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ON REGULARITIES OF STRANGE PARTICLE PRODUCTION IN HIGH ENERGY COLLISIONS



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## 1. Introduction.

Last time a lot of experimental data on multiparticle production at high energy are available from ISR (CERN), Fermilab (Batavia) and Serpukhov  $\frac{1}{\sqrt{2}}$ . Most bulk of data referes to  $\pi^{\pm}$ production and there are some data on p.  $\tilde{p}$  and  $K^{\pm}$  production/1/, <sup>/2/</sup>. However. a complete description of high energy collisions must include also information on neutral particle production. particularly on neutral strange particle production (SPP). Moreover. recent measurements /1/ suggest an appreciable contribution of the cross section for SPP to the total cross section. Even at 25 Gev/c in 7 p collisions, the total strange particle cross section is about 1/6 of the total cross section  $^{/3/}$ . The study of SPP nermits to find the dependence of particle production properties on masses and strangeness quantum numbers. Although there exist some data compilations where information on SPP at high energies is summarized (e.g. (1/1)/1/1/1/1), there are very few systematical overtitative analyses. Some regularities on  $\mathcal{I}^{\circ}$ ,  $K_{S}^{\circ}$  and A production in semiinclusive reactions have been already pointed out in ref. /5/.

In this paper we shall be concerned with model-independent presentation of data on SPP. The generating functional formalism  $\frac{6}{\frac{7}{12}}$  is particularized for processes with SPP (Sect. 2). The existing data on total cross sections for SPP are reviewed in Section 3. Section 4 deals with comparative analysis of average strange particle (SP) and non SP multiplicities data. Conditional charge distributions  $\frac{8}{\frac{7}{9}}$  for reactions with SPP are briefly illustrated. Using the charge sum rules  $\frac{9}{\frac{7}{10}}$  for SPP and some model-independent assumptions  $\frac{9}{-111}$ , asymptotic properties for these distributions are suggested (Sect.5). The main conclusions of our analysis are summarized in Sect.6.

## 2. Generating functional formalism for SPP.

Let us denote by  $d\sigma_{c_1}^{ab}$ .ex section of the exclusive reaction with hadron production

$$a+b-+c_1+a++c_n, \qquad (1)$$

where  $\vec{p}_1, \ldots, \vec{p}_n$  are the 3-momentum of particles  $c_1, \ldots, c_n$  respectively (for details see, e.g.  $\frac{12}{\sqrt{13}}$ ). The exclusive cross section for reaction (1) is obtained as

$$\sigma_{c_1\cdots c_n}^{ab,ex}(s) = (\prod_{i=1}^{n} n_i!)^{-1} \int d\vec{p}_1 \cdots d\vec{p}_n \frac{d\sigma_{c_1\cdots c_n}^{ab,c_1\cdots c_n}(s)}{d\vec{p}_1\cdots d\vec{p}_n}, \qquad (2)$$

where s denotes the squared center of mass (CM) energy, and there are  $n_i$  particles of type i. The total cross section for the ab collision is defined

$$\sigma_{\text{TOT}}^{ab}(s) = \int_{c_1, \dots, c_n} \sigma_{c_1, \dots, c_n}^{ab, ex}(s).$$
(3)

Let us introduce a projection operator  $\mathcal{J}$  on the (Hilbert) space of produced SP

$$\mathcal{J}|c_1, \ldots, c_n\rangle = \mathcal{J}_{c_1, \ldots, c_n}|c_1, \ldots, c_n\rangle = (1 - \int_{|\mathbf{S}_a|\mathbf{H}|\mathbf{S}_b|} \sum_{i=1}^n |\mathbf{S}_{c_i}|^{i}|c_1, \ldots, c_n\rangle (4)$$

Here  $|c_1, \ldots, c_n\rangle$  describes the physical state of the particles  $c_1, \ldots, c_n$ , sc denotes the strangeness of particle c, and  $\delta$  is the usual Kronecker symbol. The analogous of the total cross section (3) is the total cross section for SPP.

$$\sigma_{\text{STR}}^{\text{ab}}(s) = \underbrace{\varsigma_1, \ldots, \varsigma_n}_{c_1, \ldots, c_n} \sigma_{c_1, \ldots, c_n}^{\text{ab}, ex}(s) \mathcal{f}_{c_1, \ldots, c_n}^{c_1, \ldots, c_n}$$
(5)

For reactions with SPP it is possible to particularize the generating functional formalism  $^{6/}, ^{7/}$ . Let us introduce the exclusive functional  $^{6/}, ^{7/}$  associated with SPP

$$E_{\text{STR}}^{\text{ab}}[\underbrace{\phi_{1},\ldots,\phi_{1}}_{n_{1}},\ldots,\underbrace{\phi_{i}}_{n_{i}},\ldots, \underbrace{\phi_{i}}_{n_{i}}] = \sum_{\substack{n=2\\c_{\mu},\ldots,c_{n}}} (\int_{j}^{j} n_{j}!)^{-1} \int_{d\vec{p}_{1}} \cdots d\vec{p}_{n} \frac{d\sigma_{1}^{ab,ex}}{d\vec{p}_{1}\cdots d\vec{p}_{n}} \times \mathcal{J}_{c_{1}\cdots c_{n}} (\int_{j}^{n} n_{j}!)^{-1} \int_{d\vec{p}_{1}} (\vec{p}_{k})^{-1} \int_{d\vec{p}_{1}} \cdots d\vec{p}_{n} \frac{d\sigma_{1}^{ab,ex}}{d\vec{p}_{1}\cdots d\vec{p}_{n}} \times \mathcal{J}_{c_{1}\cdots c_{n}} (\int_{j}^{n} n_{j}!)^{-1} \int_{d\vec{p}_{1}} (\vec{p}_{k})^{-1} \int_{d\vec{p}_{1}} \cdots d\vec{p}_{n} \frac{d\sigma_{1}^{ab,ex}}{d\vec{p}_{1}\cdots d\vec{p}_{n}} \times \mathcal{J}_{c_{1}\cdots c_{n}} (\int_{j}^{n} n_{j}!)^{-1} \int_{d\vec{p}_{1}} (\vec{p}_{k})^{-1} \int_{d\vec{p}_{1}} \cdots d\vec{p}_{n} \frac{d\sigma_{1}^{ab,ex}}{d\vec{p}_{1}\cdots d\vec{p}_{n}} \times \mathcal{J}_{c_{1}\cdots c_{n}} (f_{n})^{-1} \int_{d\vec{p}_{1}} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{-1} (f_{n})^{$$

where  $\phi_i(\vec{p}_{n_{i-1}+1}), \dots, \phi_i(\vec{p}_{n_i})$  are  $n_i$  functions corresponding

to particles of type i (i=1,...,i;n,-];is the number of types of particles produced in the ab collision). Denoting by  $\phi$  the whole set of functions  $\phi_1, \ldots, \phi_v$ , we have the relations:

$$\int_{\mathrm{STR}}^{\mathrm{ab}} = E_{\mathrm{STR}}^{\mathrm{ab}} [\Phi] |_{\phi=1}, \qquad (7)$$

$$\frac{d\sigma_{c_1\cdots c_n}^{ab,ex}}{d\vec{p}_1\cdots d\vec{p}_n} \mathcal{F}_1\cdots c_n = \prod_{i=1}^{p} \left[ \prod_{k=n_{i-1}+1}^{m_i} \frac{\int}{\int \phi_i(\vec{p}_k)} \mathcal{F}_{STU_i}^{ab}[\phi] \right] \phi = o, \quad (8)$$

$$\frac{d\sigma_{c_1\cdots c_n}}{d\vec{p}_1\cdots d\vec{p}_n} = \prod_{i=1}^{\gamma} \prod_{k=n_{i-1}^{i-1}} \frac{\delta}{\delta\phi_i(\vec{p}_k)} \mathbb{E}_{\mathrm{STR}}^{\mathrm{ab}}[\phi] |_{\phi=1} =$$

$$\underbrace{\sum_{m=n}}_{m=n} \left( \prod_{i=n+1}^{m} n_{i}! \right)^{-1} \int d\vec{p}_{n+1} \cdots d\vec{p}_{m} \frac{d\vec{\sigma}_{c_{1}} \cdots c_{m}}{d\vec{p}_{1} \cdots d\vec{p}_{m}} \int_{c_{1} \cdots c_{m}} (9)$$

 $c_{n+1}, \dots, c_m$ In the last relation  $d\sigma_{c_1}^{ab,STR}/d\vec{p}_1 \dots d\vec{p}_n$  represents the differential cross section of the inclusive reaction

$$a+b \rightarrow c_1+\ldots+c_n+$$
 anything (10)

conditioned by SPP.

Further, all the formalism of the generating functional can be constructed for processes with SPP. Let us introduce the inclusive generating functional for SPP

$$\mathbf{I}_{\mathrm{STR}}^{\mathrm{ab}}[\phi] = \mathbf{E}_{\mathrm{STR}}^{\mathrm{ab}}[\phi+1], \qquad (11)$$

and the invariant cross sections

- - -

$$N_{c_1\cdots c_n}^{ab,STR}(\vec{p}_1,\ldots,\vec{p}_n) = (\sigma_{STR}^{ab}(s))^{-1} \mathbf{p}_1^{o} \cdots \mathbf{p}_n^{o} \frac{d\sigma_1^{ab,STR}}{d\vec{p}_1\cdots d\vec{p}_n}, \quad (12)$$

where  $p_{\underline{5}}^{\circ}$  is the energy of particle i (i=1,...,n). Combining eqs. (6)-(9),(11),(12), the usual expression is obtained for the inclusive generating functional  $^{6/}$  with SPP

$$I_{\text{STR}}^{\text{ab}}[\phi] = \mathfrak{o}_{\text{STR}}^{\text{ab}}[1; \sum_{\substack{n=1,2,\ldots,c_n\\c_1,\ldots,c_n}} \frac{1}{n!} \int_{i=1}^{n} \frac{d\vec{p}_{ik}}{p_i^{o}} c_1^{o} \cdots c_n^{o} (\vec{p}_1, \ldots, \vec{p}_n)$$

$$\downarrow \prod_{i=1}^{n} \frac{n_i}{k=n_{i-1}+1} [\phi_i(\vec{p}_k)]. \qquad (13)$$

The correlation functions  $^{6/}$  for inclusive reactions with SPP are introduced by

$$I_{\text{STR}}^{\text{ab}}[\phi] = \sigma_{\text{STR}}^{\text{ab}} \exp\left[\sum_{\substack{n=1,2,\ldots\\c_1,\ldots,c_n}} \int_{i=1}^{n} \frac{d\vec{p}_i}{p_i^o} c_1^{\text{ab},\text{STR}}(\vec{p}_1,\ldots,\vec{p}_n) \int_{i=1}^{n} \int_{k=n_{i-1}+1}^{n} \phi_i(\vec{p}_k) \right] .$$
(14)

With the notations

$$\langle n_{c_{i}}^{ab} \rangle^{STR} = \int_{p_{i}}^{d\overline{p_{i}}} N_{c_{i}}^{ab}, STR(\vec{p}_{i}), \qquad (15)$$

$$< \mathbf{f}_{c_1 \cdots c_n}^{ab} > \sum_{i=1}^{STR} = \int_{i=1}^{n} \frac{d\vec{p}_i}{p_i^o} \mathbf{c}_{c_1 \cdots c_n}^{ab, STR} (\vec{p}_1, \dots, \vec{p}_n), \qquad (16)$$

taking in eq. (14)  $\phi_i = z_i$  and with eq. (11), we get for the generating functional the expression

$$\mathbb{E}_{STR}^{ab}(z+1) = \mathfrak{G}_{STR}^{ab} \exp\left[\sum_{i} \langle n_{c_{i}}^{ab} \rangle^{STR} z_{i}^{i} + \underbrace{\sum_{n_{1}^{+} n_{2}^{+} \cdots + n_{j}^{-n} n_{1}!}_{n_{2}^{+} \cdots + n_{j}^{-n} n_{1}!} \frac{z_{1}^{n_{2}}}{n_{2}!} \cdots \frac{z_{j}^{n_{j}}}{n_{j}!} x$$

It is **easily** to show that eq. (15) represents indeed the associated average multiplicity of the particle  $c_i$  if SP are produced.

$$\langle n_{c}^{ab} \rangle^{STR} = \frac{1}{\sigma_{STR}^{ab}} \sum_{c} n_{c} \sigma_{c_{1}}^{ab,STR} \int_{c_{1}} \cdots c_{n} \sigma_{c_{1}}^{ab} \sum_{c} n_{c} \sigma_{nc}^{ab,STR}, \quad (17)$$

where  $\sigma_{n_c}^{ab,STR}$  is the cross section for the production of  $n_c$  particles of type c associated with SP. If the particle c is a SP, there exists the relation

$$\langle n_c^{ab} \rangle^{STR} = \langle n_c^{ab} \sigma_{TOT} / \sigma_{STR}^{ab},$$
 (18)

where

$$\langle \mathbf{n}_{c}^{ab} \rangle = (\boldsymbol{\sigma}_{TOT}^{ab})^{-1} \int \frac{d\vec{\sigma}_{p} ab}{p^{o} c} (\vec{p}) = (\boldsymbol{\sigma}_{TOT}^{ab})^{-1} \sum_{k=1,2,\dots,k} k \boldsymbol{\sigma}_{kc}^{ab} .$$
(19)

Here  $\sigma^{ab}_{kc}$  is the cross section for the production of k particles of type c in the ab collision.

Usually are studied semi-inclusive reactions with n<sub>ch</sub> charged particles and a strange particle c,

$$a+b \rightarrow n_{ab}+c + anything neutral.$$
 (20)

We have

$$\kappa_{n_{c}}n_{ch} \geq \sum_{n_{c},n_{ch}} n_{c}n_{ch}\sigma_{n_{ch},n_{c}}/\sigma_{TOT} \geq \sum_{n_{ch}} n_{ch} \langle n_{c} \rangle_{n_{ch}} \sigma_{n_{ch}}/\sigma_{TOT} = \sum_{n_{ch}} n_{ch} \sigma_{n_{ch}}/\sigma_{TOT}$$
(21)

The analogous of relation (18) is

$$\langle n_{ch} \rangle^{c} = \sum_{n_{ch}} n_{ch} \sigma_{n_{ch}}^{c} / \sigma_{c}$$
, (22)

and finally we arrive at

$$\langle \mathbf{n}_{c} \mathbf{n}_{ch} \rangle = \langle \mathbf{n}_{ch} \rangle^{c} \mathbf{\sigma}_{c} / \boldsymbol{\sigma}_{TOT} = \langle \mathbf{n}_{ch} \rangle^{c} \langle \mathbf{n}_{c} \rangle.$$
(23)

In eqs. (?o)-(?3) the index ab has been omitted for simplicity. We remember that in practice it is usual to define the average multiplicity by normalization to  $\sigma_{\text{INEL}}$ .

From eo.(23) it is seen that  $\langle n_{ch} \rangle^{c}$  is the average multiplicity for SP conditioned by the presence of a SP c. So far, little systematic experimental information is available on average multiplicity associated with SP defined by eq. (17) (see, e. p./14/). It seems that the total average charged multiplicity conditioned by SPP is smaller than the corresponding total average charged multiplicity at the given energy <sup>14</sup>. More frequently  $\langle n_{ch} n_c \rangle$  or related quantities (see, e.g.<sup>5</sup>) are analysed for the semi-inclusive reaction (20). In Sect. 4 we shall study only the experimental data of multiplicities of SP determined with eq. (19).

We have defined various quantities for SPP within the framework of the generating functional formalism for SPP. We point out that all the formalism of generating functional is applicable to processes with SPP. In particular, the following momentum-energy and charge sum rules  $ect^{/6/}$ 

$$(\mathbf{p}_{in}^{\mu} - \sum_{i=1}^{n} \mathbf{p}_{i}^{\mu}) \mathbf{n}_{c_{1} \cdots c_{n}}^{ab, STR} (\vec{p}_{1}, \dots, \vec{p}_{n}) = \sum_{c_{n+1}} \int_{\vec{p}_{n+1}}^{d\vec{p}_{n+1}} \int_{\vec{p}_{n+1}}^{\vec{p}_{n+1}} \mathbf{p}_{n+1}^{\mu} \mathbf{p}_{n+1}^{\mu}$$

$$(\mathbf{Q}_{\mathbf{in}} - \sum_{i=1}^{n} \mathbf{Q}_{\mathbf{c}_{i}}) \times_{\mathbf{C}_{1}}^{\mathrm{ab}, \mathrm{STR}} (\vec{\mathbf{p}}_{1}, \dots, \vec{\mathbf{p}}_{n}) = \sum_{\mathbf{C}_{n+1}} \int_{\mathbf{p}_{n+1}}^{d\vec{\mathbf{p}}_{n+1}} \mathbf{Q}_{\mathbf{c}_{n+1}} \times_{\mathbf{N}^{\mathrm{ab}}, \mathrm{STR}} (\vec{\mathbf{p}}_{1}, \dots, \vec{\mathbf{p}}_{n}, \vec{\mathbf{p}}_{n}, \vec{\mathbf{p}}_{n+1}).$$

$$(25)$$

Here  $p_i^{\mu}$  is the  $\mu$  component of the energy-momentum 4-vector  $p_i$ , the index in denotes the initial state, and  $Q_{c_i}$  is any charge (electrical, baryonic, strangeness) of particle  $c_i$ .

3. Total SPP cross section.

Formula (4) shows that for measuring the total cross section for SPP, all exclusive channels with produced SP must be determined. It is usual to evaluate the total SPP cross section in terms of inclusive cross sections. We remember the expression of the inclusive reaction (10)

$$\sigma_{c_1\cdots c_n}^{ab}(s) = \sum_{m=n} \sigma_{c_1\cdots c_n}^{ab,ex}(s)^{b}.$$
 (26)

Denoting by Y the hyperons and by K the K-mesons, eq. (5) takes the known form

$$\sigma_{\text{STR}}^{ab} = \sigma_{YK}^{ab} + \sigma_{K\bar{K}K\bar{K}}^{ab} + \sigma_{K\bar{K}K\bar{K}}^{ab} + \sigma_{Y\bar{K}K}^{ab} + \sigma_{Y\bar{K}K}^{ab} + \sigma_{Y\bar{K}K}^{ab} , \qquad (27)$$

where in each term of the sum the strangeness of the system is equal to the strangeness of the ab state. In principle, to find the total SPP cross section it is enough to measure the cross sections of the inclusive processes from r.h.s. of eq. (27). In practice, at high energy it is difficult to detect all the SP simultaneously (e.g.,  $K_S^0$  and  $\Lambda$  together with  $K^{\pm}$ ,  $K_L^0$ ). A number of hypotheses have to be introduced in order to evaluate the total number of SP from the observed ones. For example, supposing that  $\sigma_{K_S^0} = \sigma_{K_L^0}$  and  $\sigma_{K^+K^-} = \sigma_{K_L^0K_K^0}$ 

$$\sigma_{\mathbf{K}^{0}/\mathbf{\widetilde{K}}^{0}} - \sigma_{\mathbf{K}^{+}\mathbf{\widetilde{K}}^{0}} + \sigma_{\mathbf{K}^{-}\mathbf{K}^{0}} + 2\sigma_{\mathbf{K}^{0}\mathbf{\widetilde{K}}^{0}} + \sigma_{\mathbf{Y}\mathbf{K}^{0}} = \sigma_{\mathbf{V}\mathbf{\widetilde{K}}} + \sigma_{\mathbf{Y}\mathbf{K}^{0}}.$$
(28)

If  $\sigma_{K^0/\widetilde{K^0}}$  and  $\sigma_{YK^0}$  are measured, the  $\sigma_{KK}$  is determined. The cross section  $\sigma_{YK^0}$  can be written

$$\mathcal{T}_{YK^{\circ}} = \mathcal{T}_{K^{\circ}} + \mathcal{T}_{\Sigma^{\circ}K^{\circ}} + \mathcal{T}_{\Sigma^{+}K^{\circ}} + \mathcal{T}_{\Sigma^{-}K^{\circ}} = \mathcal{T}_{X^{\circ}} \times \mathcal{L}_{\Sigma^{\circ}} \times \mathcal{L}_{\Sigma^{\circ}} \times \mathcal{L}_{\Sigma^{\circ}}$$
(29)

because at high energy,  $\sum^{0}$  is indistinguishable from A. The relations (27)-(29) have been written just to have a more precise feeling how total SPP cross sections are practically evaluated from

measurements. Details and references could be found e.g., in /15/.

Now we display in Fig. 1 the available information on SPP in  $\pi^{-}p$ .  $\pi^{+}p$  and np collisions from thresholds till 40, 18.5, respectively 69 Gev/c. The data are from ref. /16/. Data published before 1962 have not been taken in our analysis. Evaluating  $\sigma_{v\bar{v}}$  and  $\sigma_{\rm STR}$  from  $\sigma_{\rm K^0/K^0}$  and  $\sigma_{\rm Y^0}$  at 69 Gev/c (ecs. (28),(29)), it was supposed that  $\sigma_{\rm s^2=0.45_{V^0}}$ . The lines are only to guide the eye. It may be seen that the total SPP cross sections increase with energy. The KK cross section is essentially responsible for the total SPP growing with energy, while the Y<sup>O</sup>K cross sections data present slower dependence on energy, excepting the threshold regions. It may be noted that  $\sigma_{2\pi} < \sigma_{vor}$  near threshold, but at higher energies  $\sigma_{vv} > \sigma_{vo_{r}}$ . It should, however, be remarked that in pp collisions  $\sigma_{vo_{r}}$ increases appreciably with energy, if all momentum range is considered. From threshold till ~lo Gev/ $c \sigma_{STR}^{\mathcal{F}p} > \sigma_{STR}^{\mathcal{F}p}$ , but further  $\sigma_{\text{STR}}^{\mathcal{F} p}$  and  $\sigma_{\text{STR}}^{pp}$  are comparable. It is important to test the equality  $\sigma_{\text{STR}}^{\mathcal{F} p} = \sigma_{\text{STR}}^{\mathcal{F} p}$  at higher energies (see Sect. 5), but there are no available data on SPP cross sections from  $\mathcal{T}_{p}^{p}$  interactions. We remember that  $\sigma_{TOT}^{pp} > \sigma_{TOT}^{\mathcal{F}p} > \sigma_{TOT}^{\mathcal{F}p} / 17/$ , and  $\mathrm{TH}_{STR}^{\mathcal{F}} \sim \mathrm{TH}_{STR}^{\mathcal{F}p}$  TH $_{STR}^{pp}$ , where  $\mathrm{TH}_{STR}^{ab}$  is the threshold for SPP in the ab collision. In the same ranges of momenta  $\sigma_{K\bar{K}}^{\pi p} \sigma_{K\bar{K}}^{\pi p} \sigma_{K\bar{K}}^{p}$ 

In Fig. 2 we have plotted the same data from ref. /16/ normalized to the corresponding  $\sigma_{\rm INEL}$  versus the available CM energy.  $\sigma_{\rm INRL}$ , when has not been quoted in the original papers, has been obtained by taking the difference between the interpolating curves of the data of  $V_{\rm TOT}$  and  $\sigma_{\rm EI}$  from ref. /17/. At low energies in  $\mathcal{T}_{\rm P}$ interactions, because of the variation of  $\sigma_{\rm INEL}$  and  $\sigma_{\rm TOT}$ , the errors in Fig. 2 are probably underestimated. It is observed that

1. As the orimary energy increases, the ratio  $\sigma_{\rm STR}/\sigma_{\rm INEL}$  increases.

2. The shape of the curves  $\sigma_{K\bar{K}}/\sigma_{INEL}$  is similar in  $\mathcal{T}_p$  and pp interactions, and we have  $(\widetilde{\sigma_{KK}}/\widetilde{\sigma_{INEL}})_{\mathcal{F}p} > (\widetilde{\sigma_{KK}}/\widetilde{\sigma_{INEL}})_{pp}$ . It is possible that at higher energies to be not the case (Sect. 4).

3. The increase of Y<sup>O</sup> contribution is more evident in pp collisions than in 3 p collisions.

4.  $(\sigma_{\text{STR}}/\sigma_{\text{INEL}})_{\pi^{-p}} > (\sigma_{\text{STR}}/\sigma_{\text{INEL}})_{pp}$ , and the shape of the distri-



Fig.1. SPP cross sections ( $\mu$ b) versus laboratory momentum (Gev/c). Total SPP cross sections.  $\pi^{+}p$ -O,  $\pi^{-}p$ -O, pp-O; KK cross sections.  $\pi^{+}p$ -A,  $\pi^{-}p$ -A, pp-+; Y<sup>O</sup>K cross sections.  $\pi^{+}p$ -G,  $\pi^{-}p$ -B, pp-x. The lines are to guide the eye only. Data from ref.  $^{/16/}$ .



Fig.2. Ratios of SPP cross sections on inelastic cross section versus available CM energy. The  $n_{\text{TM}}$  data and conventions like in Fig.1 are used. The inelastic cross sections are from ref. 17/. See the text.

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butions is approximately the same, the difference coming from distinct behaviour of the  $Y^{O}$  contributions.

As an overall conclusion, the total cross section for SPP has not yet reached a limiting behaviour, presenting an increase with energy. The differences in the behaviour of the SPP cross sections in various collisions seem to be a threshold effect. Several asnects of the indemendence of multiparticle production processes of the initial state at high energy are recently studied (e.g. 1/1, 1/2, 1/5/1). It is important to point out that the Y<sup>O</sup> production is also increasing with energy, because earlier analysis 1/8/1 on more reduced energy ranges suggested already a limiting behaviour.

4. Average SP multiplicities.

The average multiplicities at high energies are intensively studied experimentally and theoretically. Except the kinematical bound  $(\langle n \rangle \leq \sqrt{s/m} \rangle$ , there are no rigorous results (see,e.g.<sup>(19)</sup>). However, there are a lot of model dependent predictions for the asymptotic behaviour of the average multiplicity. Some models suggest a power dependence on s (e.g. Fermi, Satz, Landau<sup>(20)</sup>), other models even saturate the kinematical bound (e.g. Heisenberg, Pomeranchuk, Naravan, Suhonen et al.<sup>(21)</sup>), other predict a logarithmic dependence on s (multiperipheral model, Mueller-Regge model<sup>(22)</sup>) or constant values<sup>(23)</sup>. Commonly, only the average charged multiplicities are measured. A logarithmic dependence on s is acceptable at Fermilab energies<sup>(1)</sup>,<sup>(2)</sup>, but on the whole range from accelerator till cosmic rays energies the power 1/4 of s<sup>(24)</sup> or greater (e.g. <sup>(21)</sup>) describes better the average charged multiplicities. Also some empirical formula fit quite well the data (e.g.<sup>(25)</sup>).

We now proceed to examine the data on SP multiplicities. Most of available data on SPP are from pp interactions. For comparison, in Fig. 3 we have displayed the data of Antinucci et al.<sup>26/</sup> on  $\mathcal{A}_{1}^{\sharp} K_{2}^{\sharp}$ **p** and  $\tilde{p}$  production. Although some systematical errors are present in these data (for evaluation of <n> from uniparticle spectra (eq. (18)), it is supposed  $\sigma_{\text{INEL}}$ =ct, central plateau in y, factorization of the distribution in y and p ). these are considered to be not



Fig.3. Average multiplicities of particles produced in pp collisions versus s. The  $\mathcal{I}^{\dagger}$ ,  $K^{\pm}$ , p and  $\tilde{p}$  data are from ref.<sup>26/</sup>. The  $\Sigma^{\pm}$  data are from ref.<sup>16/</sup>.<sup>17/</sup>. For  $\Sigma^{\pm}$ the curves are only to guide the eye. For the other charged particles the curves represent our best fits<sup>27/</sup> to the data<sup>26/</sup> with formula (30) with 3 parameters (see Table I). The  $\Lambda/\Sigma^{\circ}$  and  $K^{\circ}/\tilde{K}^{\circ}$  curves represent our best fits to the data of Figs.4 and 5 (see Table II).

essential (c.g. (1)). The data have been fitted with the empirical formula

$$(n) = A_1^+ A_0 (lns)^{A_5} + A_3 s^A 4$$
 (30)

with ? narameters  $(A_3=0, A_5=1.0)$ , 3 parameters  $(A_4=-0.5, A_5=1.0)$ 4 narameters  $(A_5=1.0)$  and all 5 parameters free (see Table 1). The best fits for every narticle with 3 parameters are plotted in Fig. 3. The conclusions of our analysis of data<sup>26</sup> are: 1)  $A_5\approx1.0.2$ The nower  $A_4=-0.5$  does not give the best fit to all multiplicity data<sup>26</sup>, like suggested the Mueller-Regge analysis (e.g.<sup>28</sup>). Indeed, for  $\pi^*$  a logarithmic dependence on s is enough to describe the average multiplicity data and  $A_4=-0.81$  fits better than  $A_4=-0.5$ the  $\pi^-$  data. Only K<sup>±</sup> and  $\tilde{p}$  support  $A_4=-0.5$ . 3) The coefficient  $A_2$ of the logarithmic term is inverse proportional with the masses of the narticles<sup>26</sup>, 29/,30/.

The results of the fit for  $K^0/\widetilde{K}^0$  multiplicities from pp collisions in the range 3-405 Gev/c are presented in Table II. The data are from refs. /16c/,/31/,/32/. The values of the  $\chi^2$  are large proving that the data are generally inconsistent. Excepting the point at 405 Gev/c, the data from Sernukhov and Permilab are systematically unner the curve which describes all the data.

In Fig. 4 it may be seen that the curve with the corrective power term  $s^{-0.3}$  is nearer the high energy data from Serpukhov and Permilab. The point at 405 Gev/c, which is essential for the behaviour of the average multiplicity at high energy, is only preliminary.<sup>32/</sup> In Fig. 3 it is observed that  $\langle n_{K^{-}} \rangle \langle \langle n_{K} 0 \rangle \langle \langle n_{K^{+}} \rangle$ till  $s \sim 100 \text{ Gev}^2$  and at higher energies it seems to be more  $K^0/\overline{K^0}$ than  $K^+$ . This is also reflected in the values of the coefficient  $A_4$  ( $A_4^{\infty}$ -0.5 for  $K^{\pm}$  and  $A_4^{\infty}$ -0.3 for  $K^0/\overline{K^0}$ ). However, because data for  $K^0/\overline{K^0}$  production come from bubble chamber experiments and those on  $\overline{K^+/K^-}$  from counter experiments, systematical differences are not excluded.

"It is important to observe that the parametrization (30) with  $A_5=1.0$ ,  $A_4=-0.5$ , is a bad one near threshold, if the threshold is greater than  $s_0=(1/2A_3/A_2)^2$ , where a minimum occurs. This is the case for the  $\tilde{p}$  data in Fig. 3 and  $\Lambda/\Sigma^0$  in Fig. 5.

<u>Table I.</u> Values of the parameters of the best fits to  $\pi^{2}$ ,  $K^{2}$ ,  $\tilde{v}$  average multiplicity data<sup>(26)</sup> with formu a (30). If there are no errors indicated in the Table, this means that the corresponding parameter was fixed at a given value. A  $\chi^{2}$  minimalization<sup>(27)</sup> procedure has been used with the errors of the average multiplicities evaluated from ref.<sup>(26)</sup>. The best fits with 3 parameters are plotted in Fig.3. The asterisc denotes the values of the parameters quoted in ref.<sup>(26)</sup>.

	M	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A4	A <sub>5</sub>	χ²
91 <sup>+</sup> (12)	2	-1.22 ±.10	0.78 ±.01	0.	5	h.	1.91
	3	-1.7 ±.3	0.84 ±.07	1.0 ±.5	5	1.	4.42
	3	-0.974±.550	0.74 ±.091	546±1.12	5	1.	1.67
	4	-0.38 ±1.81	0.68 ±.17	-1.0 ±2.	?o±.30	1.	1.61
	5	-0.29 ±.36	0.71 ±.01	-1.1 ±.05	16±.2	0.9±.03	1.61
A <sup>-</sup> (19)	2	-1.18 ±.05	0.61 ± .02	٥.	5	1.	28.4
	3	-2.6 ±.2	0.87±.05	2.7 ±.4	5	1.	8.
	3	-2.847±.350	0.918±.064	3.133±.640	5	1.	4.47
	4	-2.35 ±.69	0.86±.09	3.95 ± .20	81±.55	1.	3.49
	5	-2.25 ±1.1	0.85±.03	3.99 ± .13	87±.09	0.981.09	3.07
K <sup>+</sup> (11)	3	-0.50 ± .03	0.13±.01	0.65±.05	5	1.	52.6
	3	<b>-0.</b> 504±.04	0.131±.02	0.661±.05	5	1.	8.57
	4	-0.498±.05	0.131±.03	0.670±.10	52±.06	1.	8 <b>.</b> 40
	5	-0.498±.05	0.131±.03	0.6701.10	52:.06	1. ±.04	8.40
(H) _X	3	-0.52 ±.04	0.11 ±.01	0.80 ± .06	5	1.	168.
	3	-0.453±.121	0.105±.021	0.613±.280	5	1.	5.43
	4	-0.453±.122	0.105±.023	0.615±.280	5±.05	1.	5.43
	5	-0.453±.130	0.105±.024	0.616±.300	51±.06	1.0±.05	5.43
(OF) d	3	-0.37 ±.04	0.059±.006	0.75±.12	5	1.	15750.
	3	-0.331±.034	0.059±.006	0.746±.094	5	1.	8.4
	4	-0.332±.036	0.059±.007	0.745±.10	5 ±.05	1.	8.4
	5	-0.333±.037	0.059 <u>±.00</u> 8	0.745±.10	5 ±.00	1.0 ±.02	8.4

<u>Table II.</u> Values of the best fits to  $K^0/\tilde{K}^0$  and  $\Lambda/\Sigma^0$  average multiplicities with formula (30). The pp data are from refs./16c/, /31/,/32/. In the fit 16 points for the reaction pp- $\Lambda/\Sigma^0$ , the data of Blobel et al, and Bartke et al. are excluded. For  $\mathcal{T}$  p interactions the data are from refs./ $\Omega/A^3/A^3/A^4$ . See Figs 4,5,6.

	-						
	F	A1	<b>A</b> 2	A.3	A <sub>4</sub>	۸ <sub>5</sub>	x <sup>2</sup>
PP → K°(R)	3	598±.030	.146±.029	.839±.070	5	1.	44.5
	5	898±.045	.168±.072	1.080±.060	335±.07	1.041.04	40.3
PP→A/5°	3	106±.018	.035±.004	.102±.030	5	1.	32.9
	4	065±.020	.028±.005	.48 ±.30	-1.95 ±.41	1.	30.2
	5	068±.030	.032±.007	2.59±1.12	-2.8 ±.14	0.94±.02	29.4
(6F) 03/V-dd	3	088±.017	.033±.004	.051±.028	· <b>÷</b> •5	1.	117.
	4	064±.02?	.030±.004	.145±.120	-1.62 ± .42	1.	115.
	5	066±.025	.032±.005	.80 ±.35	-2.4 ±.4	0.99±.03	94.
1 - 0- 1 K - 1 K - 1	2	070±.007	.061±.003	٥.	<del>.</del> .5	1.	8.6
	3	022±,075	.0521.016	076±.121	5	1.	8.2
120	1	0.036	D.	o <b>.</b>	5	1.	49.2
7 p-4 (16)	2	020±.003	.006±.001	o <b>.</b>	5	1.	22.8
	3	0?3±.0?3	.015±.005	.07?±.038	5	1.	19.2

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Т. • с. - The  $\Lambda/\Sigma^{\circ}$  production data are even more inconsistent than those of  $K^{\circ}/\overline{K^{\circ}}$ . The data of ref. <sup>(18)</sup> suggest a weaker increase of  $\langle n_{\Lambda/\Sigma^{\circ}} \rangle^{\text{with energy}}$  than those of Blobel et al. <sup>(16</sup> c). A fit to all the data yields very high  $\chi^2$ . The exclusion of data of Blobel et al. <sup>(16c)</sup> improves the fit, but these data have good statistics and are nearer the high energies data. In any case, a logarithmic increase plus a very low corrective term ( $\sim s^{-1.5}$ ) should describe the average multiplicity  $\Lambda/\Sigma^{\circ}$  data. It may be remarked that the  $A_2$ coefficient for  $\Lambda/\Sigma^{\circ}$  is smaller than the corresponding one for  $\widetilde{p}$ , confirming the  $3^{\underline{rd}}$  observation, and also  $A_1^{\widetilde{p}} < A_1^A$ , reflecting that  $TH_4 < TH_{\pi^*}$ .

In Fig.3 we compare the fits of the  $\Lambda/\Sigma^0$  average multiplicities ties with the other particle multiplicities data. The corridor represents the possible behaviours of  $\langle n_{\Lambda/\Sigma 0} \rangle$  between the fits with all data (upper curve), and without the data at 12, 24, 24.5 Gev/c (lower curve).  $\Lambda/\Sigma^0$  having a common threshold with K<sup>\*</sup>,  $\langle n_{\Lambda/\Sigma} 0 \rangle$  and  $\langle n_{K^+} \rangle$  increase similarly with energy, but after this threshold blowup,  $\langle n_{\Lambda/\Sigma} 0 \rangle$  has a more slower increase, remembering the behaviour of  $\langle n_p \rangle$ . This represents a kind of "inertia" of the baryonic number, but the presence of the strangeness determines differences in the  $\langle n_{\Lambda} \rangle$  and  $\langle n_{p} \rangle$  behaviours.

We have also plotted in Fig.4 our best fit to the 3-205 Gev/c data of average  $K^0/\overline{K}^0$  multiplicities from  $\pi^- p$  interactions  $^{16a/},^{33/}$   $^{34/}$ . A logarithmic dependence on s is enough to describe the data. At higher energies it seems that  $< n_K o_{/\overline{K}} o_{/$ 

We show in Fig.5 (and TableII) the trials to describe the  $A/\Sigma^{0}$  average multiplicities data from  $\mathcal{T}_{p}$  interactions in the range 1.59-205 Gev/c. The data are inconsistent. The logarithmic fit dies not describe data from loo<sup>13</sup> and 205 Gev/c<sup>134</sup>. A somewhat better fit for high energy data is obtained with a corrective term s<sup>-0.5</sup>, but the coefficient  $A_{1}$  is consistent with 0, and the low energy data are not described by the fit. In any case, the increase of  $\langle n_{A} \rangle$  with s is to be noted.



Fig.4. Average  $K^0/\overline{K}^b$  multiplicities versus  $P_{LAB}$  (Gev/c).  $\mathscr{T}^-p$  interactions -• and pp interactions-0. Data are from r9fs./16c/:/31/for pp collisions and /16a/:/33/:/34/ for  $\mathscr{T}^-p$  collisions The curves are results of our best rits with 3 and 5 parameters (see Table II and the text).



Fig.5. Average  $\Lambda/\Sigma^0$  multiplicities from pp collisions versus laboratory momentum. • data of Oh and Smith  $^{16c}, ^{18}, o$  - the other data  $^{16c}, ^{31}, ^{32}$ . The upper curve is the best fit with all the points, the lower curve is the best fit without the points at 12, 24 Gev/c (Blobel et al.) and 24.5 Gev/c (Bartke et al.). See Table II. 5. Conditional charge distributions for reactions with SPP.

In refs.  $\frac{9}{10}$  it has been emphasized that charge and energy sum rules have non trivial consequences on the charge and conditional charge distributions (CCD). With the generating functional formalism  $\frac{6}{10}$  for reactions with SPP sketched in Sect. 2, we extend some of conclusions of refs.  $\frac{9}{10}$  to reactions with SPP.

We remember the definition of the CCD (introduced  $in^{/8/}$  for single-particle inclusive reactions and generalized in  $^{/9/}$ ) for m-particle inclusive reactions

$$\langle Q_{\mathbf{m}}^{\mathbf{ab}}(\mathbf{s},\mathbf{k}) \rangle = (\underbrace{\sum_{c_1,\ldots,c_m}}_{\mathbf{c_1},\ldots,\mathbf{c_m}} \frac{\mathrm{d} \sigma_{c_1}^{\mathbf{ab}}\ldots c_m}{\mathrm{d} \mathbf{k}})^{-1} \underbrace{\sum_{c_1,\ldots,c_m}}_{\mathbf{c_1},\ldots,\mathbf{c_m}} (Q_{c_1}^{+} \cdots Q_{c_m}) \frac{\mathrm{d} \sigma_{c_1}^{\mathbf{ab}}\ldots c_m}{\mathrm{d} \mathbf{k}},$$

where k is a kinematical variable. For inclusive reactions with SPP we define analogously

$$\langle Q_{\mathbf{m}}^{\mathbf{a},\mathbf{b}}, \mathrm{STR}(\mathbf{s},\mathbf{k}) \rangle = \left( \underbrace{\sum_{\mathbf{c}_{1}}, \ldots, \mathbf{c}_{\mathbf{m}}}_{\mathbf{c}_{1}}, \underbrace{\mathrm{d} \sigma_{\mathbf{c}_{1}}^{\mathrm{a},\mathbf{b},\mathrm{STR}}}_{\mathbf{d},\mathbf{k}} \right)^{-1} \times \frac{\mathrm{d} \sigma_{\mathbf{c}_{1}}^{\mathrm{a},\mathbf{b},\mathrm{STR}}}{\sum_{\mathbf{c}_{1},\ldots,\mathbf{c}_{\mathbf{m}}}} \left( Q_{\mathbf{c}_{1}}^{+}, \ldots + Q_{\mathbf{c}_{\mathbf{m}}} \right) \underbrace{\mathrm{d} \sigma_{\mathbf{c}_{1}}^{\mathrm{a},\mathbf{b},\mathrm{STR}}}_{\mathrm{d}\mathbf{k}}, \qquad (31)$$

where the cross section of  $c_1, \ldots, c_m$  particles generated together with SP is defined by eq. (9). Let us introduce the total number and charge of the m-particle system, conditioned by SPP, generated in the region R of the phase space

$$\begin{split} \boldsymbol{\gamma}_{m}^{ab,STR}(s,R) &= \underbrace{\sum_{c_{1},\dots,c_{m}} \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} \mathbf{x}_{1}^{ab,STR}(s,\vec{p}_{1},\dots,\vec{p}_{m}), \quad (32)}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{n}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{m}}) \int_{R} \frac{d\vec{p}_{1}}{p_{1}} \cdots \frac{d\vec{p}_{m}}{p_{m}^{o}} x}_{\boldsymbol{\chi}_{m}^{ab,STR}(s,R) = \underbrace{\sum_{c_{1},\dots,c_{m}} (Q_{c_{1}}^{\dagger} \cdots + Q_{c_{m}}) \int_{R} \underbrace{\sum_{c_{1},\dots,c_{m$$

Now we briefly illustrate some CCD with SPP and analyse them. <u>Central regions</u>. Applying the technics from ref.  $^{9/}$  to the sum rules (24), (25) and supposing that  $\lim_{\mathbf{S} \to \mathbf{C}} a^{ab}$ ,  $STR(s) = \mathbf{C}$  then



Fig.6. Average  $\Lambda/\Sigma^{\circ}$  multiplicities from  $\mathcal{T}^{-}$ p collisions versus s (p<sub>LAB</sub>) in Gev<sup>2</sup> (respectively Gev/c). Data from refs. /16a/,/33/,/34/. For the curves see Table II and the text.

$$\lim_{\mathbf{s} \neq \mathbf{e}} \mathbf{x}_{\mathbf{m}}^{\mathrm{ab}, \mathrm{STR}}(\mathbf{s}, \mathbf{R}_{o}) \quad (\mathbf{v}_{\mathbf{m}}^{\mathrm{ab}, \mathrm{STR}}(\mathbf{s}, \mathbf{R}_{o}))^{-1} \approx \mathbf{o}, \tag{34}$$

where  $R_o$  denotes a "central region"<sup>/9/</sup>. Supposing that the total average multiplicities with SP are asymptotically increasing indefinitely (Sect. 4), the CCD with SPP approaches a <u>zero limiting</u> value in the central region at asymptotic energies.

In Fig. 7 we have constructed for illustration the electrical CCD for the reaction  $\pi^- p \rightarrow YK^{0+}$  pions at 25 Gev/c. The single-particle spectra are read from the projections of the Peyrou plots from ref. /15/. The  $\pi^0$  spectrum has been taken a half of the sum of the  $\pi^+$  and  $\pi^-$  spectra  $^{1/} \cdot ^{0/-/10'}$ . The general character of the distribution is similar with that of the CCD of the reaction  $\pi^-$ n at 16 Gev/c<sup>/8/</sup>, /9/. In the central region the distribution changes the sign having a minimum (in module). A similar central minimum is present in Fig. 8 (9) for the baryonic (strangeness) CCD of the reaction  $\pi^-$ p-YK<sup>0</sup> pions at 25 Gev/c.

 $\frac{\text{Pragmentation regions.In Pig. 7 it may be remarked that in every fragmentation region the CCD with SPP "follows" the initial charge <math>\binom{8}{.}\binom{9}{.}$ . We explain this property, at least partially in some regions, using the sum rule (25) and assuming the validity of the Pomeranchuk hypothesis of Cornille-Martin/11/,/9/,/10/ (PCM) for SP limNab,STR (s,x\_1,p\_1,...,x\_m,p\_m\_)=limNab,STR (s,x\_1,p\_1,...,x\_m,p\_m\_), s d\_i = \begin{cases} c\_i, \text{ if } x\_i < 0 \\ c\_i, \text{ if } x\_i > 0 \end{cases} (35)

where x is the Feynman variable  $(x=p_{\parallel}/p_{CM})$  and  $p_{\parallel}(p_{\perp})$  is the longitudinal (transverse) CM momentum. The use of the PCM hypothesis at accelerator energies is not completely supported by the data for SFP (e.g. Stuntebeck et al<sup>/16a/</sup>). However, it is exnected that at higher energies the PCM hypothesis to be better verified by SP experimental data. Since, if the compensative central charges<sup>/10/,/11/</sup> for SPP are negligible, the left (right) total charge is expected to be  $Q_{\rm b}$ (respectively  $Q_{\rm a}$ ). At accelerator energies the data do not present a spectacular verification of this prediction<sup>/10/</sup>. Anyhow, this hypothesis explains partially the sign of the CCD in every hemisphere.

Now in Fig. 8 we compare the baryonic CCD for the reaction  $\mathcal{F}_{D} \rightarrow YK^{O}$  pions at 25 Gev/c with previously compiled data from 16 Gev/c  $\mathcal{F}_{P}$ 



Fig.7. Electric CCD versus cos at 25 Gev/c for the reaction  $\mathcal{F}^{-}v \rightarrow YK^{\circ}$  pions ( $\theta$  is in CM). Data from ref. (15/. See the text.



Fig.8. Baryonic CCD for the reactions  $\mathcal{T}$  p-YK<sup>0</sup> pions at 25 Gev/c and 16 Gev/c  $\mathcal{T}^{\pm}$ p, 24, 1500 Gev/c pp collisions (p<sub>1</sub> =0.4 Gev/c) versus reduced CM longitudinal momentum. The non SP data are from ref.<sup>9</sup> and the SP data are from ref.<sup>15</sup>. The errors are statistical only. See the text.

and 24, 1500 Gev/c pp collisions  $^{9/}$ . The errors in Fig. 8 are statistical only. The approximate concordance of both kinds of CCD shows that in average the baryons production is not dependent on their strangeness.

Finally, Fig. 9 displays the strangeness CCD for the reaction  $\mathcal{T} p \rightarrow YK^{\circ}$  mions at 25 Gev/c. Although a minimum is observed in the central region, the strangeness in both hemispheres is different from o, contrary to expectations earlier suggested. Indeed, at 25 Gev/c we are not in the conditions of the PCM hypothesis for SPP. However, for higher energies a diminution of the total strangeness in both fragmentation regions is expected. Also we expect for the reaction  $\pi^+ p \rightarrow YK^{\circ}$  pions at similar energies, in the forward (backward) hemisphere, a symmetrical curve with respect to the y exit (the same curve) for GCD comparative with the corresponding distribution of the reaction  $\pi^- p \rightarrow YK^{\circ}$  pions.

With hypothesis (35) and zero compensative central charges /9/, /10/, a Pomeranchuk type theorem /10/ is predicted for asymptotic energies

$$\lim_{s \to \infty} \overline{STR}(s) = \lim_{s \to \infty} \overline{STR}(s).$$
(36)

From Sect. 3 we remember that there are no data at high energy to verify the prediction (36). New data from Fermilab furnishes good experimental evidence for the Pomeranchuk theorem  $^{35/}$ , so it would be interesting to verify the Pomeranchuk theorem for SPP.

6. Conclusions.

We now briefly summarize and discuss some of the main conclusions of our paper. The generating functional formalism has been particularized to SPP and related quantities have been defined within the framework of this formalism.

1. The analysis of the available data shows that till ~loo Gev/c laboratorv momentum, the cross section for SPP do not present a limiting behaviour; the  $K\tilde{K}$  cross sections still increase with energy, and also the  $Y^0$  cross section. When plotted versus adequate variable, the cross sections for SPP normalized to the corresponding

inelastic cross sections seem to have a shape independent of the colliding particles.

2. The data of average multiplicity of particles can be parametrized with a logarithmic plus a negative corrective power of s, which is not necessary -0.5, how was pointed out in ref.  $^{/26/}$ .

3. A more abundant production of neutral than charged K-mesons seems to occur in pp collisions. For  $\tilde{\pi}$  p collisions, a logarithmic increase is enough to describe the  $\langle n_{\mathbf{r}} \circ / \tilde{\mathbf{K}} \circ \rangle$  data.

4. The average  $\dot{A}/\Sigma^0$  multiplicities from pp and  $\mathcal{F}$  p collisions increase with energy. More accurate data are needed for establishing the exact shape of the increase. Again further data at momenta greater than 200 Gev/c would be of great interest in order to test the apparent plateau in  $\langle n_A/r_{\rm c}^0 \rangle$  and  $\langle n_{\rm KO}/\bar{\rm KO} \rangle$ .

5. Appling the generating functional formalism to processes with SPP, we extended the conclusions of the papers 9/,10/ to CCD with SPP. With model independent hypotheses, predictions on SPP at high energies were presented, particularly a Pomeranchuck-type theorem.

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Fig.9. Strangeness CCD for the reaction  $\mathcal{T}^{-}p \rightarrow YK^{0}$ +pions at 25 Gev/c versus reduced CM longitudinal momentum. Data from ref. (16/The errors are statistical only. See the text.

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