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VERTEX RECONSTRUCTION WITHOUT TRACK RECONSTRUCTION

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1. INTRODUCTION

In most cases the determination of the vertex position requires previous reconstruction of all trajectories [1]. Besides, the check of an association of trajectories into the vertex once found is rater complicated.

The basis of such an approach is the analysis of chi-square (χ^2) criterion [2,3] as the element of mathematical model of multitrack event reconstruction procedure [4]. Many authors [4,5] note the limitation of mathematical resourses of χ^2-model, especially for great multiplicity events, which make the vertex reconstruction difficult.

In [7] a function of parameters $\vec{p} \equiv \{p_1 ; \ell = 1, \dots, L\}$ of trajectories $\phi(z;\vec{p})$ is suggested which is determined as trajectory integral with weighed function associating alternatively trajectory value and all measured coordinates:

$$R(\vec{p}) = N \cdot \frac{\sum\limits_{n=1}^{N} \Delta\ell(z_n) \sum\limits^{M_n} \sigma_{mn} \cdot G[\phi(z_n;\vec{p}) - a_{mn};\sigma_{mn}]}{\sum\limits_{n=1}^{N} \Delta\ell(z_n)} \tag{1}$$

$$\Delta\ell(z) \equiv [1 + \mathring{\phi}^2(z;\vec{p})]^{1/2} .$$

Particularly, for linear tracks $x = \phi(z;A,B) = A*z + B$ in XZ plane

$$R(A,B) = \sum_{n=1}^{N} \sum_{m=1}^{M_n} \sigma_{mn} \cdot G(A \cdot z_n + B - a_{mn};\sigma_{mn}), \tag{2}$$

where N is total number of detectors, a_{mn} are coordinates from z_n detector; M_n, count number of each detector; $G(x;\sigma)$, coordinates precision (σ) function, for instance:

$$G(x;\sigma) = \exp(-x^2/2\sigma^2)/\sigma\sqrt{2\pi}, \tag{3a}$$

or

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$$G(x;\sigma) = \begin{cases} 1/2\sigma, & |x| \le \sigma \\ 0, & |x| > \sigma. \end{cases} \tag{3b}$$

Functions (2) have many extrema in parameters of trajectories (A, B) space: main maxima correspond to real tracks, lower level maxima occur due to the noise or random "tracks". The analysis of such functions is extremely complicated task/6/ , so the exotic possibility of finding the tracks without their coordinate identification is hardly practical.

2. VERTEX FUNCTIONS

However this mathematical model commits a different approach to the task of vertex coordinates determination without previous track reconstruction.

If a particle decays to several other ones at a point $(x = u, z = v)$ and their trajectories are registered by detectors as counts $a_{mn} = \phi_m(z_n;\vec{p})$, then it is possible to construct a "decay" function $D(u,v)$. Main maxima locations of these functions correspond to vertices of interactions or decays (if there are more than one of them in an event). The rule of constructing $D(u,v)$ for any open curve is evaluted from the recipe given in/7/ for linear trajectories:

Step 1. It is necessary to determine the analytical form of correspondence between parameters of trajectory $\vec{p}$ and L — set of points: (u,v) is a possible vertex, $\{(c_\ell, z_\ell); \ell = 1, \dots, L-1\}$ are any formal detector counts. If this step is possible then the function

$$\phi(z;\vec{p}) \equiv \phi[z;(u,v),\{(c_\ell,z_\ell);\ell = 1,\dots,L-1\}],$$

corresponds to the trajectory.

Step 2. Take detector counts $a_{k\ell}$ (for instance those with maximum multiplicity — M_ℓ) as formal coordinates. Then:

$$D(u,v) = \underbrace{\sum_{k=1}^{M_\ell} \cdots \sum_{m=1}^{M_n}}_{L-1} R[(u,v),\underbrace{(a_{k\ell},z_\ell),\dots,(a_{mn},z_n)}_{L-1}], \tag{4}$$

As opposite to euristic rule of constructing of $D(u,v)$ from/7/, multiple $(L-1)$ summation in (4) converts local maxima $R[(u,v)\{(c,z)\}]$ to global maximum of decay vertex (u,v). Similarly we may construct "decay" function for 3-dimensional vertex $D(x,y,z)$.

The value of global maximum decay function is taken as associativity criterion of trajectories and the vertex. If Q trajectories begin at the vertex, then for ideal detectors (absolute accuracy, 100% efficiency) $D(u,v)$ value at global maximum must be $N \cdot Q^{L-1}$.

The certain form of "decay" functions depends on the detector layout and $\phi(z;\vec{p})$ trajectory type. For instance, for linear tracks in plane

$$D(x,z) = \sum_{k=1}^{M_N} \sum_{n=1}^{N} \sum_{m=1}^{M_n} \sigma_{mn} \cdot G\left[\frac{x - a_{kN}}{z_N - z} \cdot (z_N - z_n) + a_{kN} - a_{mn}; \sigma_{mn}\right] \tag{5}$$

(here last detector counts a_{kN} are taken as formal coordinates c_ℓ).

In the particular case when one trajectory is already known the decay functions may be of great practical significance. This situation is typical for one-beam accelerators, where the single trajectory of the accelerated particle $\phi_0(z;\vec{p}_0)$ can be reconstructed simply and quickly. Global maximum of the function $C(z)$ for primary interaction vertex $u = \phi_0(v;\vec{p}_0)$

$$C(z) = D[\phi_0(z;\vec{p}_0), z] \tag{6}$$

corresponds to z coordinate of the interaction. The function (6) has one argument only for both one-dimensional and two-dimensional coordinates. In particular, if parameters (A,B,P,Q) of the "input" linear trajectory $x = A \cdot z + B$, $y = P \cdot z + Q$ are defined and each detector registers space coordinates (a_{mn}, b_{mn}), then $C(z)$ is

$$C(z) = \sum_{k=1}^{M_N} \sum_{n=1}^{N} \sum_{m=1}^{M_n} G\{f[z, v(A,B,a_{kN}), h(A,a_{kN},a_{mn})]; \sigma_{mn}\} \times$$

$$\times\, G\{f[z, v(P,Q,b_{kN}), h(P,b_{kN},b_{mn})]; \sigma_{mn}\} \cdot \sigma_{mn}^2, \tag{7}$$

where

$$f(z,v,h) = v/(z_N - z) - h, \tag{7a}$$

and

$$v(A,B,a) = (z_N - z_n) \cdot (A \cdot z_N + B - a), \tag{7b}$$

$$h(A,c,a) = a - c - A \cdot (z_N - z). \tag{7c}$$

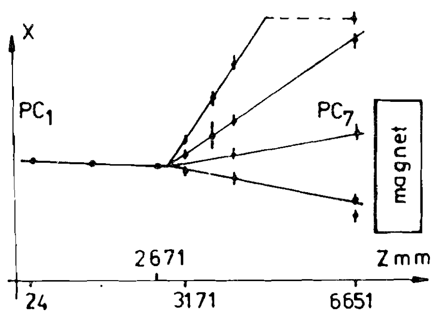

Fig. 1. The experimental set-up for the research of the proportional chambers efficiency. An event of the interaction of Mg-nucleon on the Carbon target is shown. The coordinates of Magnum fragments are shown by the points.

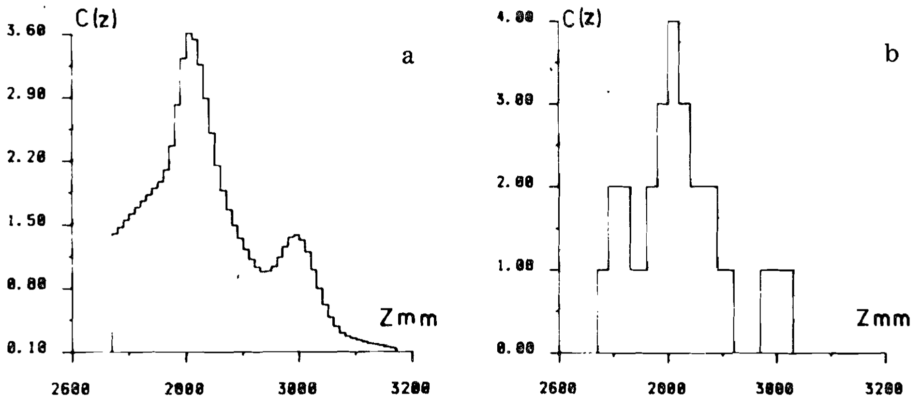

Fig. 2. The graphic of C(z) function (7) for the event at fig. 1 is shown. The function of an accuracy is smooth (3a) at fig. 2a, and the function of an accuracy is discrete (3b) at fig. 2b.

Therefore, the founded location of the main maximum of $C(z)$ provides us with nontraditional schedule of the track pattern reconstruction: as we know z-coordinate of the interaction vertex, we get additional ("virtual") detector with single (!) count, so the further determination of "exit" trajectories is simplified substantially.

The experimental set-up of proportional chambers for studying the efficiency of registration of nucleus induced by synchrophasotron JINR and nuclear fragments is shown on fig.1. There are shown XZ components of space points measured in any event of the nucleus of Magnium on Carbon target interaction. Each block of chambers measures X and Y components of space point and cluster size (s_{mn}) — some neighbour wires detecting the fragment. The function $C(z)$ for this event is shown on fig. 2a, $G(x;\sigma)$ is used here in the form (3a), $\sigma_{mn} = s_{mn}$. The same function (5) is shown on fig. 2b,

but $G(x;s)$ — (3b). The problem consists in the determination of the z-position of the global maxima $C(z)$ (fig. 2a).

3. GLOBAL MAXIMA SEARCH

This complicated problem has consummated solution for smooth functions only [6]. However this solution needs in a big number of calculations of the local values of a function, that for a big number of events (like shown on fig. 1) may lead to unacceptable cost of computer's time.

The integral approach in the determination of the global maxima position is suggested in paper [8]. For one-dimensional positive function $f(x)$ ($A < x < B$, A and B may have an infinitive values) that has a global maxima (GM) near a point "u" the function, symmetrized around "u", is constructed

$$F(x,u) = 1/2\,[f(x) + f(2u-x)]. \tag{8}$$

If "u" is chosen near the GM position, function (8) has narrow peak at "x" in GM region, otherwise — $F(x,u)$ is spread. If it is possible to calculate analytically the momenta of $f(x)$:

$$\mu_n(f) = \mu_0(f)^{-1} \cdot \int_A^B (x-c)^n \cdot f(x)\,dx, \tag{9}$$

where

$$\mu_0(f) = \int_A^B f(x)\,dx, \quad c \equiv \mu_1(f) = \mu_0(f)^{-1} \cdot \int_A^B x \cdot f(x)\,dx, \tag{9a,b}$$

then the demand for choosing "u" near GM position is reduced to the analysis of momenta of $F(x,u)$, that are determined by initial momenta $\mu_n(f)$ easily:

$$\mu_1(F) = u, \quad \mu_2(F) = \sigma^2 + t^2 \ (t \equiv u - c), \quad \mu_3(F) = 0, \tag{10a,b,c}$$

$$\mu_4(F) = \mu_4(f) + t^4 + 4\gamma t + 6\sigma^2 t^2, \tag{10d}$$

where: σ^2, γ — dispersion and asymmetry of $f(x)$.

The minimization of the $\mu_2(F)$ leads to simple condition: $u = c$.

It is more constructive to analyse the excess, defined by the following

$$E(t) = (\sigma^2 + t^2)^{-2} \cdot (t^4 - 2\gamma t + \sigma^2 R_0^2), \tag{11}$$

where

$$2R_0^2 \equiv 3\sigma^2 - \sigma^{-2}\mu_4(f) \tag{11a}$$

is the parameter that characterised a distance between two peaks of $f(x)$.

The minimization of the excess (11) on variable "t" leads to the cubic equation

$$2\sigma^2 t^3 + 3\gamma t^2 - 2\sigma^2 R_0^2 t - \gamma\sigma^2 = 0. \tag{12}$$

One of three possible roots (12) places u the most near to GM position of $f(x)$ and one may use its value as the initial point for the local method of exact determination of GM position. The more cautious attitude to cubic-roots (12) leads to the necessity of creating the iterational procedure — the successive diminution of the integrational region of $f(x)$ that is done by the analysis of (10b, 11). As the criterion of the iterations termination the value of a parameter (11a) is used: if $R_0^2 \le 0$, then one can be in a sure that $f(x)$ has unimodal character in last integration region, otherwise — $R_0^2 > 0$.

For multidimensional functions, for instance — (4), analogical analysis of the excess as the function of vector $\vec{u}$ is suggested in [8]. This amalogy has been successfully tested for complicated functions, but the realization of GM-position determination for "decay" functions (4) is not yet performed now. The analysis of $D(x,y,z)$ must follow after the $C(z)$ analysis, when all detector counters for tracks emitted from the primary vertex are deleted.

The main difficulty in the realization of such kind of GM determination consists in the analytical integration (9, 9a,b). However if the discrete function (3b) will be used in $C(z)$ then this problem of analytical integration will be reduced to the accurate determination of lower (z_ℓ) and upper (z_u) limits of z, where function (3b) has nonzero value.

The limits z_ℓ, z_u depend on parameters v, h in the function $f(z, v, h)$ (7a) in the following way:

1) At $v = 0$ $z_{\ell,u} = \pm\infty$ if $h \le s$, otherwise — $G(f, s) = 0$.

2) For $v \ne 0$ let it be $R = h/v$ and $S = s/|v|$;

a) when $S \ge R > 0$

$$z_\ell = -\infty, \quad z_u = z_N - 1/(R + S); \tag{13a}$$

b) if $R > S$

$$z_\ell = z_N - 1/(R - S), \quad z_u = z_N - 1/(R + S); \tag{13b}$$

c) if $R < 0$ and $|R| < S$ then limits are the same as (13a);

d) if $R < 0$ and $|R| \ge S$ $G(f; s) = 0$.

Then one can compare given above joint limits (for XZ and YZ components of a space point) with initial boundaries, in particular $Z_L = z_3$ and $Z_U = z_4$ (position of PC_3 and PC_4 on fig. 1). Thus the problem of calculation (9-9b) consists in a simple integration of a power-like functions.

One of three roots equation (12) (that gets the munimum to excess) has value $Z = 2780$ mm for event on fig. 1. This point was used as initial for the precise determination of GM position of the smooth function $C(z)$ (shown on fig. 2a). One has got $Z = 2809$ mm by the parabolic approximation.

4. RESULTS

This described above method has been used for the determination of z-position of a primary vertex for 10000 events. The z-region of maximal values of the histogram of this z-vertices (the bright part on fig. 3) is according to the cardon target position. The histogtam of the z-crossing "input" trajectory with trajectories of nuclear fragments is also shown on fig. 3 (the dark part) for the same 10000 events. The trajectories of fragments were found by the traditional exhaustive search.

It must be noted that only for fragments passed the narrow window of the magnet (fig. 1) trajectories were reconstructed, i.e. z-crossings were determined for trajectories with insignifficant angle difference.

Nevertheless, the physical criterion of the quality of the information — the image of the target, evidenced for the benefit of the bright part of fig.3. This z-vertex determination method may improve the reconstruction of parameters of excited tra-

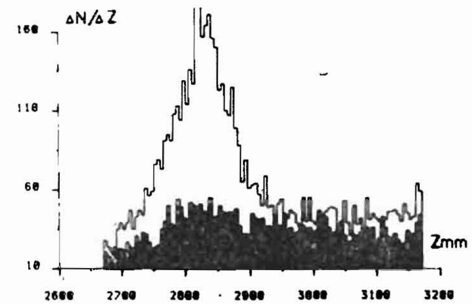

Fig. 3. The z-vertices distribution as the global maxima position of the function C(z) (7) is the bright part of the figure. The dark part of the figure is the z-crossing histigram of Mg nucleon trajectory with fragment trajectories passed through the narrow window of the magnet.

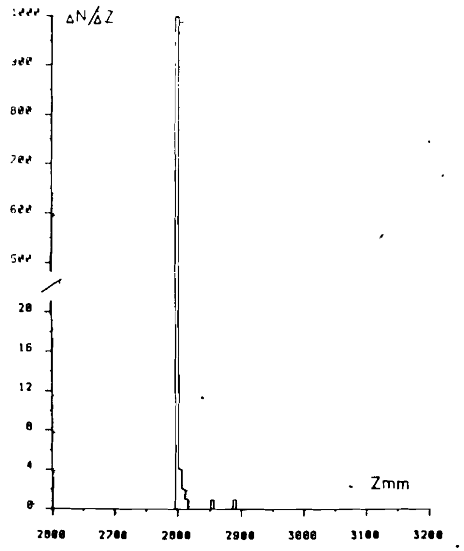

Fig. 4. The z-vertices distribution as the global maxima position of the function C(z) (7) is shown for 1000 Monte-Carlo events of ideal straight tracks generated randomly from the fixed point-like target (z= = 2800 mm).

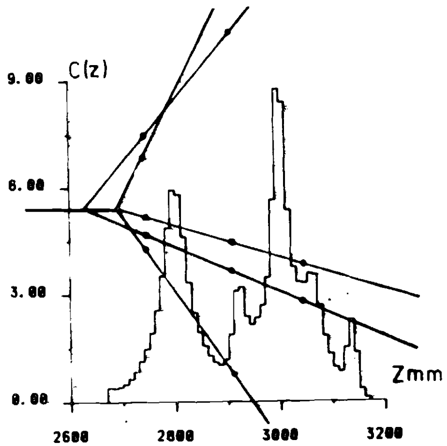

Fig. 5. The graphic of C(z) function (7) for the Monte-Carlo event of Mg interaction at z = 2800 mm and the secondary interaction at z = 3000 mm.

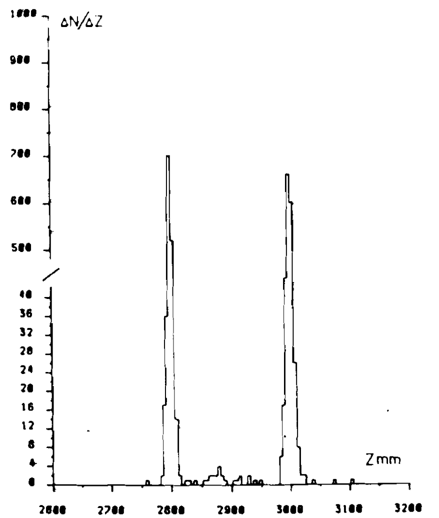

Fig. 6. The z-vertices destribution as the global maxima position of the function C(z) (7) is shown for 1000 Monte-Carlo events which are similar to those shown on fig. 5.

jectories essentially and histograms on fig. 3 would not have so significant difference.

It was expended 11 minutes CPU-time of the EC-1061 computer for the z-vertex histogram. The procedure of the trajectories reconstruction for the z-crossing expends about 20 minutes of CPU-time. It must be noted that for the approximate determination of GM-position by scheme (11-12) 6 minutes of CPU-time was expended.

The Monte-Carlo data were analysed for the test of this method GM-position determination. Ideal straight tracks were generated randomly from the fixed point-like target (z = 2800 nn) according to geometry of fig. 1. The number of this tracks was generated in limits from 2 to 8. The statistical distribution of determining z-vertices for 1000 events is shown on fig. 4.

If in any event the physical pattern is possible: a primary interaction produces some secondary particles, one of them has the parameters of the trajectories the same as primary particle and interacts (or decays) with the particles creation, too, then the C(z) function may have two prominent peaks. For such type of a function two cubic-roots of (12) may show the maxima positions with sufficient precision. The C(z) for such type of Monte-Carlo event is shown on fig. 5. The cubic-roots (12) (the approximation of the discrete function of an accuracy was used) have not so bad discrepance to real vertices (2800 mm, 3000 mm) after the first iteration: z = 2838 mm, z = 3017 mm. The following precise determination has almost absolute coincidence — z = 2802 mm, z = 3000 mm. The combinatorial part of the C(z) distorts somewhat the GM-position. Because of this, the z-vertices distribution (fig. 6) has bigger full width of peaks at background than the peak shown on fig. 4. In 1000 of such kind of Monte-Carlo events the primary interaction had a multipticity from 2 to 6, a second vertex - 3 tracks (fixed number). This analysis was rough enough, i.e. the initial z-point (for the local specify) was determined after first iteration only, without the final localization of GM-region, i.e. without condition $R_o^2(11a) \leq 0$. Besides, for such kind of tasks it is necessary to delete all coordinates of trajectories emitted from one (primary) vertex.

5. CONCLUSION

Undoubtedly, suggested vertex functions have a complicated structure. But it is also undoubted, that a problem of vertex reconstruction is complicated, especially for a curve in magnetic field and for an event with a big multiplicity.

However one may have a hope that the described scheme of C(z) — analysis for straight trajectories has shown any possibilities of vertex functions.

It is possible, that the GM-position will be determined most exactly for a multiplicity of the primary interaction ~ 100, the GM-amplitude for this is about $\sim 100 \cdot N$ (N — total number of detectors), since the more probable background would have an amplitude $\sim 2 \cdot N$.

The relatively small CPU-time for the approximate GM — determination of C(z) allows one to have any realistic hope in the creation of a programmable trigger for the primary z-vertex detection in a fixed z-limits.

The simplest variant of the trigger needs the integration of the C(z) only — if this integral is greater than assumed, then this event is accepted.

Another way consists in the analysis of z-centre of the C(z) (the first momentum — (9b)).

The more complicated variant of the trigger will need to a fast solution of the cubic equation (12) for the realization of the suggested primary z-vertex reconstruction scheme.

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Восстановление вершин
без восстановления траекторий

Предложены правила построения функций, позволяющих определять координаты вершин без восстановления траекторий. Изложена реализация варианта этих функций для прямых траекторий, когда параметры одной траектории известны.

Работа выполнена в Общеинститутском научно-методическом отделении ОИЯИ.

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Vertex Reconstruction without Track Reconstruction

Rules for the creation of the functions for vertex position determination without previous track reconstruction are suggested. The variant for straight tracks, when one trajectory is known, is described.

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