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**THE DETERMINATION  
OF THE HADRON MEAN FREE PATH  
FOR PARTICLE-PRODUCING COLLISIONS  
IN INTRANUCLEAR MATTER  
BY MEASUREMENT**

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## 1. INTRODUCTION

The subject matter in this work is the determination of the hadron mean free path  $\langle \lambda_r \rangle$  for a given reaction  $r$  in intranuclear matter, by direct measurement.

In analyzing collisions of high energy hadrons with massive target nuclei, at energies above the production threshold, the mean free path  $\langle \lambda_r \rangle$  of the incident hadron in intranuclear matter for a definite collision reaction  $r$  is useful to apply instead of corresponding hadron-nucleon collision cross-section; but, nobody knows for sure whether the hadron-nucleon cross section for a given reaction  $r$  - for the particle-producing collision, for example, is the same when the target nucleon is free or when it is among many nucleons inside a massive nucleus. The answer to the question "What is the cross section or the mean free path of a hadron for a process in collisions with nucleons inside a target nucleus or, in other words - in intranuclear matter?" must find its answer primarily in experiment; it is not obvious a priori that the cross section for a hadron-nucleon collision process with free nucleon is the same as that for the same process in hadron-nucleon collisions with nucleons inside a target nucleus.

The problem to be discussed here is concerned mainly with the method of experimental determination of the mean free path for particle-producing process in hadron collisions with objects inside target nuclei, and it is also concerned with finding of the relation between cross section for hadron collisions with free nucleons and corresponding mean free path in intranuclear matter.

This paper closes the series of articles on the subject published in JINR Communications by one of the authors /Z.S./ 1.2/

## 2. HEURISTIC CONSIDERATIONS

An atomic nucleus, characterized by the mass number  $A$  and the charge number  $Z$ , can be considered as a loosely bound collection of nucleons within a spherical volume of finite size and definite radial distribution of the nucleon density. At a given distance  $b$  from the nuclear diameter of a straight

line traversing the nuclear sphere, one can determine the thickness  $\ell(b)$  of the intranuclear matter layer and the mean nucleon density  $\langle \rho(b) \rangle$  inside a cylindrical volume  $v = \pi D_0^2 \ell(b)$ .  $\ell(b)$  and  $\langle \rho(b) \rangle$  can be determined simply and uniquely<sup>3,4</sup> - from the well-known Fermi distribution<sup>5</sup> for any of atomic nuclei; indeed, the determination is possible due to the experimental data on the nuclear structure obtained by R. Hofstadter<sup>6-8</sup>.

When the radius  $D_0$  of the volume  $v$  is as large as the strong interaction range of the nucleon, the hadron interacts with all the nucleons contained within it, in passing along the cylinder axis  $\ell = \ell(b)$ ; it has been found experimentally<sup>9,10</sup> that  $D_0$  should be taken approximately as large as the nucleon diameter  $D_0$  is.

This way, arrived at identical target nuclei along straight line courses and hit them at identical distance  $b$  from nuclear diameters, or at definite impact parameters  $b$ , hadrons have to cover definite intranuclear matter layer thicknesses  $\ell(b)$  and interact with definite number  $n_N(b)$  of the nucleons met  $n_N(b) = \pi D_0^2 \ell(b) \langle \rho(b) \rangle$ . In other words, any target nucleus may be considered as a collection of a coin-like intranuclear matter slabs of various radii and of definite thicknesses; for a given impact parameter  $b$  the thickness of the collection is  $\ell(b)$ . On the other side, any sample of collisions of  $I_0$  identical hadrons with identical target nuclei  $(A, Z)$  at a given impact parameter  $b = \text{constant}$  may be considered as a collision of a beam of hadrons of the intensity  $I_0$  with an intranuclear matter slab of a thickness  $\ell(b)$ .

Consider then a hadron flux  $I$ , incident on a layer thickness  $d\ell$  within a target nucleus. The differential intensity loss due to a reaction  $r$  is given by

$$dI = -I \sigma_r \langle \rho \rangle d\ell, \quad (1)$$

where  $\sigma_r$  is the hadron-nucleon cross section for the reaction  $r$ ,  $\langle \rho \rangle$  is the volume density of nucleons along the hadron path  $\ell$ . So,

$$I = I_0 e^{-\sigma_r \langle \rho \rangle \ell} = I_0 e^{-\ell / \langle \lambda_r \rangle}, \quad (2)$$

where  $\ell$  in fm,  $\sigma_r$  in fm<sup>2</sup>/nucleon,  $\langle \rho \rangle$  in nucleons/fm<sup>3</sup>,  $I$  is the intensity or the number of hadrons which have travelled a distance  $\ell$  in intranuclear matter, when the intensity of the incident beam was  $I_0$ .

From the formula (2), simple relation between the mean free path  $\ell_r$  in fm, the cross section  $\sigma_r$  in fm<sup>2</sup>/nucleon, and the mean nucleon density  $\langle \rho \rangle$  in nucleons/fm<sup>3</sup> along  $\ell$  exists:

$$\ell_r = \frac{1}{\sigma_r \langle \rho \rangle}. \quad (3)$$

It is useful to express the mean free path in units nucleons per some area  $S$  in fm<sup>2</sup>, and to name it  $\langle \lambda_r \rangle$  whereas the thickness in this unit to name  $\lambda$ . Naturally and conveniently it will be to use  $S = \pi D_0^2$ , where  $D_0 = 1.81$  fm is as large approximately as the nucleon diameter, or - exactly - as the nucleon strong interaction range<sup>9,10</sup>. In such units, the mean free path  $\langle \lambda_r \rangle$  is  $\langle \rho \rangle$ -independent, and related simply to the cross section  $\sigma_r$ :

$$\langle \lambda_r \rangle = \ell_r \cdot \langle \rho \rangle = \frac{1}{\sigma_r}, \quad (4)$$

where  $\langle \lambda_r \rangle$  we will express in nucleons/ $S$  units and  $\sigma_r$  in  $S$ /nucleon,  $S \approx 10.3$  fm<sup>2</sup>.

Here, it should be borne in mind that in analysing pion-xenon nucleus collisions it has been found that the number  $n_N$  of the emitted nucleons of kinetic energies from about 20 MeV up to about 400-500 MeV equals the number  $n_N = \pi D_0^2 \langle \rho \rangle \ell$  of nucleons contained within the volume  $v = \pi D_0^2 \ell$  centered on the hadron path  $\ell(b)$  fm inside intranuclear matter and involved in the collision<sup>9-12</sup>.

As the basis for any measurement of the mean free path  $\langle \lambda_r \rangle$  of a hadron inside a medium before to come in it into a reaction  $r$ , the formula

$$I = I_0 e^{-\lambda / \langle \lambda_r \rangle} \quad (2')$$

is taken, where  $I$  is the number of the beam particles covering the medium layer of a thickness  $\lambda$  without the reaction  $r$ ,  $I_0$  is the number of particles in the incident beam falling on this layer. In hadron-nucleus collisions,  $I_0$  is simply the number of events with a given collision impact parameter  $b \pm \Delta b = \text{constant}$ , where  $\Delta b = D_0$ , and  $I$  is simply the number of the events among  $I_0$  in which incident hadron traversed the target nucleus without causing the reaction  $r$ .

Let the reaction  $r$  is the particle-producing collision at the impact parameter  $b \pm D_0$  inside the target nucleus. Then, the intensity  $I$  is the number of hadrons which covered the

thickness  $\lambda = \lambda(b \pm D_0)$  without causing particle production, whereas the intensity  $I_0$  is the number of hadrons which fell on the intranuclear matter layer of the thickness  $\lambda = \lambda(b \pm D_0)$  at the impact parameters  $b \pm D_0$ .

For the measuring of the mean free path  $\langle \lambda_r \rangle$ , the quantities  $I_0, I, \lambda(b \pm D_0)$  should be determined experimentally, and we will show how it is possible to do it.

### 3. SELECTION CRITERIA FOR COLLISIONS AT DEFINITE IMPACT PARAMETERS

Firstly, the selection criteria for the hadron-nucleus collision events must be worked out in which the collision impact parameter  $b \pm D_0$  is recognized uniquely. As physical basis for the criteria, some experimental facts must be used: 1. Hadrons are braked and can be stopped inside intranuclear matter; definite range-energy relation exists<sup>/13-15/</sup>. 2. For a given incident hadron and for a given target nucleus, there exists such definite value of the hadron momentum  $P_{hs}$  at which the hadron  $h$  can pass through the intranuclear matter layer as thick as the target nucleus diameter  $D$  is without causing particle production and stop at the end of this layer. For the  $^{131}_{54}\text{Xe}$  nuclei and for incident pions it happens at the collision impact parameters within the value limits of  $b$  from  $0 - D_0$  to  $0 + D_0$  at the momentum value  $P_{\pi s} \approx 3.5 \text{ GeV/c}$ . This momentum can be found experimentally from the momentum-dependence of the probability  $P_{st}$  of the stoppings, fig.1. Such stoppings within the intranuclear matter layers as thick as the xenon nucleus diameter  $D$  happen only at uniquely definite  $P_{\pi s}$  value and at corresponding impact parameters  $b = 0 \pm D_0$ ; at larger impact pa-

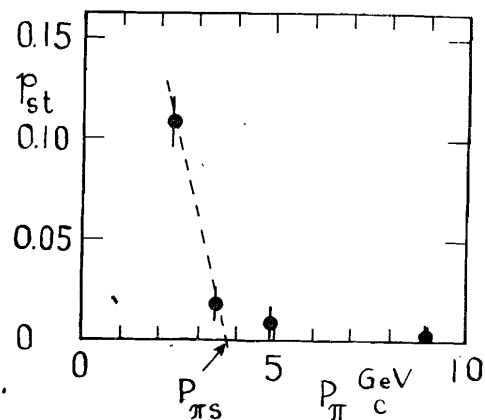


Fig. 1. Incident pion momentum  $P_{\pi}$ -dependence of the probability  $P_{st}$  of the occurrence of pion stoppings in intranuclear matter. At the point  $P_{\pi s}$  on the pion momentum axis the value of  $P_{\pi s}(D)$  which corresponds to stoppings on the matter layer thicknesses as large as the target nucleus diameter  $D$  nucleons/S lies; for  $^{131}_{54}\text{Xe}$  target nucleus this value is<sup>/13/</sup>  $P_{\pi s} \approx 3.5 \text{ GeV/c}$ .

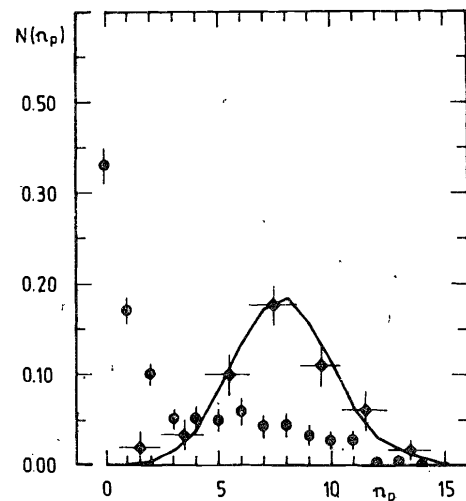


Fig. 2. The proton multiplicity  $n_p$  distribution  $N(n_p)$  for the stoppings -  $\blacklozenge$ , and for the passages -  $\bullet$  without causing particle production in pion-xenon nucleus collisions at 3.5 GeV/c momentum; according to previous works<sup>/10/</sup>. Polygon represents the binomial distribution of the proton multiplicities in stoppings.

rameters, the stoppings do not happen at all<sup>/18/</sup> and only passages at unknown impact parameters  $b > D_0$  happen - it

is due to spherical shape of the target nucleus; at higher  $P_{\pi}, P_{\pi} > P_{\pi s}(D)$ , the passages occur only. 3. The proton multiplicity  $n_p$  distribution  $N(n_p)$  in a sample of the stoppings, after the hadron passages through the intranuclear matter layer thickness as large as the nucleus diameter is, has a characteristic symmetric bell-like shape with the symmetry axis located at the mean proton multiplicity  $\langle n_p \rangle = \frac{Z}{A} \pi D_0^2 D \langle \rho \rangle = \frac{Z}{A} SD$ , where  $S = \pi D_0^2$  and  $D$  is in nucleons/S;  $\langle n_p \rangle$  in the stoppings can be predicted for a given nucleus from the experimentally known information about nucleon emission process in hadron-nucleus collisions<sup>/11/</sup> and from the experimentally known charge distribution in nuclei<sup>/3-5/</sup>; for the xenon nucleus  $\langle n_p \rangle \approx 8$ , fig.2. In any other class of events, at any other incident pion momentum the shapes of the  $N(n_p)$  distribution are different. 4. Any hadron causes the emission of nucleons with kinetic energies from about 20 MeV up to about 400-500 MeV from the target nucleus in passing through it; the number  $n_N$  of the nucleons equals the number  $n_N$  of nucleons contained within the volume  $v = \pi D_0^2 \ell$  centered on the hadron path in intranuclear matter, and interacted with the projectile<sup>/11,12/</sup>. Among the nucleons emitted is some number  $n_p$  of the protons and the ratio  $n_p/n_N$  changes from 0 to 1, in average, it equals  $\langle n_p/n_N \rangle = Z/A$ . Such fluctuations of the  $\langle n_p/n_N \rangle$  are found when the projectile hadrons were electrically charged, it would be of great importance for the knowledge about the nucleus structure to test it for collisions of neutral hadrons with nuclei.

And so, it is possible to single out the events in which the incident hadrons, pions for example, fell on the target nucleus at the impact parameter  $b = 0 \pm D_0$  and passed through the nuclear matter layer  $\lambda$  in nucleons/S as thick as the nucleus diameter  $D$  in nucleons/S is and do not cause the particle production. In fact these "passages" are stoppings on the matter layers as thick as the target nucleus diameters, but they can be treated as passages through the intranuclear matter layers as thick as the target nucleus diameter  $D$  in nucleons/S is. In order to single out such events, one should analyse stoppings in hadron-nucleus collisions at definite projectile momentum  $P_{hs}$  determined by the above described procedure, using  $\langle n_p \rangle - P_h$  distribution shown in fig.1; now, it is possible to determine  $P_{hs}$  from the range-energy relation for hadrons in intranuclear matter.

Secondly, the events should be selected in which incident hadrons fell on identical thicknesses  $\lambda(b \pm D_0) = \lambda(0 \pm D_0) \approx D$  in nucleons/S inside intranuclear matter, causing any-type collisions; in other words, the events should be selected in which incident hadrons collided with definite target nuclei at definite impact parameters  $b = 0 \pm D_0 = \text{constant}$ .

The number  $I_0$  of the events in which incident hadron fell on the target nucleus at the impact parameter  $b = 0 \pm D_0$  can be determined<sup>3,4/</sup> - from the measurable target nucleus radius  $R$ , known for nuclei well enough<sup>5/</sup>, and from the total number  $N_0$  of the hadrons which fell on the nucleus - known as the total number of the any-type events among which the "stoppings" on the nucleus diameter  $D$  were selected:

$$I_0 = N_0 \frac{\pi D_0^2}{\pi R^2}; \quad (5)$$

as  $R$  such impact parameter  $b$  has been taken at which the intranuclear matter layer thickness<sup>3,4/</sup>  $\lambda(b \pm D_0) = 0.5$  nucleons/S, or  $b = R = 7.59$  fm; for  $D_0$  the value 1.8 fm was taken.

Thirdly, the collision events should be selected, in the sample  $I_0$  of the above ones, in which the incident hadrons traversed the target nuclei along their diameters without causing particle production. These events are the stoppings on the intranuclear matter layers as thick as the target diameter is. The stoppings at definite incident hadron momentum  $R_{hs}$  are just the events looked for. This way we have the value for  $I$  as well.

#### 4. MEASUREMENT OF THE PION MEAN FREE PATH IN INTRANUCLEAR MATTER

In this measurement, xenon target nuclei were used bombarded by negatively charged pions at 3.5 GeV/c momentum in the 180 litre xenon bubble chamber at the Moscow Institute for Theoretical and Experimental Physics<sup>16/</sup>.

Out of  $N_0 = 6301$  any-type collision events  $I = 96 \pm 10$  stoppings on  $D$  nucleons/S were found in the photograph scanning and analysis<sup>10/</sup>. Simple calculations<sup>3,4/</sup>, based on the data on the nuclei structure<sup>5/</sup>, lead to the value  $\lambda(0 \pm D_0) = 19$  nucleons/S of the intranuclear matter layer thickness the incident pions traversed. Formula (5) gives the value  $I_0 = 358 \pm 19$ .

From formula (2'), we have the mean free path  $\langle \lambda_{in} \rangle$  for the particle-producing collisions of hadrons in intranuclear matter:

$$\langle \lambda_{in} \rangle = - \frac{\lambda(0 \pm D_0)}{\ln(I/I_0)}; \quad (6)$$

this is the measured value for pions, we call it  $\langle \lambda_{\pi in} \rangle_{meas}$ . Using the values for  $\lambda(0 \pm D_0)$ ,  $I_0$ , and  $I$  obtained in measurements, we have:

$$\langle \lambda_{\pi in} \rangle_{meas} = 14.4 \pm 1.8 \text{ nucleons/S.} \quad (7)$$

The value of the mean free path calculated from formula (4) using the data on pion-nucleon inelastic cross section<sup>17/</sup> at the incident pion momentum 3.5 GeV/c,  $\sigma_{\pi p in} = 21.6 \pm 1.10$  mb or  $0.21 \pm 0.01$  S/nucleon, is:

$$\langle \lambda_{\pi in} \rangle_{calc} = 4.8 \pm 0.2 \text{ nucleons/S.} \quad (8)$$

The ratio  $\langle \lambda_{\pi in} \rangle_{meas}$  and  $\langle \lambda_{\pi in} \rangle_{calc}$  is:

$$k = \frac{\langle \lambda_{\pi in} \rangle_{meas}}{\langle \lambda_{\pi in} \rangle_{calc}} = 3.0 \pm 0.15. \quad (9)$$

It means that formula for the determination of the pion mean free path  $\langle \lambda_{\pi in} \rangle$  in intranuclear matter, for the particle-producing collisions, should be

$$\langle \lambda_{\pi in} \rangle = k \frac{1}{\sigma_{in}} \text{ nucleons/S,} \quad (10)$$

where  $\sigma_{in}$  is the cross section for elementary pion-nucleon inelastic collision in S/nucleon units.

There are indirect experimental indications<sup>/18/</sup> that relation (10) is valid for both the pion- and the proton-nucleus collisions within wide interval of the projectile momentum values - from about 2 up to about 1500 GeV/c; various quantities characterizing hadron-nucleus collision outcome, calculated using free-parameterless formulas in which the relation (10) plays fundamental role, are in agreement with corresponding experimental data<sup>/18/</sup>.

## 5. CONCLUSIONS AND REMARKS

The value  $\langle \lambda_{\pi in} \rangle_{exp}$  obtained experimentally is definitely larger than the value  $\langle \lambda_{\pi in} \rangle_{calc}$  calculated using the formula (4), commonly applied, and the known data on the inelastic cross section  $\sigma_{in}$  for elementary hadron-nucleon collisions<sup>/17/</sup>. The pion mean free path in intranuclear matter for particle-producing collisions obtained experimentally is larger than the mean thicknesses  $\langle \lambda \rangle$  of the heaviest nuclei, which are smaller than about 11 nucleons/S<sup>4/</sup>; it is comparable with the maximum thicknesses of the heaviest nuclei as well<sup>/4/</sup>.

Obviously, relation (9) for the particle-producing collisions, and others similar which might be obtained for other reactions  $r$ , is valid for special type of collisions - for the particle-producing collisions in the case under discussion here. For the any-type collisions - for the total cross sections - formula (4) which is commonly in use should be valid; the mean free path  $\langle \lambda_t \rangle$  for any-type collisions should be related to the hadron-nucleon total cross section  $\sigma_t$  in S/nucleon as:

$$\langle \lambda_t \rangle = \frac{1}{\sigma_t} \text{ nucleons/S.} \quad (11)$$

In this work, the selection criteria of the stoppings and of the any-type events included to value  $I_0$  and to value  $I$  were reexamined<sup>/10/</sup> and the values obtained for  $\langle \lambda_{\pi in} \rangle_{exp}$  and for  $k$  are determined more accurately. Previous paper contain less accurate data, which lie within experimental errors, although.

Here, the mean free path  $\langle \lambda_{\pi in} \rangle_{exp}$  was obtained for pions at  $P_{\pi} = 3.5$  GeV/c only. The questions arise therefore: Will the relation (9) be valid for other values of  $P_{\pi}$ ? Will it be the same relation for other hadrons - for protons, kaons, and other? The conclusive answer lies in experiments, but it will be reasonably to expect the answer "yes".

The method worked out here and in previous papers<sup>/1,2/</sup> can be applied in future experiments by various techniques, as well.

## 6. ACKNOWLEDGMENT

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## REFERENCES

1. Strugalski Z. JINR, E1-80-799, Dubna, 1979.
2. Strugalski Z. JINR, E1-83-563, Dubna, 1983.
3. Strugalski Z., Pawlak T., JINR, E1-81-378, Dubna, 1981.
4. Pawlak T. et al. JINR, E1-86-643, Dubna, 1986.
5. Elton L.R.B. Nuclear Sizes. Oxford University Press, 1961.
6. Hofstadter R. - Rev.Modern Phys., 1956, 28, p.214.
7. Hofstadter R. - Ann.Rev. Nucl.Sci., 1957, 7, p.231.
8. Hofstadter R. Nuclear and Nucleon Structure. Frontiers in Physics, W.A.Benjamin, Inc., New York, 1963.
9. Strugalski Z. JINR, E1-11976, Dubna, 1978.
10. Strugalski Z., Pawlak T., Pluta J. JINR, E1-82-718, E1-82-719, E1-82-841, Dubna, 1982; E1-83-234, Dubna, 1983; E1-85-888, Dubna, 1985.
11. Strugalski Z. JINR, E1-84-853, Dubna, 1984.
12. Strugalski Z. Proc. of the 20th International Cosmic Ray Conference, Moscow, August 2-15, 1987, vol.5, p.46.
13. Strugalski Z. JINR, E1-83-850, Dubna, 1983.
14. Strugalski Z. JINR, E1-84-194, Dubna, 1984.
15. Strugalski Z., Pawlak T., Pluta J. JINR, E1-84-855, Dubna, 1985.
16. Kuznetsov E.V. et al. - Instrumentation and Experimental Technique (Russian), 1970, 2, p.56.
17. Flaminio V. et al. CERN-HERA 79-01, 02, 03, 28 March, 1979.
18. Sredniawa B. Strugalska-Gola E. JINR, E1-83-277, Dubna, 1983.

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Определение среднего свободного пробега адрона в ядерной материи до столкновения, ведущего к рождению частиц

Показано, как возможно определить на опыте средний свободный пробег  $\langle \lambda_{in} \rangle$  для столкновений адронов в ядерной материи, ведущих к рождению частиц. Определен средний свободный пробег для пионов в ядрах  $^{131}_{54}\text{Xe}$  при импульсе 3,5 ГэВ/с. Найдено соотношение между  $\langle \lambda_{in} \rangle$  в единицах нуклон/S и сечением  $\sigma_{in}$  в единицах S/нуклон для неупругих столкновений адронов с нуклонами:  $\langle \lambda_{in} \rangle = k \frac{1}{\sigma_{in}}$ , где  $S \approx 10 \text{ фм}^2$ ,  $k = 3,0 \pm$

$0,15$ ; приводится физический смысл величины S.

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The Determination of the Hadron Mean Free Path for Particle-Producing Collisions in Intranuclear Matter by Measurement

It is shown how it is possible to determine the hadron mean free path  $\langle \lambda_{in} \rangle$  for particle-producing collisions in intranuclear matter by measurement. The mean free path for the collisions of pions inside  $^{131}_{54}\text{Xe}$  nuclei at 3.5 GeV/c momentum has been measured. The relation between  $\langle \lambda_{in} \rangle$  in units nucleons/S and the hadron-nucleon inelastic cross section  $\sigma_{in}$  in units S/nucleon is found:  $\langle \lambda_{in} \rangle = k \frac{1}{\sigma_{in}}$ ,

where  $S \approx 10 \text{ фм}^2$ ,  $k = 3.0 \pm 0.15$ ; physical meaning of S is given in this paper.

The investigation has been performed at the Laboratory of High Energies, JINR.

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