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# EVALUATION OF THE POSSIBILITY FOR DETERMINING THE PARAMETERS OF THE $t_{\mu}$-ATOMS CHARGE EXCHANGE ON HELIUM

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## 1. INTRODUCTION

Occurrence of the $^3He$ and $^4He$ in $D_2/T_2$ mixtures leads to the reduction of the number of fusion cycles catalyzed by one muon. The losses are due to [1]:

- helium atomic capture of the muon,
- muon transfer from excited muonic hydrogen to helium,
- muon exchange process from the $d\mu$ and $t\mu$ in the ground state to helium.

So, helium, produced in fusion reactions in $dt\mu$, $dd\mu$, and $tt\mu$ mesic molecules and from the tritium $\beta$ decay, should be taken into account as the primary fuel poison factor.

The probability of the atomic capture by a hydrogen atom in H/He mixtures is expressed as:

$$W_H = (1 + Ac)^{-1}, \qquad (1)$$

where c is the relative helium concentration defined as the ratio of the number of helium atoms to the number of hydrogen atoms, and A is the ratio of the capture rates for helium and hydrogen. Till now no experimental data concerning the value of A in a H/He mixture have been available, but theoretical considerations lead to the conclusion that $\pi^-$ and $\mu^-$ atomic capture are the same [2,3], therefore the value of A can be taken from experiments on $\pi^-$ capture in which A = 1.84 was obtained [4].

The muon transfer from excited $p\mu^*$-atoms to helium was observed for the first time by Bystritsky et al. [5]. The dependence of the probability of the $p\mu$-atom formation in the ground state, $W_0$, on relative helium concentration, c, has been measured. The estimated rate of this process is about $2\cdot 10^{12}\ s^{-1}$ * [6], i.e., of the same order as mesic atoms de-excitation rates [7]. There are no experimental data for D/He and T/He mixtures, nor theoretical considerations about this process, but one can expect that it takes place analogously for $d\mu^*$ and $t\mu^*$ atoms changing considerably kinetics of the muon catalyzed fusion.

The direct transfer of the muon from mesic hydrogen in the ground state to helium is strongly suppressed [8]; its rate is

* Throughout the paper the values of transition rates are referred to the liquid hydrogen density ($4.25\cdot 10^{22}\ cm^{-3}$).

about $10^6$ $s^{-1}$ . It is well established now that transfer occurs mainly via formation of the intermediate mesic molecules [9,10]

$$H\mu + He^{++} \rightarrow [(H\mu He)^*]^{++} \rightarrow H^+ + [He\mu]^+, \qquad (2)$$

where H = p, d, t.

The rates, $\lambda^0_{He}$ , of processes (2) are about $10^8$ $s^{-1}$ . This phenomenon was investigated experimentally in the case of $p\mu$ + $^4He$ [5] , $d\mu$ + $^3He$, and $d\mu$ + $^4He$ [11] at room temperature. From $dt\mu$ muon catalyzed fusion experiments [12] the dependence of the $\lambda^0_{^3He}$ on temperature (from 100 to 400 K) was extracted but with experimental error about 30%.

To understand the results of the experiments on muon catalyzed fusion in $D_2/T_2$ , having in mind that isotope exchange in excited states $(d\mu)_n + t \rightarrow (t\mu)_n + d$ [13,14] augments considerably the population of $t\mu$ -atoms, it is desirable to have independent information about $t\mu$ + He transfer rates from the experiments with targets containing only $T_2$ and He . In this case the transfer rate can be extracted from the time distributions and yields of neutrons from the synthesis in $tt\mu$ molecule [15] . In this paper we present the kinetic formulae describing the processes in $T_2$/He mixtures and we give some recommendations concerning the tritium density $\phi$ and the helium concentration at which an experiment aimed at the joint determination of the values of the probability $W_0$ and the rate $\lambda^0_{He}$ should be performed.

## 2. KINETICS OF THE PROCESSES IN $T_2$/He MIXTURES

The processes in a $T_2$/He mixture are shown schematically in Fig.1. The muon entering the target or the muon liberated in a fusion, after slowing down, forms an excited $t\mu^*$ atom $(n \geq 14)$ [3] or $He\mu^*$ . The rate of this process is about $10^{12} s^{-1}$ .

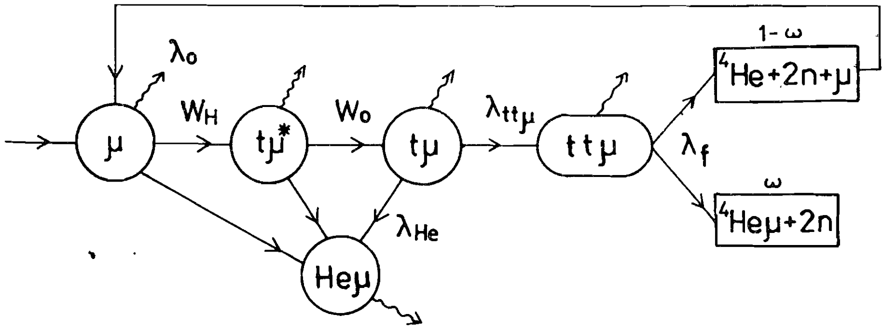



*Fig.1. Scheme of processes in the* $T_2$/He *target.*



Next, in the collisions with the mixture atoms $\mu$ -atom deexcitation to the ground state or muon transfer to helium takes place. The cascade in the muonic atom is very fast ($\approx 10^{11}$ $s^{-1}$ ). The $t\mu$ -atom can have two spin states in the ground state: $s = 1$ and $s = 0$. The transition from the upper to lower spin state is rapid [1] ($10^9$ $s^{-1}$ ), the reverse transition can be neglected because of the hyperfine splitting energy $E_{hfs}$ = $= E_{t\mu(1)} - E_{t\mu(0)} = 0.24$ eV. In the $t\mu$ ground state two processes are possible: the $tt\mu$ -molecule formation via a non-resonant Auger-electron-ejection mechanism [16] with the rate $\lambda^0_{tt\mu}$ = $=3\cdot10^6$ $s^{-1}$ and the molecular exchange of $\mu^-$ from $t\mu$ to He . Since both of them are independent of the spin state of the $t\mu$ atom, it can be easily shown that the kinetic graph with the hyperfine structure reduces to the graph without it, as presented in Fig.1. This also means that the spin-flip transition will not be observed in targets containing only $T_2$ or $T_2$ + Z. After the $tt\mu$ -molecule formation the nuclear synthesis takes place in it with the rate $\lambda_f$ = $10^7 \div 10^8$ $s^{-1}$ [17]. In this process the muon can be released with probability $f = 1 - \omega$, $\omega = 0.1$ [18] and reenters the chain to initiate the next cycle.

From the quite realistic assumption that measurements are realised at times $t > 0.01$ $\mu s$ after the muon stops, it follows that the changes of the shape of the measured neutron time distributions due to muon capture and $t\mu^*$ atom cascade are negligible, therefore these two processes can be described only by $W_H$ -the probability of $t\mu^*$ formation and $W_0$ -the probability that $t\mu^*$ -atom reaches its ground state.

Within the above approximation the kinetic equations describing the muon-catalyzed fusion in $T_2$/He are:

$$\frac{dN_{t\mu}}{dt} = W_H W_0 S(t) - \Lambda_t N_{t\mu},$$

$$\frac{dN_{tt\mu}}{dt} = \lambda_{tt\mu} N_{t\mu} - \Lambda_f N_{tt\mu}, \qquad (3)$$

$$\frac{dN_n}{dt} = 2\lambda_f N_{tt\mu}.$$

With the source term defined by:

$$S(t) = \delta(t) + \lambda_f f N_{tt\mu}, \qquad (4)$$

where

$$\Lambda_t = \lambda_{t\mu} + \lambda_{He} + \lambda_0 = \lambda^0_{tt\mu}\phi + \lambda^0_{He}\phi c + \lambda_0, \quad \Lambda_f = \lambda_f + \lambda_0. \qquad (5)$$

The initial conditions are: $N_i(t=0) = 0$ . $\delta(t)$ represents the external muon source. Index 0 denotes rates normalized to LHD. $\lambda_0 = 0.455\ \mu s^{-1}$ is the muon decay rate.

The Laplace transform of the first real cycle is simply

$$F_1(s) = \frac{W_H W_0 \lambda_{tt\mu} \lambda_f}{(s+\Lambda_t)(s+\Lambda_t)} \tag{6}$$

and, according to prescription given in our earlier paper [15] the all-cycles (AC) time distribution registered with the registration efficiency $\epsilon$ is:

$$F(t) = \frac{dN_n(t)}{dt} = \frac{2\epsilon W_H W_0 \lambda_{tt\mu} \lambda_f}{\sqrt{\Delta}} (e^{-r_1 t} - e^{-r_2 t}) , \tag{7}$$

where

$$\Delta = (\Lambda_f - \Lambda_t)^2 + 4 W_H W_0 \lambda_{tt\mu} \lambda_f f , \tag{8}$$

$$r_{1,2} = \frac{1}{2}(\Lambda_t + \Lambda_f \pm \sqrt{\Delta}) .$$

The A-C yield of neutrons from fusion is given by:

$$Y = \frac{2\epsilon W_H W_0 \lambda_{tt\mu} \lambda_f}{\Lambda_t \Lambda_f - W_H W_0 \lambda_{tt\mu} \lambda_f f} . \tag{9}$$

The time distributions for consecutive cycles are of great importance for the analysis of experimental data. The time distribution of neutrons in the k-th registered cycle is expressed as:

$$F_k(t) = 2f^{k-1}(\epsilon W_H W_0 \lambda_{tt\mu} \lambda_f)^k \frac{(-1)^k}{(k-1)!} \sum_{j=1}^{k} \frac{(k+j-2)!}{(j-1)!(k-1)!} \times \tag{10}$$

$$\times \left[\frac{e^{-s_1 t}}{(s_1 - s_2)^{k+j-1}} + \frac{e^{-s_2 t}}{(s_2 - s_1)^{k+j-1}}\right] t^{k-j},$$

where

$$s_{1,2} = \frac{1}{2}(\Lambda_t + \Lambda_f \pm \sqrt{(\Lambda_f - \Lambda_t)^2 + 4f(1-\epsilon) W_H W_0 \lambda_{tt\mu} \lambda_f}) .$$

Consequently, the yield of the k-th registered cycle looks like:

$$Y_k = 2f^{k-1}\left[\frac{\epsilon W_H W_0 \lambda_{tt\mu} \lambda_f}{\Lambda_t \Lambda_f - f(1-\epsilon) W_H W_0 \lambda_{tt\mu} \lambda_f}\right]^k . \tag{11}$$

## 3. RESULTS AND DISCUSSION

The kinetic formulae given in the previous Section were applied to predict the results of the experiments in $T_2$/He mixtures with different tritium densities $\phi$ and helium concentrations c .



The probability of the $t\mu$ formation in the ground state, $W_0$ , is a complex function of the rates of transitions (radiation, Auger and Stark) between $t\mu^*$ levels and the rates of transitions from $t\mu^*$ levels to a helium atom. It means that $W_0$ depends on both $T_2$ density and He concentration. However, the parametrization given in [5] takes into account only $W_0$ dependence on concentration c . In order to obtain dependence $W_0 = W_0(\phi, c)$ it was assumed that $W_0$ are the same for $t\mu^*$/He and for $p\mu^*$/He , and that the transitions of $\mu$ occur from the $t\mu^*$ -levels n=5 and lower. Using the scheme of the cascade in excited muonic hydrogen given in [13] we obtained the transfer rates from $p\mu^*$ to He fitting the experimental points $W_0$ taken from [5]. The transfer rates for levels n=5,4,3,2p,2s are 4.8, 1.0,0.4,2.1 and 0.002 (all in $10^{12}$ $s^{-1}$), respectively.

The dependence of $W_0$ on helium concentration and $T_2$ density is shown in Fig.2. One can see that $W_0$ is sensitive to concentration changes mainly in the range c=0 to c=0.3.

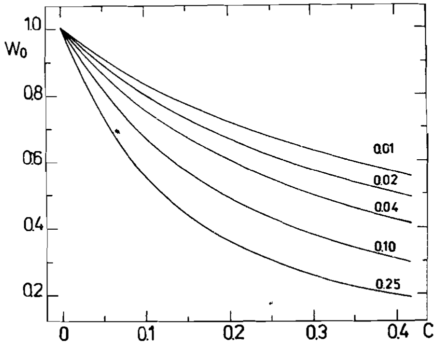



*Fig. 2. Hypothetical dependence of* $W_0$ *(the probability of reaching the ground state by an excited* $t\mu^*$ *atom) on the relative helium concentration* c *at different tritium densities* $\phi$ *indicated at the curves.*

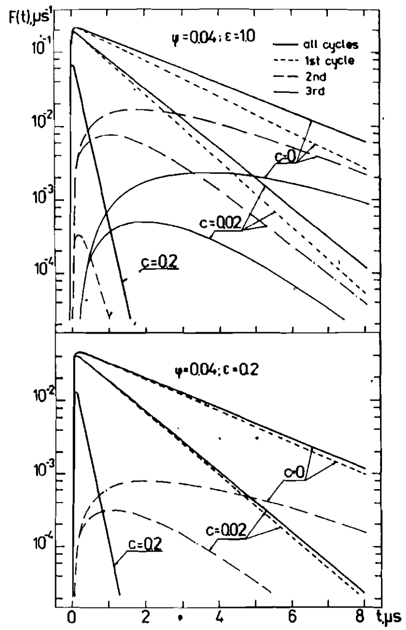



*Fig.3. Time distributions of registered neutrons from the nuclear synthesis in a* $tt\mu$*-molecule for tritium density* $\phi$ *=0.04 and helium concentrations* c *indicated at the curves. Besides all-cycles time distributions, the distributions of the first three cycles are presented. The top part corresponds to the neutron registration efficiency* $\epsilon$ *=1, the bottom part –* $\epsilon$ *=0.2. The chain parameters are:* $\lambda^0_{tt\mu} = 3\mu s^{-1}$, $\lambda^0_{He} = 600\,\mu s^{-1}$, $\lambda_f = 50\ \mu s^{-1}$ *and* $\omega$ *=0.1.*

So far no experimental data exist concerning the values of $\lambda^0_{tt\mu}$, $\lambda_f$, and $\omega$. If not stated otherwise, the following theoretically predicted values are used: $\lambda^0_{tt\mu} = 3\,\mu s^{-1}$ [16], $\lambda_f = 50\,\mu s^{-1}$ [17], $\omega = 0.1$ [18]. The neutron time distributions were constructed using two values of the rate of muon transfer from $t\mu$ in the ground state to helium: $\lambda^0_{He} = 200\,\mu s^{-1} (t\mu^*/{}^4He)$ and $\lambda^0_{He} = 600\,s^{-1} (t\mu^*/{}^3He)$, both values calculated for room temperature [9].

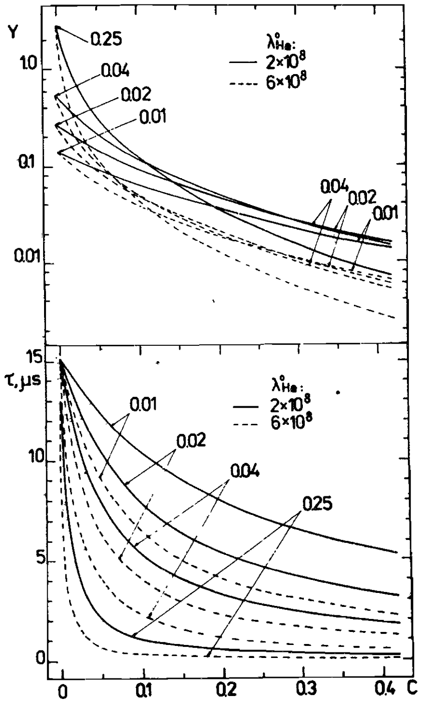
Fig.3 represents a typical time distribution of neutrons from fusion reaction in a $T_2/He$ mixture for all-cycles and for the first, second, and third registered cycles. As is seen, increase in helium concentration leads to a rather dramatic change of the slope of time distributions, which restricts the available time range. Let us remark that application of the cycle-by-cycle analysis for $c > 0.02$ is strongly limited because of the very poor statistics of the second and third cycles even for the case with the neutron registration efficiency $\epsilon = 1$. The analysis of the second cycle yield (Eq.11) confirms this conclusion.

At the planning stage of the experiment the number of neutrons produced by one muon and the slope of time distributions are of special interest. Fig.4 shows the dependence of the all-cycles yield of neutrons and the time $\tau$ after which $F(t)$ decreases $10^3$ times with respect to its maximum value on helium concentration $c$. Analysis of these two factors leads to the conclusion that measurements of $W_0(\phi, c)$ and $\lambda^0_{He}$ should be performed at the densities $\phi \lesssim 0.2$.

To get some information about errors, the simulated results of $tt\mu$-muon catalyzed fusion experiments were analysed for several values of $\phi$ and $c$. Time distributions (7) were constructed by random dispersion of events in each time interval ($\approx 0.1\,\mu s$) according to Poisson distribution. Next, the "experimental" distributions, obtained in this way, were fitted [19] using formula (7) in order to extract values of $W_0$ and $\lambda^0_{He}$. It was assumed that parameters $\lambda^0_{tt\mu}$, $\lambda_f$, $\omega$, and $\epsilon$ are found earlier in experiments with a pure tritium target as suggested in [20]. The data obtained in each case with specified values of $\phi$ and $c$ were analyzed independently. The results of the experiment modelling are shown in Fig.5. It is seen that even with moderated statistics (e.g., 2500 registrated neutrons) the errors of $W_0$ and $\lambda^0_{He}$ are less than 5%. Of course, a joint fit to a set of data obtained at different $\phi$ and $c$ will give smaller values of errors, e.g. from the joint analysis of 6 neutron time distributions at $\phi = 0.02, 0.04$ and $c = 0.02, 0.2, 0.4$ with statistic $N = 2500$ the errors were: $\Delta\lambda^0_{He}/\lambda^0_{He} \approx 1.2\%$ and $\Delta W_0/W_0 \approx 2.5\%$. Practically the same values of errors were obtained when $\lambda^0_{He} = 200\,\mu s^{-1}$ was used and in the case with $\lambda^0_{tt\mu} = 1\,\mu s^{-1}$. In all those cases the confidence level was $\geq 90\%$.



*Fig.4. The dependence of the all-cycles yield of neutrons, Y, and the time $\tau$ after which F(t) decreases $10^3$ times with respect to its highest value, on the helium concentration for different values of tritium density $\phi$ indicated at the curves.*



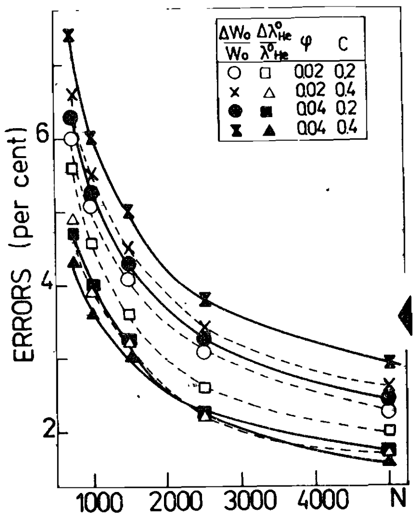




*Fig.5. Estimates of errors $\Delta W_0/W_0$ and $\Delta\lambda^0_{He}/\lambda^0_{He}$ as a function of the number of registered neutrons N for different values of tritium density $\phi$ and helium concentration c. The points are obtained with $\lambda^0_{He} = 600\,\mu s^{-1}$ and $\epsilon = 0.2$.*

The above results indicate that it is possible to determine the parameters of muon transfer from $t\mu$-atoms to helium: $W_0$ and $\lambda^0_{He}$, from neutron time distributions obtained in experiments with tritium density $\phi \lesssim 0.2$ and helium concentration c in range: $0 \div 0.4$. The required statistic (about $10^4$ registered neutrons) can be obtained in less that 10 hours, therefore the $^3He$ build-up from the tritium $\beta$ decay during exposition can be kept below $c \approx 7 \cdot 10^{-6}$. In order to find the values of transfer rates from different levels of an excited $t\mu^*$-atom the measurements should be performed at least at two tritium densities (e.g., $\phi = 0.01$ and $\phi = 0.1$). These conclusions are also valid for D/He mixture ($\lambda^0_{dd\mu} = 2.76\,\mu s^{-1}$, $\lambda_f = 10^3\,\mu s^{-1}$ [1]) except that the shape of time distributions at $t \lesssim 0.3\,\mu s$ will be slightly different due to different values of $dd\mu$-formation rate from two spin states of the $d\mu$-atom.

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Оценка возможности определения параметров процесса перехвата мюонов от $t\mu$-атомов к гелию

Приводятся уравнения кинетики, описывающие мезоатомные и мезомолекулярные процессы, происходящие в смеси $T_2$-He. На основании этих формул выполнено моделирование эксперимента по измерению вероятности перехода $t\mu^*$ -атома из возбужденных состояний в основное ($W_0$) и скорости перехвата мюона от $t\mu$ -атома, находящегося в основном состоянии к ядрам гелия ($\lambda^0_{He}$). Для экспериментального определения величин $W_0$ и $\lambda^0_{He}$ с требуемой точностью найдены диапазоны значений плотности трития и относительной концентрации гелия с ($0 < \phi < 0{,}2$, $0 < c < 0{,}4$).

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Evaluation of the Possibility for Determining the Parameters of the $t\mu$-Atoms Charge Exchange on Helium

The kinetic formulae describing $\mu$-atomic and $\mu$-molecular processes in a $T_2$/He target are presented. Those formulae were applied to predict the results of experiments aimed at determining the values of probability of the $t\mu$-atom formation in its ground state and the rate of muon transfer from $t\mu$ in the ground state to helium. It is estimated that variation of the helium concentration in the range 0 - 0.4 at two tritium densities ($\phi \leq 0.2$) is sufficient in this kind of experiment.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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