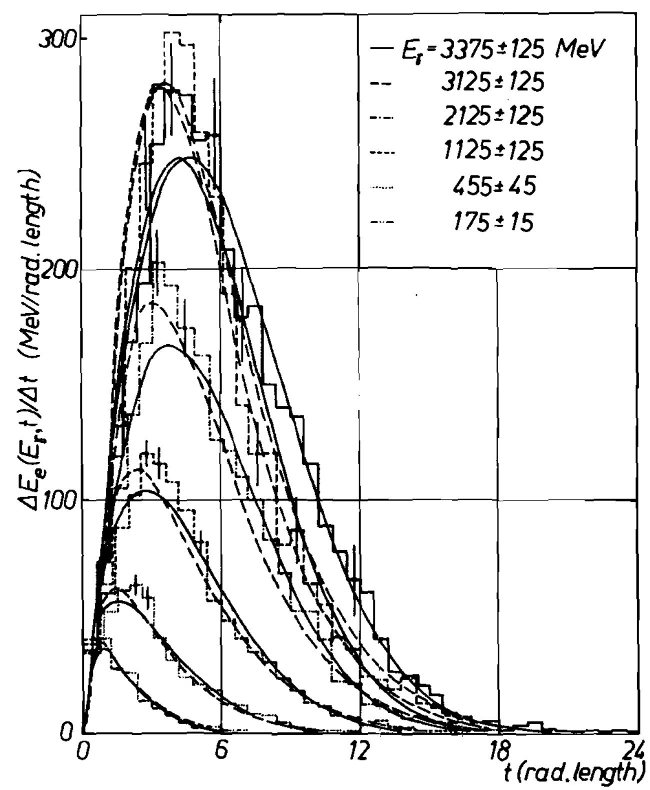

Fig.2. Longitudinal distribution of the average partial ionization loss between 0 and Lpot(E_γ) (bins of 0.6 RL) for showers produced in liquid xenon by gamma-quanta with the energy E_γ. Solid lines correspond to the Weibull approximating function, dashed ones display the fit to the form (2). Numerical values of Lpot are collected in the table.

It should be noted that the DD parametrized by the functions (1) and (2) are both normalized in such a way that $\int_0^\infty f(E_\gamma,t)dt = E_\gamma$, where

$$f(E_\gamma,t) = \frac{\Delta\Sigma E(E_\gamma,t)}{\Delta t}. \qquad (3)$$

Hence it follows, in particular that the parameter A_0 in eq.(1) should be equal to E_γ.

Inspection of the table indicates that the function (1) is somewhat preferable, especially at higher energies, i.e. at $E_\gamma \gtrsim 1000$ MeV. Nevertheless, within the shower maximum the experimental data are systematically over both the fitting functions suggesting the measured PER are perhaps overestimated in this region because of, for instance, the peculiar compression of shower electron tracks (fig.1). In order to clear up this question the parametrization has been repeated using experimental data without the aforesaid critical regions of extent of 2-3 RL. The obtained results do not differ significantly from the preceding ones, although the χ^2/NDF values for both functions are markedly lower, particulary in case of the Weibull distribution (1).

Table.

Fitted values of parameters A_i (i=0,1,2) appearing in eqs. (1) and (2) vs GQ energy E_γ, and corresponding χ^2/NDF values. Units are such that the shower depth t is expressed in RL and the DD - in MeV/ 0.6 RL. N_γ is the number of shower events in a sample at a given GQ energy centred at E_γ. L_{pot} is the potential length.

E_γ(MeV)	eq.	A_o(MeV)	$100 \cdot A_1$	$100 \cdot A_2$	χ^2/NDF	N_γ	L_{pot}(RL)
3375 +/-125	(1)	3450+/-76	190+/-2	693+/-7	18/36	15	24.0
	(2)	286+/-14	176+/-5	49+/-1	69/35		
3125 +/-125	(1)	3052+/-62	191+/-2	620+/-7	51/34	12	24.0
	(2)	299+/-10	180+/-5	52+/-1	62/34		
2875 +/-125	(1)	2734+/-69	192+/-3	624+/-8	28/35	11	24.0
	(2)	278+/-13	164+/-6	49+/-1	47/35		
2625 +/-125	(1)	2474+/-52	193+/-2	576+/-9	46/35	12	22.6
	(2)	324 +/-9	153+/-3	50+/-1	71/35		
2375 +/-125	(1)	2320+/-52	178+/-2	624+/-8	15/31	20	22.6
	(2)	258+/-12	153+/-6	47+/-1	44/31		
2125 +/-125	(1)	2060+/-37	180+/-2	599+/-6	29/35	35	21.3
	(2)	254 +/-8	150+/-4	48+/-1	65/35		
1875 +/-125	(1)	1782+/-36	173+/-2	582+/-7	28/32	33	20.5
	(2)	248 +/-8	135+/-4	46+/-1	31/32		
1625 +/-125	(1)	1554+/-26	170+/-1	562+/-6	42/34	66	20.0
	(2)	241 +/-6	123+/-3	45+/-1	41/33		
1375 +/-125	(1)	1313+/-24	167+/-2	515+/-6	36/30	58	20.0
	(2)	243 +/-7	130+/-4	51+/-1	52/30		
1125 +/-125	(1)	1095+/-16	164+/-2	487+/-5	38/29	80	20.0
	(2)	225 +/-5	117+/-3	50+/-1	61/29		
875 +/-125	(1)	854+/-14	154+/-2	465+/-5	25/30	86	20.0
	(2)	208 +/-5	100+/-3	49+/-1	29/30		
680 +/-70	(1)	652+/-13	150+/-2	433+/-6	27/27	60	20.0
	(2)	178 +/-5	89+/-3	49+/-1	18/27		
555 +/-55	(1)	542+/-13	147+/-2	397+/-6	20/29	58	20.0
	(2)	169 +/-6	80+/-4	50+/-2	11/29		
455 +/-45	(1)	443+/-11	143+/-2	345+/-5	18/26	59	18.6
	(2)	181 +/-7	80+/-4	59+/-2	18/26		

Table (continued)

E_γ(MeV)	eq.	A_o(MeV)	$100 \cdot A_1$	$100 \cdot A_2$	χ^2/NDF	N_γ	L_{pot}(RL)
375 +/-35	(1)	365 +/-9	131+/-2	342+/-6	17/27	61	17.3
	(2)	141 +/-5	59+/-4	51+/-2	13/27		
310 +/-30	(1)	300 +/-9	137+/-2	291+/-6	10/22	53	16.0
	(2)	159 +/-8	69+/-5	66+/-2	16/22		
255 +/-25	(1)	241 +/-7	130+/-3	261+/-5	13/19	55	14.9
	(2)	133 +/-7	54+/-5	63+/-3	14/19		
210 +/-20	(1)	195 +/-6	125+/-3	215+/-6	10/15	47	14.8
	(2)	136+/-15	47+/-9	80+/-7	9/15		
175 +/-15	(1)	166 +/-6	124+/-4	192+/-7	10/15	29	11.4
	(2)	136+/-15	47+/-9	80+/-7	9/15		
145 +/-15	(1)	138 +/-6	113+/-4	190+/-8	6/15	24	12.6
	(2)	97+/-11	27+/-9	70+/-7	6/15		
120 +/-10	(1)	115 +/-8	117+/-7	161+/-9	5/13	20	10.6
	(2)	111+/-22	38+/-15	90+/-12	4/12		
100 +/-10	(1)	95 +/-5	124+/-7	119+/-6	5/6	14	10.1
	(2)	188+/-47	61+/-17	144+/-20	4/6		

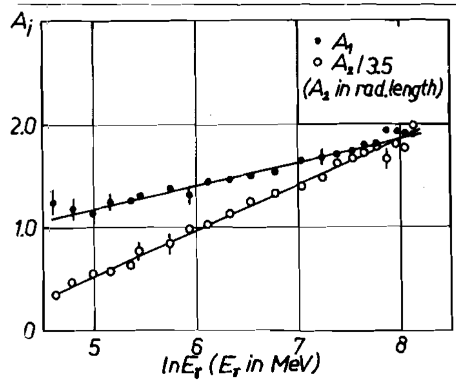

Fig.3. Fitted values of parameters of the Weibull distribution (1) as a function of the energy E_γ.

III. CUMULATIVE DISTRIBUTION

Figure 4 shows the cumulative distribution function of the average ionization loss (ID), viz.

$$A(E_\gamma,t) = \frac{1}{E_\gamma}\int_0^t f(E_\gamma,t^*)\,dt^* \qquad (4)$$

for several values of the energy E_γ. It follows from this equation that $0 \le A(E_\gamma,t) \le 1$ because practically the whole of the primary photon energy E_γ is released on the average, as ionization loss of shower electrons up to the depth t being equal to about L_{pot}. The ID

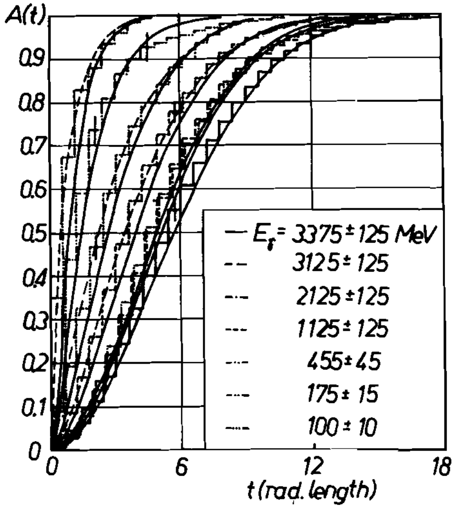

Fig.4. Same as fig.2, for the cumulative distribution.

defined by eq.(4) is useful in practice indicating, for example, how large should be the average thickness t of active layers of calorimeters (or shields) to measure (or simply to absorb) the given part $A(E_\gamma,t)$ of shower electron ionization loss when exposed to a beam of GQ with the energy E_γ. A better parametrization, i.e. the Weibull function (1) is indicated by smooth curves of fig.4.

It is suggestive to find such a simple demensionless and natural variable $x(E_\gamma)$ depending on E_γ only that, being used instead of t, would make possible to build a more general description of the LDS within sufficiently wide energy interval of GQ producing e.m.showers in dense enough media. To this end the ratio $x(E_\gamma)=t/\bar{t}(E_\gamma)$ has been used, where $t(E_\gamma)$ is the average value of the length t, i.e.

$$\bar{t}(E_\gamma) = \frac{1}{E_\gamma}\int_0^\infty t\cdot f(E_\gamma,t)dt. \qquad (5)$$

Fig.5 shows our appropriate results obtained earlier on a part of experimental material /7/. Smooth linas correspond to the Weibull parametrization of the cumulative distribution function, viz.

$$A(E_\gamma,t) = 1 - \exp(-(\frac{t}{A_2})^{A_1}) \qquad (6)$$

for two extreme values of energy: 210 and 3375 MeV.

One can see (fig.5) that in this way (if any) the experimental data cannot be reduced to the one universal distribution, although there are two common points for all energies: at $x(E_\gamma)\cong 1.2$ and the point of the shower origin (if media are not too dense). It should

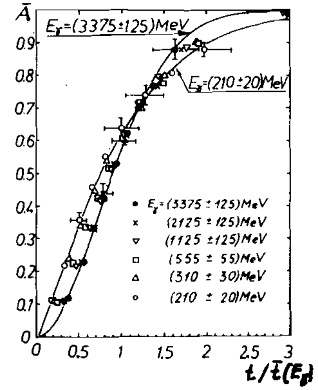

Fig.5. Cumulative longitudinal distribution of the average ionization loss in showers initiated by gamma-quanta with the energy E_γ. Solid lines show the best fit to the form (1) for two extreme values of the energy E_γ. $\bar{t}$ is the average value of t.

be also noticed that the higher is the energy E_γ the more compact are the showers. The energy dependence of the average length $\bar{t}(E_\gamma)$ is plotted in fig.6. This dependence can be parametrized by a simple linear function of $\ln E_\gamma$:

$$\bar{t}(E_\gamma) = b\ \ln E_\gamma - a, \qquad (7)$$

where a = 4 RL and b = 1.2, t is expressed in RL.

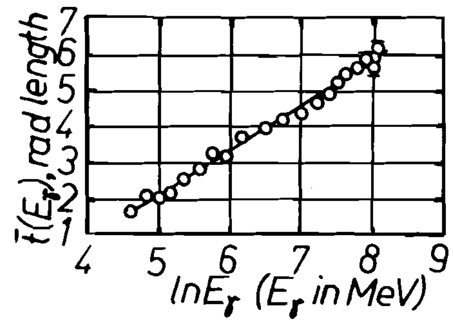

Fig.6. Energy dependence of the average thickness $\bar{t}(E_\gamma)$ of showers produced in liquid xenon (1 RL ≅ 4 cm) by photons with the energy E_γ.

IV. CONCLUSIONS

The average partial ranges of electrons observed along the shower axis within a layer Δt=0.6 RL as a function of depth t in liquid xenon for photon initiated e.m.showers with energies between 100 and 3500 MeV have been measured using pictures of the 180 l XeBC of ITEP (Moscow). This chamber permits to analyse many details of a shower structure within the absorber thickness as large as 25 RL. The obtained experimental results were fitted both to the Weibull (1) and to the gamma (2) distribution. It turned out that the

Weibull parametrization is preferable, especially at the energy range of primary photons $E_\gamma \geq 1000$ MeV and the parameters A_1 and A_2 of this distribution can be described by simple functions of $\ln E_\gamma$:

$$A_i = a_i + b_i \ln E_\gamma, \qquad (8)$$

where: $a_1 = 0.014 +/- 0.067$, $b_1 = 0.23 +/- 0.10$, $a_2 = -5.93 +/- 0.08$, $b_2 = 1.55 +/- 0.01$.

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Электромагнитные ливни, вызываемые гамма-квантами с энергией E_γ = 100-3500 МэВ.
II. Продольное развитие

Приведены результаты исследования электромагнитных ливней, инициируемых гамма-квантами с энергией E_γ=100-3500 МэВ в жидком ксеоне. Получены распределения средних ионизационных потерь каскадных электронов вдоль оси развития ливней. Обсуждается вопрос об аппроксимации экспериментальных данных распределениями Вейбула и гамма-функциями. В работе использованы стереофотографии со 180-литровой ксеноновой пузырьковой камеры ИТЭФ /Москва/, облученной в пучке π^--мезонов с импульсом 3,5 ГэВ/с.

Работа выполнена в Лаборатории высоких энергий ОИЯИ и в Институте физики Варшавского политехнического института.

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Electromagnetic Cascades Produced by Gamma-Quanta with the Energy E_γ=100-3500 MeV.
II. Longitudinal Development

The longitudinal development of photon-initiated showers of primary energy between 100 and 3500 MeV in liquid xenon has been studied using pictures of the 180 l xenon bubble chamber of ITEP /Moscow/. The distribution of the average ionization loss of shower electrons along the shower axis is obtained and fitted both to the Weibull and gamma functions.

The investigation has been performed at the Laboratory of High Energies, JINR and the Institute of Physics of the Warsaw University of Technology.

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