сообщения
объединенного
института
ядерных
исследований
дубна

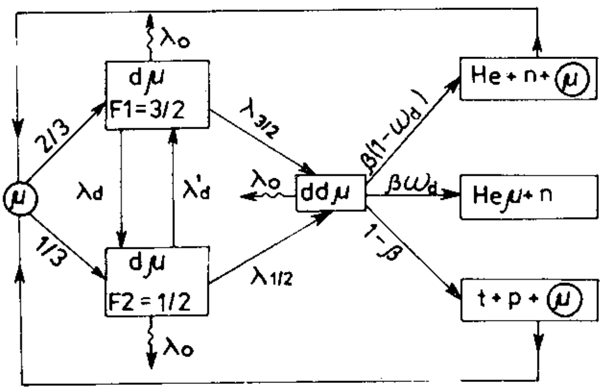
E1-89-57

V.V.Filchenkov

ON THE DETERMINATION OF THE PARAMETERS OF THE μCF REACTION d + d

1989

Contrary to muon catalyzed fusion (μCF) of the d+t reaction the process of ddμ-molecule formation is described rather simply by the Vesman resonance mechanism /1/. Hence the accurate comparison of the experimental and theoretical data on the ddμ-molecule formation rate ($\lambda_{dd\mu}$) is possible which allows one to determine with a high accuracy the energy of the weakly bound level (J = v = 1) in the ddμ-molecule and the contribution to this energy due to the vacuum polarization effects /2/. The present paper is devoted to the method of extraction of physical parameters from the experimental data on the process of μCF of d+d reaction.



The scheme of the processes occuring when negative muons are stopped in deuterium.

The scheme of the processes occuring when negative muons are stopped in deuterium is given in the Figure. Muonic atoms dμ are formed in two spin states: F1 = 3/2 with the probability 2/3 and F2 = 1/2 with the probability 1/3. Between these spin states the transitions dμ(F1) + d — dμ(F2) + d can occur with the rate of λ_d.

Muonic molecules $dd\mu$ can be formed from each $d\mu$-atom spin state with the rates of $\lambda_{3/2}$ and $\lambda_{1/2}$. In the figure the process of the back decay of ($dd\mu$, d, 2e) complex is not shown. This process leads to some changes in $d\mu$-atom spin states population. Taking it into account we shall consider λ_d as an effective value of the transition rate /2/.

An important peculiarity of $dd\mu$-molecule formation process is the dependence of its rate on $d\mu$-atom spin state /2/. It follows from paper /2/ that the value of $\lambda_{3/2}$ and $\lambda_{1/2}$, especially the value of $\lambda_{3/2}$ at low temperatures, are very sensitive to the value of the energy level (J = v = 1) in the $dd\mu$-system.

The fusion reaction d+d occurs in the $dd\mu$-molecule. In most cases the muon is released in the reaction and forms a $d\mu$-atom again. The probability of the muon sticking to the helium-3 nucleus is $\omega_d \approx$ 12% and the effective probability of the muon being lost in two channels of the fusion reaction is $\omega \approx 7\%$ /3/. From each of the spin states of the $d\mu$-atom and $dd\mu$-molecule the muon can decay with the rate $\lambda_0 = 0.455\ \mu s^{-1}$.

In the experiment the yield and time distribution of the fusion reaction products are detected with the efficiency ε. As is shown in papers /4-7/ in the general case the procedure of obtaining the expressions for the time distributions of the detected events is rather complicated, since one muon can initiate many cycles of fusion reaction and the registration efficiency is not equal to one ($\varepsilon<1$). Some time ago we had suggested the effective way to simplify this procedure /5/. Namely, to obtain the expression for the time distribution of the first detected events (dn1/dt) one should at first find a relatively simple formula for the time distribution of all detected events (dN/dt) and then replace the parameter ω by $\alpha=\varepsilon+\omega-\varepsilon\omega$ in it. The expressions for the second, the third and etc.,events can be found by the sequential algorithm:

$$f_K(t) = dn_K/dt = (1-\omega)\int_0^t f_{K-1}(x)\, f_1(t-x)\, dx .$$

The formula for all detected events is usually obtained by theoreticians when kinetics concrete of μCF process is considered. For μCF of the d+d reaction this expression has been obtained in paper /2/ with an accuracy $\sim \omega^2$ according to the authors statement *). When using this formula to obtain the expression for the

*) An identical (from the mathematical point of view) task was considered in /8/ to elucidate the question about the role of $t\mu$-atom thermalization process in the form of the time distribution of d+t reaction products.

experimental time distribution, especially for the case $\varepsilon \simeq 1$, the error due to this approximation should increase. That is why there is a task to obtain an accurate expression for the experimental time distribution dN/dt.

Following the papers /2,9/ we shall find the formula for all detected events in the form

$$dN/dt = \lambda_{3/2} n_{3/2}(t) + \lambda_{1/2} n_{1/2}(t), \tag{1}$$

where $n_{3/2}$ and $n_{1/2}$ are the relative populations of $d\mu$-atom spin states with F1 = 3/2 and F2 = 1/2 ($n_{3/2}(t=0) = 2/3$ and $n_{1/2}$ (t=0) = 1/3). The functions $n_{3/2}$, $n_{1/2}$ satisfy the following differential equation system (see the figure)

$$dn_{3/2}/dt = -(\lambda_0+\lambda_d+\lambda_{3/2})n_{3/2} + 2/3(1-\omega)\lambda_{3/2}n_{3/2} + [2/3(1-\omega)\lambda_{1/2} + \lambda_d']n_{1/2} \tag{2}$$

$$dn_{1/2}/dt = -(\lambda_0+\lambda_d'+\lambda_{1/2})n_{1/2} + 1/3(1-\omega)\lambda_{1/2}n_{1/2} + [1/3(1-\omega)\lambda_{3/2} + \lambda_d]n_{3/2}.$$

The solutions of system (2) are

$$n_{3/2}(t) = a_{11}\exp(\gamma 1\cdot t) + a_{12}\exp(\gamma 2\cdot t)$$
$$n_{1/2}(t) = a_{21}\exp(\gamma 1\cdot t) + a_{22}\exp(\gamma 2\cdot t). \tag{3}$$

The exponential factors are:

$$\gamma 1 = -(2\lambda_0+b+c) - [(b-c)^2 - 4d]^{1/2},$$
$$\gamma 2 = -(2\lambda_0+b+c) + [(b-c)^2 - 4d]^{1/2}, \tag{4}$$

where b, c, d are the following combinations of parameters,

$$b = \lambda_d + 1/3\lambda_{3/2} + 2/3\lambda_{3/2}\omega, \quad c = 2/3\lambda_{1/2} + 1/3\lambda_{1/2}\omega + \lambda_d',$$
$$d = (\lambda_d + 1/3\lambda_{1/2} - 1/3\lambda_{1/2}\omega)(2/3\lambda_{3/2} - 2/3\lambda_{3/2}\omega + \lambda_d'). \tag{5}$$

Amplitudes a_{ik} are

$$a_{11} = [2/3(\gamma 2 - c_{11}) - 1/3 c_{12}]/(\gamma 2 - \gamma 1), \quad a_{12} = 2/3 - a_{11},$$
$$a_{22} = a_{12}(\gamma 2 - c_{11})/c_{12}, \quad a_{21} = 1/3 - a_{22}, \tag{6}$$

where $c_{11} = \lambda_0 + b$, $c_{12} = 2/3\lambda_{1/2}(1-\omega) + \lambda_d'$.

The exact solution for the time distribution of all detected events may be presented in the form

$$dN/dt = A_f\exp(-\lambda_f t) + A_s\exp(-\lambda_s t), \tag{7}$$

where $\lambda_f \equiv -\gamma 1$, $\lambda_s \equiv -\gamma 2$ and

$$A_f = \lambda_{3/2}a_{11} + \lambda_{1/2}a_{21}, \quad A_s = \lambda_{3/2}a_{12} + \lambda_{1/2}a_{22}. \tag{8}$$

The yield of all detected events is

$$\eta = \varepsilon (A_f/\lambda_f + A_s/\lambda_s) .$$

As we have mentioned above the expression for the time distribution of the first detected events dn1/dt is obtained when the parameter ω is replaced by $\varepsilon + \omega - \varepsilon\omega$ in the expression for dN/dt.

Now let us compare our results for the function dn1/dt with the approximate formulae from /2,8/. At first it is more convenient to do it on the assumption that inverse transitions $1/2 \to 3/2$ are absent (it corresponds to a real case for low temperatures $T \lesssim 100$ K). It follows from papers /2,8/ that the expressions for γ_f, A_f, A_s correspond to the solutions of system (2) in approximation $\omega = 0$:

$$A_f = 2/3(\lambda_{3/2} - \lambda_{1/2})(\lambda_d + 1/3\lambda_{1/2} - 1/3\lambda_{3/2})/(\lambda_d + 1/3\lambda_{3/2} + 2/3\lambda_{1/2}),$$
$$A_s = \lambda_{1/2}(\lambda_d + \lambda_{3/2})/(\lambda_d + 1/3\lambda_{1/2} + 2/3\lambda_{3/2}), \qquad (9)$$
$$\lambda_f = \lambda_0 + \lambda_d + 1/3\lambda_{3/2} + 2/3\lambda_{1/2} .$$

For the exponential factor of the slow component the following expression is used in /2,8/:

$$\lambda_s = \lambda_0 + \omega A_s , \qquad (10)$$

where A_s is determined from eq. (9)

Accurate values of λ_f, A_f, λ_s, A_s were calculated using formulae (4-8). In the calculations the values $\overset{\circ}{\lambda}_d = 30\ \mu s^{-1}$, $\overset{\circ}{\lambda}_{3/2} = 4\ \mu s^{-1}$, $\overset{\circ}{\lambda}_{1/2} = 0.05\ \mu s^{-1}$ ($T \simeq 30$ K) and $\overset{\circ}{\lambda}_{1/2} = 2\ \mu s^{-1}$ ($T \simeq 250$ K) were used. (As usual, $\lambda_x = \overset{\circ}{\lambda}_x \varphi$, where $\overset{\circ}{\lambda}_x$ is normalized to the liquid hydrogen density $4.25 \cdot 10^{22}$ nucl/cm^3, φ is the relative deuterium density). Note that the results of comparing practically do not depend on φ .

Calculations were done for the set of ε values in the range ε = 1-100%. It follows from the calculations, that for $\varepsilon \sim 1\%$ the results obtained according to approximate formulae (9-10) are in agreement with the exact solutions within an accuracy of 1%. When ε increases, the discrepancy becomes larger and for $\varepsilon \simeq 1$ it amounts 10-15%. As an example the calculation results are given in table 1 for ε =0.5. As is seen, the discrepancy in A_s values is equal to 5-8%, and in the A_f/A_s ratio, from which the value of $\lambda_{3/2}$ is extracted, it is $\simeq 8\%$. One should bear in mind this fact when the experimental data are compared with theoretical values of $\lambda_{dd\mu}$ obtained in ref. /2/ (the results of comparison depend on ε).

At the next stage the inverse transitions F2 = 1/2 → F1 = 3/2 were accounted. According to ref. /2/ their rate was taken to be $\lambda_d' = \xi\lambda_d$, where ξ = 2exp(-ΔE/kT), ΔE = 0.045 eV. The calculation results are presented in table 2 for $\overset{\circ}{\lambda}_{1/2} = 2\,\mu s^{-1}$, which corresponds to the temperature T ≃ 250K (for $\overset{\circ}{\lambda}_{1/2} = 0.05\ \mu s^{-1}$ at T ≃ 30K ξ = 0). It follows from the data of table 2 that the comparison results are qualitatively the same as for $\lambda_d' = 0$ – again the meanings of the A_S/A_S ratio differ by ≃ 10%.

Table 1. The parameters of time distributions of the first detected events of d+d fusion reaction. The values $\overset{\circ}{\lambda}_d = 30\ \mu s^{-1}$, $\overset{\circ}{\lambda}_{3/2} = 4\ \mu s^{-1}$, $\varphi = 0.33$, $\omega = 0.07$ and $\varepsilon = 0.5$ were used. The rate of the inverse transitions F2=1/2 → F1=3/2 is assumed to be zero

$\overset{\circ}{\lambda}_{1/2}$		The values of the parameters of function (7) (μs^{-1}) According to ref. /2,8/	Present paper
0.05	A_s	0.0180	0.0165
	λ_s	0.465	0.464
	A_f	0.876	0.877
	λ_f	10.90	11.37
2	A_s	0.693	0.665
	λ_s	0.823	0.818
	A_f	0.423	0.445
	λ_f	11.23	11.56

Table 2. The same as in table 1 but with allowance to the inverse transitions 1/2 → 3/2

$\overset{\circ}{\lambda}_{1/2}$		The parameters of function (7) in μs^{-1} According to ref. /2/	Present paper
2	A_s	0.798	0.777
	λ_s	0.882	0.876
	A_f	0.312	0.333
	λ_f	13.46	13.63

To check the calculation algorithm the Monte-Carlo computer code developed to simulate the processes caused by negative muons in deuterium. Using this code the time distributions of the d+d reaction events, consecutively detected with an efficiency ε , were stored. The parameters of these distributions were found by the least squares analysis. For the first detected events the expression (7) was used in the analysis. The results of this analysis are presented in table 3 for $\lambda^0_d=30\ \mu s^{-1}$, $\lambda^0_{3/2}=4\ \mu s^{-1}$, T=300K, ω = 0.07, φ=0.33, ε=0.9. The value of $\lambda_{1/2}$ was deliberately underestimated comparing with the theoretically expected one for T=300K to enhance the role of inverse transitions. One can see from the table, that the Monte-Carlo results are in satisfactory agreement with the calculations based on formulae (4-8).

Table 3. The comparison of the calculated parameters of the expression $dn1/dt = \varepsilon\ (A_f \exp(-\lambda_f t)+A_s \exp(-\lambda_s t)$ for the first detected events of the d+d fusion reaction according to formula (4-8) and those found from the analysis of distribution obtained by the Monte-Carlo code. The values $\lambda^0_d=30\ \mu s^{-1}$, $\lambda^0_{3/2}=4\ \mu s$, $\lambda^0_{1/2}=1\ \mu s^{-1}$, φ=0.33, ω=0.07, ε=0.9 and T=300K were used

Parameters	According to formula (4-8)	Obtained by the Monte-Carlo code
A_s	0.541	0.539(1)
λ_s	0.961	0.957(2)
A_f	0.458	0.454(6)
λ_f	14.61	14.48(28)

Conclusions

1. Exact expressions have been obtained for the parameters of time distributions of the d+d fusion reaction events detected with an efficiency $\varepsilon < 1$.

2. The use of expressions obtained in ref. /2,8/ for the parameters of the time distribution of all detected events with replacement $\omega \rightarrow \varepsilon + \omega - \varepsilon\omega$ is shown to lead to a 5-10% error in the values of $\lambda_{3/2}$, $\lambda_{1/2}$ extracted from the analysis of this distribution.

3. When comparing the experimental data with theory one should take into account that the values of $\lambda_{dd\mu}$ (T) are correct with an accuracy of 1% only for the small values of detection efficiency. Therefore one should correct the conclusion of ref. /2/ about the optimum value of the energy level in the ddμ-system.

The author is indebted to P.Kammel for fruitful discussions.

References

1. Vesman E.A. Zh.Eksp.Teor.Fis. Pisma,1967, 5, p. 113.
Engl.transl.: Sov.Phys.JETP Lett.,1967, 5, p. 91.
2. Menshikov L.I., Ponomarev L.I., Strizh T.A., Faifman M.P.
Zh.Eksp.Teor.Fis., 1987, 91, v. 91, p. 1173.
3. Balin D.V. et al. Zh.Eksp.Teor.Fis. Pisma,1984, v. 40, p. 318.
Phys.Lett., 1984, v. 141B, p. 173.
4. Zinov V.G., Somov L.N., Filchenkov V.V. Preprint JINR,
P15-82-478, Dubna, 1982.
5. Filchenkov V.V., Somov L.N., Zinov V.G. Nucl.Instr.and Meth.,
1984, v. 228, p. 174.
6. Bystritsky V.M., Gula A., Wozniak Y. Atomkernenergie/Kerntechnik,
1984, 45, p. 197.
7. DeW Van Siclen C. J.Phys.C: Nucl.Phys., 1985, 11, p. 267.
8. Menshikov L.I., Ponomarev L.I. Zh.Eksp.Teor.Fis. Pisma,1987,
45, p. 329.
9. Kammel P. et al. Phys.Rev., 1983, A28, p. 2611.

Received by Publishing Department
on January 30, 1989.