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RIDGE ANALYSIS
AND DEEP INELASTIC SCATTERING DATA

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1. Introduction

A significant amount of experimental information about the structure of nucleons and nuclei is obtained from deep inelastic interaction of muons.

The experimentally measured distribution of events exhibits, besides statistical errors, certain distortions due to the finite resolution of the detector. In the following we propose a method to reconstruct the original distribution. The method is quite general and involves solving a Fredholm integral equation of the first kind. There is an extensive body of literature [1-5] on this problem. The approach used here was first proposed by Tikhonov [1] and Phillips [2] and the statistical interpretation performed by Zhigunov [4] and Pyt'ev [5]. Methods of regression and ridge analysis [6] are also used in this work. The main difficulties in our case are that (1) the integral equation is two-dimensional and (2) the resolution function is known with statistical errors of the same order as the experimental distribution errors.

This work was initiated for the analysis of data taken with the NA-4 experiment at CERN. Without limiting the generality of the discussion, we shall describe the method as applied to this experiment.

The NA-4 set-up was conceived for muon-nucleon interaction research and is a toroidal iron spectrometer with a focusing magnetic field of azimuthal symmetry [7]. The spectrometer is ≈ 55m long and has a diameter of 2.75m. The extended target is inside the central bore of the torus. A scattered muon oscillates inside the spectrometer due to the field. Multiple scattering of the muons in the iron of the magnet, statistical fluctuations of their energy loss, and the limited number and comparatively coarse spatial resolution of the multiwire chambers used to measure the track are the principal sources of distortion of the experimental distribution.

In section 2 we derive the form of the integral that describes our problem. In section 3 we describe the system of linear equations obtained by algebraization of the integral equation, and in section 4 we propose a method of solving this system. The generalization of this method to a system with coefficients that have statistical errors is proposed in section 5. The problem of choosing the basic parameters of the method is discussed in section 6. All our steps are illustrated by computer experiments.

2. The problem

The deep inelastic muon-nucleon cross section can be written as

$$d^2\sigma/dxdQ^2 = F_2(x,Q^2) \cdot K(x,Q^2,E) \cdot \Delta(x,Q^2,E), \qquad (1)$$

where

F_2 is the nucleon structure function,

K is the kinematical factor,

Δ^{-1} is the radiative correction factor,

E is the muon beam energy,

Q^2 is the four - momentum transfer,

$x = Q^2/(2M_p v)$,

M_p is the proton mass,

v is the transferred energy.

The measured cross section $(d^2\sigma/dxdQ^2)_{exp}$ is related to the true cross section (1) by

$$(d^2\sigma(x,Q^2)/dxdQ^2)_{exp} = \int [d^2\sigma(x',Q^{2\prime})/dx'dQ^{2\prime}] P(x',Q^{2\prime};x,Q^2)dx'dQ^{2\prime}, \qquad (2)$$

where $P(x',Q^{2\prime};x,Q^2)$ is the resolution function of the experimental set-up. Here x, Q^2 are the measured parameters and x', $Q^{2\prime}$ the true ones. In (2) we integrate over the kinematically allowed region.

We know the left-hand side of (2) and the functions K, Δ. The resolution function can be obtained by direct measurement, if possible, or by Monte Carlo simulation of the experiment.

Our problem is to extract the structure function F_2 from eqs.(1) and (2). It should be noted that the actual incident muon beam is not monoenergetic, therefore eq. (2) must be integrated over the beam energy distribution. We do not write this more complicated equation here but assume the beam to be monochromatic for simplicity.

3. Basic equation

After algebraization of equation (2) we obtain a system of linear equations

$$\vec{f} = P\vec{\phi} + \vec{\varepsilon}, \qquad (3)$$

$\vec{f}$ is an m - dimensional vector representing to the measured distribution,

$\vec{\phi}$ is an n - dimensional vector representing to the structure function,

P is the m × n matrix representing the resolution function multiplied by K and Δ,

$\vec{\varepsilon}$ is an m - dimensional random vector (noise) with an average value $E\vec{\varepsilon} = 0$ and diagonal variance matrix V,

$V = \mathrm{Var}\,\vec{\varepsilon} = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_m^2)$,

where the σ_i are the statistical errors of the measured distribution.

Without limiting the general nature of the discussion, we describe the variables defined above (and, later, the solution of the problem) as applied to experimental data measured with a 280 GeV beam. The number of experimental events is $8 \cdot 10^4$. The number of simulated events is $4 \cdot 10^5$, among which $13 \cdot 10^4$ are reconstructed. The x,Q^2 distribution of the simulated events is $Q^2(1-x)^{-1}$. Figs.1 and 2 show binnings for the experimental data and the structure function, respectively.

The binning for the structure function is defined by the kinematic parameters of the events detected in the experimental set-up. The boundaries are determined through the distribution of the reconstructed simulated events by their original kinematic parameters.

The binning for the experimental data is chosen such that the maximum of available experimental information could be used. The size of the bins is chosen to allow at least 20 events within each, but, where statistics are sufficiently

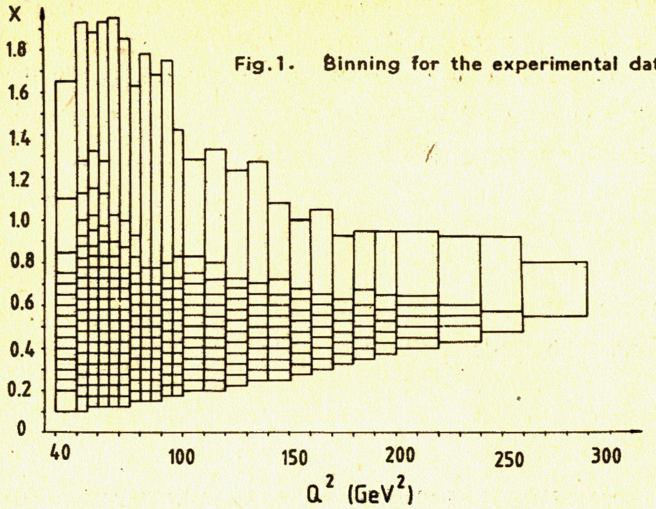


Fig.1. Binning for the experimental data.

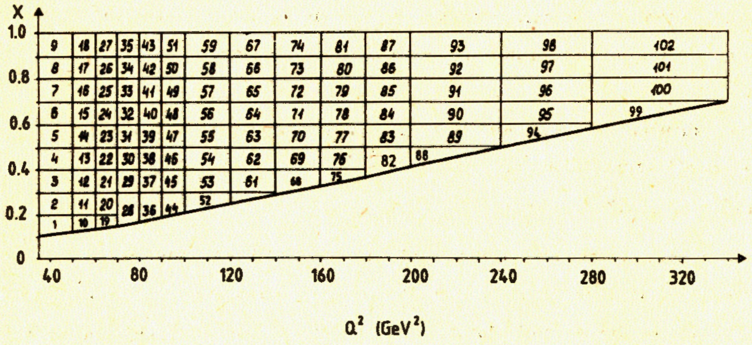


Fig.2. Binning for the structure function.

large, they are chosen such that up to 4 bins of experimental data correspond to one structure function bin. As f_i we take the number of events in the i-th bin of the experimental data, i.e. $\sigma_i^2 = f_i$.

The structure function in the j-th bin is calculated at the average values of the kinematic parameters in this bin. These mean values are defined as

$$x_j = \sum_i u^i x^i / \sum_i u^i, \quad Q^2_j = \sum_i u^i Q^{2i} / \sum_i u^i,$$

where the summation extends over all simulated events in the bin j.

x^i, Q^{2i} are kinematical variables of the simulated events,

u^i are the weights of the simulated events,

$u^i = Q^{2i}(1 - x^i)^{-1}F_2(x^i,Q^{2i})$.

For $F_2(x,Q^2)$ we use the parametrization

$$F_2(x,Q^2) = p_1(1 + p_2x)(1 - x)^{p_3}(Q^2/5)^{p_4 \ln 4x}, \tag{4}$$

where $p_1 = .52$, $p_2 = .9$, $p_3 = 3.2$, $p_4 = -.155$. This describes satisfactorily the data on structure functions obtained in other experiments and ensures good agreement between the spectrum of the reconstructed simulated events and the experimental spectrum.

The elements of the matrix P are calculated as

$$P_{ij} = W_j \cdot \sum_l g^l / \sum_l w^l, \tag{5}$$

where the summation extends over all events simulated in the j-th structure function bin. w^l is defined as

$$w^l = u^l \cdot v^l \cdot K(x^l,Q^{2l},E) \cdot \Delta(x^l,Q^{2l},E)$$

and

$g^l = w^l$, if the event was reconstructed and its reconstructed parameters fall into the i-th bin of the experimental data;

or

$g^l = 0$, if this condition is not fulfilled.

W_j is defined as

$$W_j = K(x_j,Q^2_j,E) \cdot \Delta(x_j,Q^2_j,E) \cdot s_j \cdot a_N,$$

where

s_j is the bin area,

a_N is a normalization constant.

$a_N = N \cdot \rho \cdot t \cdot A$, where

N is the muon flux,

ρ is the target density,

t is the target length,

A is Avogadro's number.

Use of the weights w^l helps in reducing somewhat the approximation error due to the algebraization of eq. (2). The error can be reduced even more by repeating the whole procedure upon calculation of F_2. The FWHM x and Q^2 resolutions of the NA-4 set-up are shown in figs. 3 and 4 (see ref. [7], [8]).

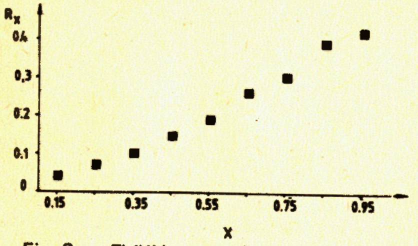


Fig.3. FWHM x resolution vs x of the NA-4 set-up.

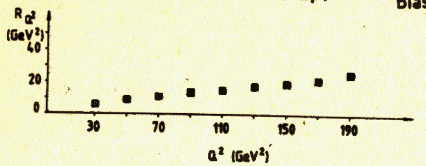


Fig.4. FWHM Q^2 resolution vs Q^2 of the NA-4 set-up,

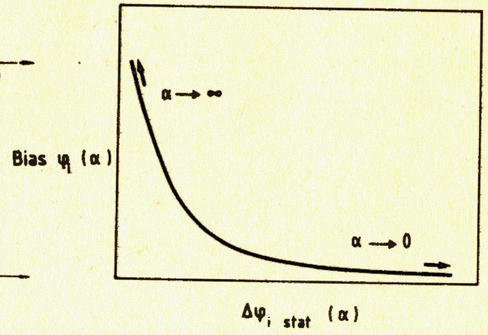


Fig.5. Dependence of statistical error and bias on α

4. Solution of the basic equation

Equation (3) can be solved in the least-squares sense by minimizing

$$\chi^2 = \sum_{i=1}^{m} (f_i - \sum_{j=1}^{n} P_{ij}\hat{\phi}_j)^2/\sigma^2_i .$$

The $\hat{\vec{\phi}}$ that yields the minimal χ^2 is taken as the estimator of $\vec{\phi}$ [6]:

$$\hat{\vec{\phi}} = (P^T V^{-1} P)^{-1} P^T V^{-1} \vec{f}, \qquad (6)$$

where T indicates transposed matrices. By substitution of (3) into (6) we obtain the expression:

$$\hat{\vec{\phi}} = \vec{\phi} + (P^T V^{-1} P)^{-1} P^T V^{-1} \vec{\varepsilon},$$

which shows that the least-squares method estimator is unbiased. The complete matrix of statistical errors for $\hat{\vec{\phi}}$ is [6]

$$\mathrm{Var}\, \hat{\vec{\phi}} = (P^T V^{-1} P)^{-1}.$$

It is well known [1] that a system of linear equation obtained from the integral equation (2) is a poorly determined system, i. e., its solution exhibits an extraordinarily strong dependence on a variation of the nonhomogeneous terms, as well as on the errors of the matrix elements and the computer round-off errors. Therefore, when solving equation (3) by the least-squares method, we may obtain $\hat{\vec{\phi}}$ with very large errors or may not be able to solve the equation at all if there is no inverse matrix for $P^T V^{-1} P$.

For the χ^2 functional this means the existence of a direction along which χ^2 varies too little and for which the minimum is therefore very poorly determined.

It is reasonable to assume that the stucture function is a smooth function, and this condition can help us to find a stable method of solution of (3). For this purpose, we shall search for the solution of (3) in the form of a minimum of the functional

$$\chi^2_r = \chi^2 + \alpha \cdot \sum_{i=1}^{n} (\hat{\phi}_i - \hat{\phi}_{i_1})^2 (x_i - x_{i_1})^{-2} s_i \qquad (7)$$

$$+ (\hat{\phi}_i - \hat{\phi}_{i_2})^2 \beta^{-2} (Q_i^2 - Q_{i_2}^2)^{-2} s_i$$

$$+ (\hat{\phi}_i - \hat{\phi}_{i_3})^2 [(x_i - x_{i_3})^2 + \beta^2 (Q_i^2 - Q_{i_3}^2)^2]^{-1} s_i$$

$$+ (\hat{\phi}_i - \hat{\phi}_{i_4})^2 [(x_i - x_{i_4})^2 + \beta^2 (Q_i^2 - Q_{i_4}^2)^2]^{-1} s_i ,$$

where,

α and β are regularization parameters,

i_1, i_2, i_3, i_4 indicate bins adjacent to the i-th bin, where

i_1 is the nearest upper neighbour bin,

i_2 is the nearest right neighbour bin,

i_3 is the nearest upper right neighbour bin,

i_4 is the nearest upper left neighbour bin.

For example, for $i = 40$, $i_1 = 41$, $i_2 = 48$, $i_3 = 49$ and $i_4 = 33$ (cf. fig. 2). If a bin does not exist, the corresponding term in eq. (7) is equal to zero. For example, for $i = 1$ there exists no upper left neighbour, and the addend with i_4 does not exist for $i = 1$.

As can be seen from (7), an increase of α leads to an increase in the smoothness of the solution, while β regulates the relation between the smoothness in the x and Q^2 directions.

Expession (7) can be represented in the form

$$\chi^2_r = \chi^2 + \alpha \cdot \sum_{i,j=1}^{n} \hat{\phi}_i \Omega_{ij}(\beta) \hat{\phi}_j, \qquad (8)$$

where $\Omega(\beta)$ is an $n \times n$ symmetric non-negative definite matrix and

$$\Omega(\beta)\vec{g} = 0, \text{ if } \vec{g} = (c, c, \dots, c)^T, \; c = \text{const.} \qquad (9)$$

As shown in [1], the minimum of χ^2_r is given by

$$\hat{\vec{\phi}} = (P^T V^{-1} P + \alpha\Omega(\beta))^{-1} P^T V^{-1} \vec{f}. \qquad (10)$$

By substitution of (3) into (10) we obtain the expression

$$\hat{\vec{\phi}} = \vec{\phi} - (P^T V^{-1} P + \alpha\Omega(\beta))^{-1} \alpha\Omega(\beta)\vec{\phi} + (P^T V^{-1} P + \alpha\Omega(\beta))^{-1} P^T V^{-1} \vec{\varepsilon}$$

which gives us the bias of the estimator with regularization

$$\text{Bias } \hat{\vec{\phi}} = -(P^T V^{-1} P + \alpha\Omega(\beta))^{-1} \alpha\Omega(\beta)\vec{\phi}. \qquad (11)$$

The complete matrix of statistical errors for $\hat{\vec{\phi}}$ [4]

$$\text{Var } \hat{\vec{\phi}} = (P^T V^{-1} P + \alpha\Omega(\beta))^{-1} P^T V^{-1} P (P^T V^{-1} P + \alpha\Omega(\beta))^{-1}. \qquad (12)$$

The least-squares method excludes distortions, and as a result we have a big noise in the solution. The solution obtained from the regularization method is distorted by the stabilizer $\alpha\Omega(\beta)$, but here the noise component can be made reasonably small. The dependence of the bias and the statistical error on α is sketched in fig.5.

As can be seen from (11), the bias decreases if α decreases. The bias can be considerably diminished if one uses some a priori information in the form of a parametrization of $F_2 = F_2(x, Q^2, p_1, p_2, \dots)$ which follows from theory or from other experimental data; we will call this F_2 the trial function. For this purpose let us represent equation (3) in the form

$$\vec{f} = PDD^{-1}\vec{\phi} + \vec{\varepsilon},$$

where

$$D = \text{diag}(\phi_{1\,tr}, \dots, \phi_{n\,tr}),$$

and

$$\phi_{i\,tr} = F_2(x_i, Q_i^2, p_1, p_2, \dots).$$

We now obtain a new equation

$$\vec{f} = P'\vec{\phi}' + \vec{\varepsilon}, \qquad (13)$$

where

$$P' = PD \text{ and } \vec{\phi}' = D^{-1}\vec{\phi}.$$

For a properly chosen trial function $\vec{\phi}'$ has equal components. As follows from (9), $\Omega(\beta)\vec{\phi}' \approx 0$, therefore the bias in $\hat{\vec{\phi}}'$ is small.

After solving equation (13), $\hat{\vec{\phi}}$ is readily obtained from the equation

$$\hat{\vec{\phi}} = D\hat{\vec{\phi}}',$$

and we can find the structure function in the bin centres, with the approximate formula

$$\hat{\vec{\phi}}_c = D_c\hat{\vec{\phi}}'$$

where D_c is a matrix with the trial function values in the bin centres. The bias of the estimator $\hat{\vec{\phi}}$ obtained using a trial function is

$$\text{Bias}\ \vec{\hat{\phi}} = -D(P'^T V^{-1} P' + \alpha\Omega(\beta))^{-1} \alpha\Omega(\beta)\vec{\hat{\phi}}'$$

and the complete matrix of errors

$$\text{Var}\ \vec{\hat{\phi}} = D(P'^T V^{-1} P' + \alpha\Omega(\beta))^{-1} P'^T V^{-1} P' (P'^T V^{-1} P' + \alpha\Omega(\beta))^{-1} D.$$

5. Generalization of the regularization method for a matrix P with errors

Any of the two methods mentioned in section 2 allows to obtain the resolution function, or the matrix P, with statistical errors. The real matrix element p_{ij} equals

$$p_{ij} = \bar{p}_{ij} + \xi_{ij},$$

where $\bar{p}_{ij}$ is the true value of the matrix element and ξ_{ij} is a random value with

$$E\xi_{ij} = 0,\ \text{Var}\ \xi_{ij} = d_{ij}\ ,\ \text{Cov}\ \xi_{ij}\xi_{kl} = 0.$$

If we calculate p_{ij} by the Monte Carlo method, then

$$d_{ij} = W_j^2 \cdot \sum_l (g^l)^2 / (\sum_l w^l)^2.$$

Here, the summation convention is the same as for the corresponding matrix element p_{ij} (5). When the matrix elements in eq. (3) have statistical errors, it cannot be solved satisfactorily in the least-squares sense [6], but it may be solved in the framework of the maximum likelihood method by minimizing the likelihood function logarithm [9]

$$\ln L = \sum_{i=1}^{m} (f_i - \sum_{j=1}^{n} p_{ij}\hat{\phi}_j)^2 \times (\sum_{j=1}^{n} d_{ij}\hat{\phi}_j^2 + \sigma_i^2)^{-1} + \text{const}.$$

As shown in [10], a good approximate solution of this problem is

$$\vec{\hat{\phi}} = (P^T V^{-1} P - \Xi) P^T V^{-1} \vec{f}, \tag{14}$$

where

$$V = \text{diag}(\sum_{i=1}^{n} \phi_i^2 d_{1i} + \sigma_1^2, \ldots, \sum_{i=1}^{n} \phi_i^2 d_{mi} + \sigma_m^2),$$

$$\Xi = \sum_{j=1}^{m} \text{diag}(d_{j1}, \ldots, d_{jn}) / (\sum_{i=1}^{n} \phi_i^2 d_{ji} + \sigma_j^2).$$

Since the true value of ϕ_i which enters into the right-hand part of (9) is unknown, the values of $\vec{\phi}$ from the previous experiment can be employed, or $\vec{\hat{\phi}}$ can be found by an iteration. This procedure is convergent as is shown in ref. [10]. Generalization of (13) by the regularization method leads to the final expression for the estimator of $\vec{\hat{\phi}}$:

$$\vec{\hat{\phi}} = D(P'^T V'^{-1} P' - \Xi' + \alpha\Omega(\beta))^{-1} P'^T V'^{-1} \vec{f}$$

and for the bias and the complete error matrix

$$\text{Bias}\ \vec{\hat{\phi}} \equiv -D(P'^T V'^{-1} P' - \Xi' + \alpha\Omega(\beta))^{-1} (-\Xi' + \alpha\Omega(\beta))\vec{\phi}, \tag{15}$$

$$\text{Var}\ \vec{\hat{\phi}} \equiv D(P'^T V'^{-1} P' - \Xi' + \alpha\Omega(\beta))^{-1} P'^T V'^{-1} P' \times (P'^T V'^{-1} P' - \Xi' + \alpha\Omega(\beta))^{-1} D, \tag{16}$$

where

$$V' = \text{diag}(\sum_{i=1}^{n} d_{1i} + \sigma_1^2, \ldots, \sum_{i=1}^{n} d_{mi} + \sigma_m^2),$$

$$\Xi' = \sum_{j=1}^{m} \text{diag}(\phi_{1\ tr} d_{j1}, \ldots, \phi_{n\ tr} d_{jn}) / (\sum_{i=1}^{n} d_{ji} + \sigma_j^2).$$

6. Choice of regularization parameters and of the trial function

For choosing α we used the graphical method first applied in ridge analysis [11]. By plotting the $\hat{\phi}'_i(\alpha)$ (ridges) versus α we can obtain some idea as to the ill-conditioning of (3). The value of α can then be chosen to be the minimal one below which the system (3) becomes unstable. For illustration purposes we have performed a computer experiment. In this case pseudodata are obtained as follows:

1. A parametric formula for the true function F_2 is assumed and the vector $\vec{\phi}$ is formed.

2. The actual matrix P is multiplied by the vector $\vec{\phi}$ and as a result we obtain a pseudoexperimental vector $\vec{f}$.

3. Using a random number generator with normal distribution, fluctuations are added to the vector $\vec{f}$. These fluctuations correspond to the actual statistical errors of the experimental distribution.

4. In the same way fluctuations are added to the elements of the matrix P. These fluctuations correspond to the actual statistical errors of the matrix elements.

Since we know the true F_2 in the simulation problem, its solution allows us to understand, in general, the influence of choosing the α, β and trial functions on the quality of the reconstruction. As a true F_2, we chose the following parametrization [12]

$$F_2(x,Q^2) = cx^{\alpha_0+\alpha_1 s}(1-x)^{\beta_0+\beta_1 s},$$

where

$$s = -\ln[g^2(Q^2)/g^2(Q^2_0)],\quad g^2(Q^2) = 16\pi^2(\beta_0 \ln(Q^2/\Lambda^2))^{-1},$$

with the following parameters: Q^2_0 = 110 GeV², c = 1.46, α_0 = 0.49, α_1 = - 0.81,

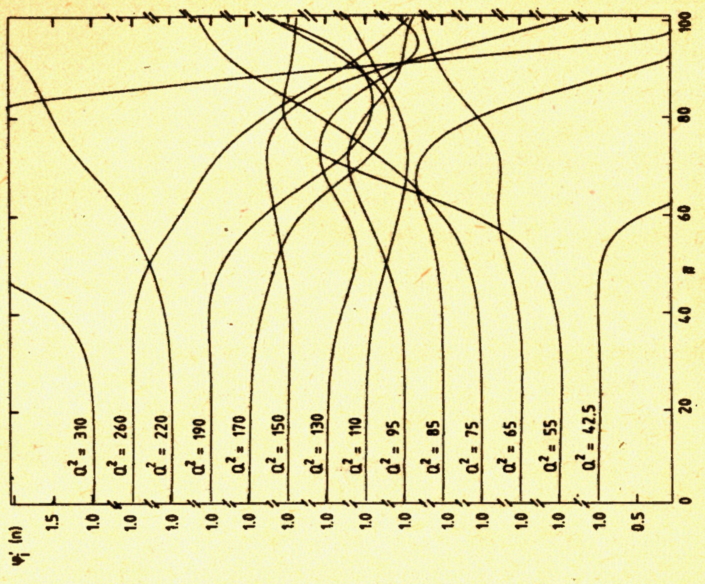
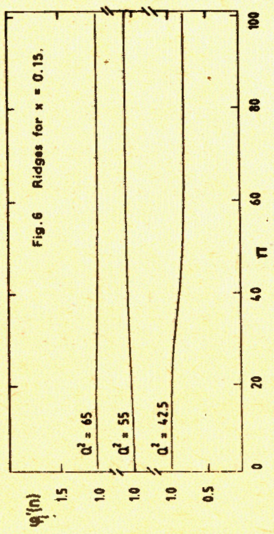


Fig.8. Ridges for x = 0.75.

Fig.6 Ridges for x = 0.15.



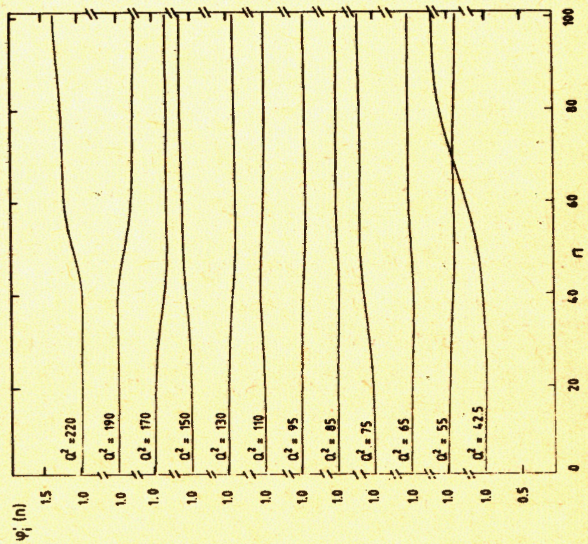


Fig.7 Ridges for x = 0.45.

β_0 = 11 - 2/3f, f = 4, Λ = 0.26 GeV, β_1 = 1.57, β_0 = 4.08. This parametrization satisfactorily describes the NA-4 data published in ref. [13]. In figs. 6 - 9, ridges are shown for different values of x. We use β = 1/30 and a trial F_2 equal to the true F_2. The ridges are shown as a function of n, which is related to α simply by $\alpha = 100/1.2^n$. We have chosen $\alpha = 100/1.2^{35} = 0.17$, above which the system (3) is stable. Fig.10 shows the reconstructed F_2 for this α. Fig.11 represents the correlation with the reconstructed F_2 for different bin pairs. For comparison, figs.12 and 13 show the reconstructed F_2 with $\alpha = 100/1.2^{25} = 1.05$ and the corresponding correlation matrix. Figs.14 and 15 represent the reconstructed F_2 and the correlation matrix with $\alpha = 100/1.2^{45} = 0.03$. A large positive correlation and the resulting smooth F_2 correspond to a reconstruction with α = 1.05. In the case where the trial function does not coincide with the true function, the reconstructed F_2 for α = 1.05 has a larger bias than in the case of α = 0.17 and α = 0.03. The noise component increases if α decreases, as we have see from fig.15. If α is smaller than $100/1.2^{70}$, F_2 can even become negative in some bins due to the behaviour of the ridges. The large positive correlation between the F_2's in adjacent bins in the region of large x (x > 0.65) is due to the insufficient experimental information (events, bins) in this region.

The choice of the parameter β mainly influences the character of the correlations for different parameters x and Q^2, and the value of α for which system (2) becomes stable. The best β, in our opinion, does not give large correlations in any particular direction. This means that the smoothness in the x and Q^2 directions is the same. The best β is considered to be the one for which different ridges become stable at the same value of α. Figs.16 and 17 are show F_2 and the corresponding correlation matrix for β = 1, and figs.18 and 19 show F_2 and the corresponding correlation matrix for β = 1/60. We chose α in both cases by the method described above. In the first case, we have a large positive correlation in the x direction, and the smoothness in the x direction is larger than in the Q^2 direction. In the second case, we have large positive correlations in the Q^2 direction and a large smoothness in this direction. In the case of β = 1/30, the smoothness is approximately the same in both directions.

It follows from (15) that the choice of the trial function influences the value of the bias of the reconstructed structure function; the dependence of the result on the choice of the trial function is different for different regions.

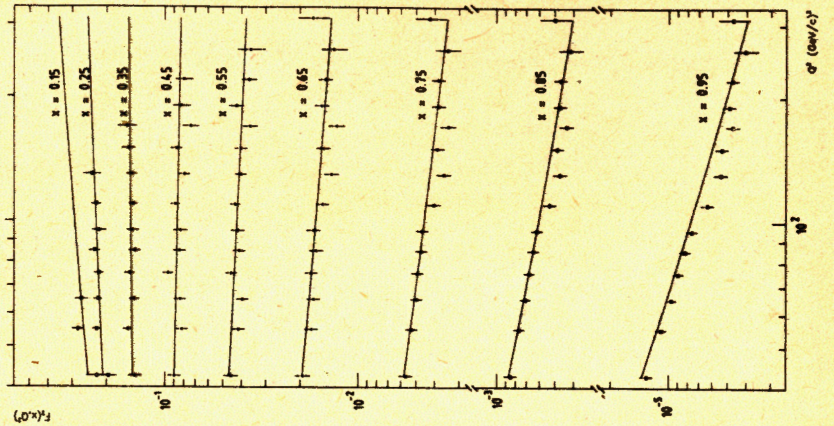


Fig.10. The reconstructed F_2 for trial function equal to the true function (solid line), α = 0.17, β = 1/30.

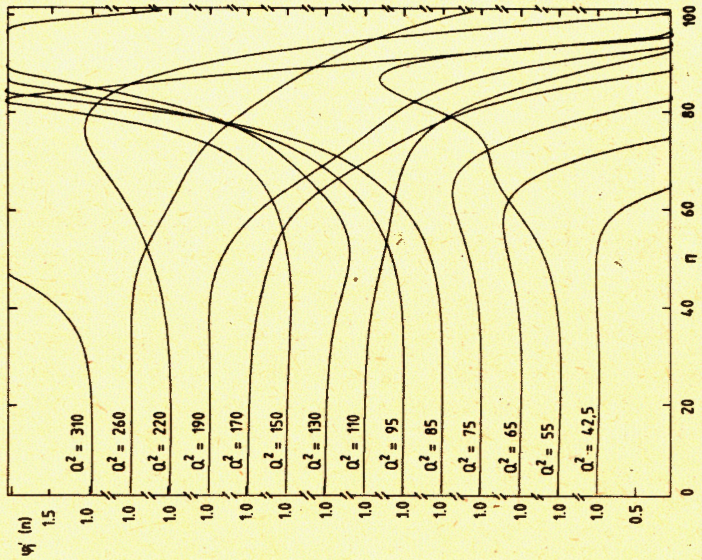


Fig.9. Ridges for x = 0.95.

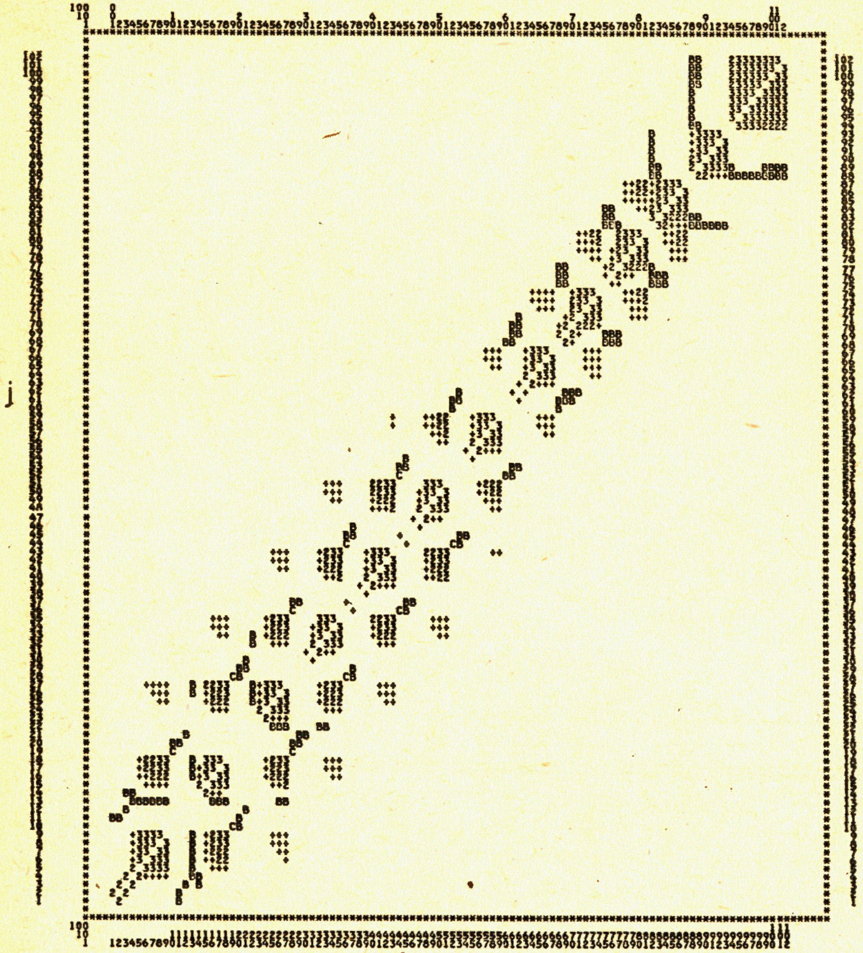


Fig.11 The correlation matrix for F_2 with trial function equal to the true function, $\alpha = 0.17$, $\beta = 1/30$. The only correlations shown are those greater than 0.25 in absolute value. The notation is the following:

$+ \Rightarrow 0.25 < Cor(\hat{\phi}_i, \hat{\phi}_j) \leq 0.5$, $2 \Rightarrow 0.5 < Cor(\hat{\phi}_i, \hat{\phi}_j) \leq 0.75$,

$3 \Rightarrow 0.75 < Cor(\hat{\phi}_i, \hat{\phi}_j) \leq 1.$, $B \Rightarrow -0.25 > Cor(\hat{\phi}_i, \hat{\phi}_j) \geq -0.5$,

$C \Rightarrow -0.5 > Cor(\hat{\phi}_i, \hat{\phi}_j) \geq -0.75$, $D \Rightarrow -0.75 > Cor(\hat{\phi}_i, \hat{\phi}_j) \geq -1$.

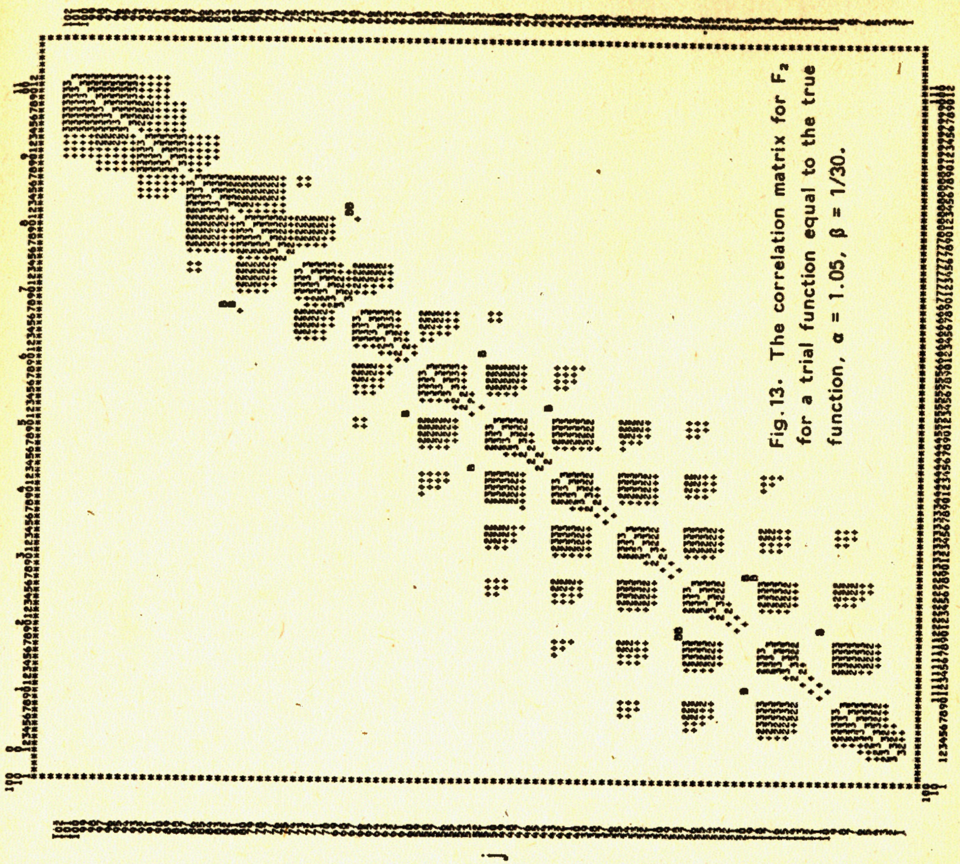


Fig. 13. The correlation matrix for F_2 for a trial function equal to the true function, $\alpha = 1.05$, $\beta = 1/30$.

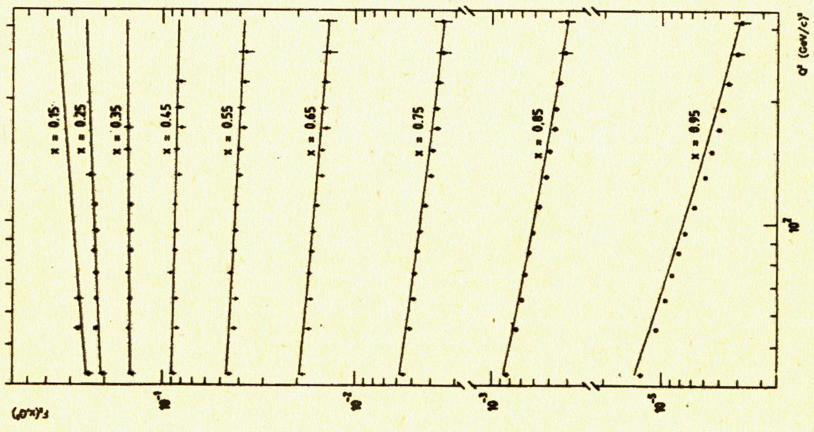


Fig. 12. The reconstructed F_2 for a trial function equal to the true function (solid line), $\alpha = 1.05$, $\beta = 1/30$.

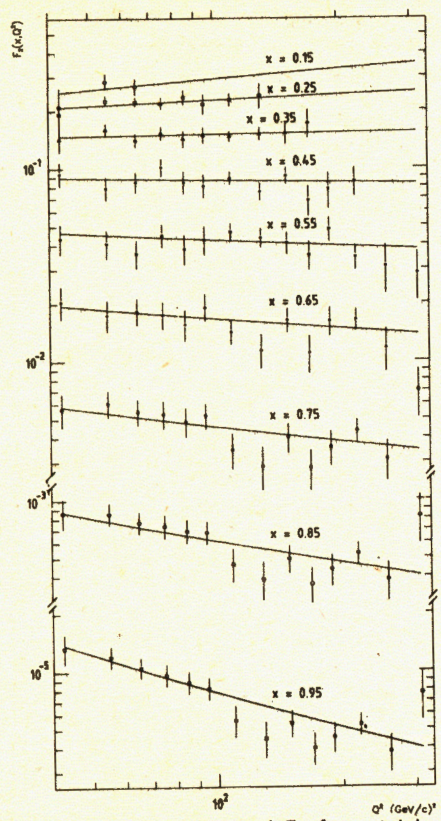


Fig.14. The reconstructed F_2 for a trial function equal to the true function (solid line), $\alpha = 0.03$, $\beta = 1/30$.

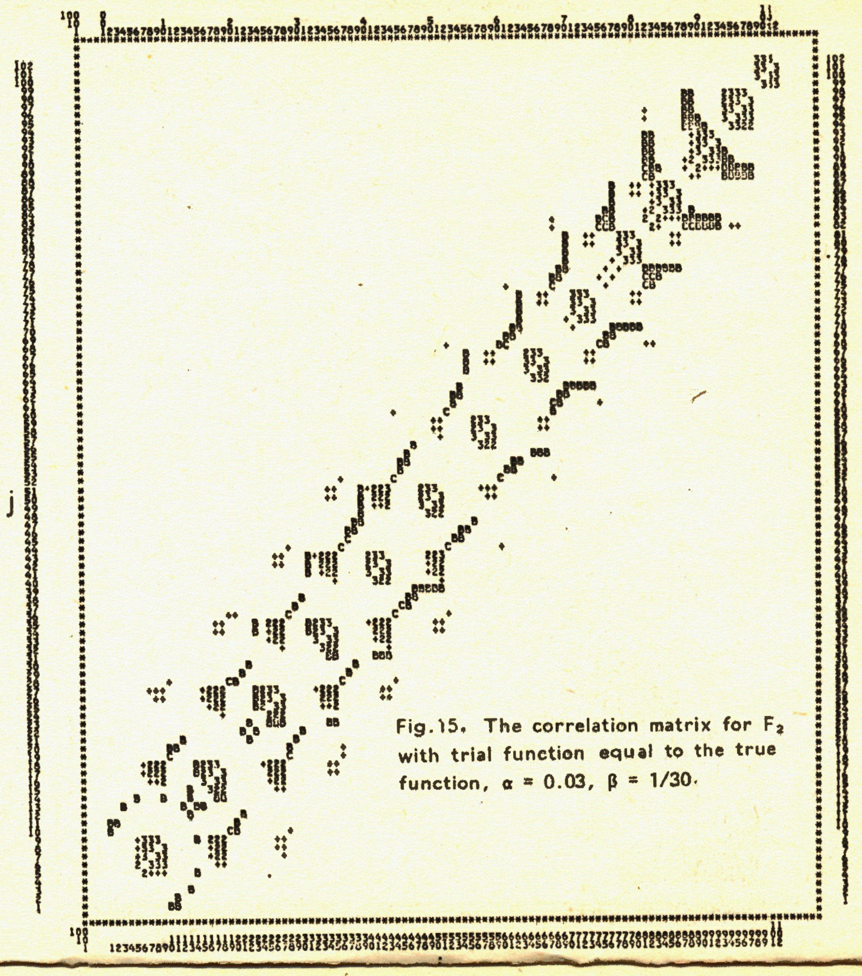


Fig.15. The correlation matrix for F_2 with trial function equal to the true function, $\alpha = 0.03$, $\beta = 1/30$.

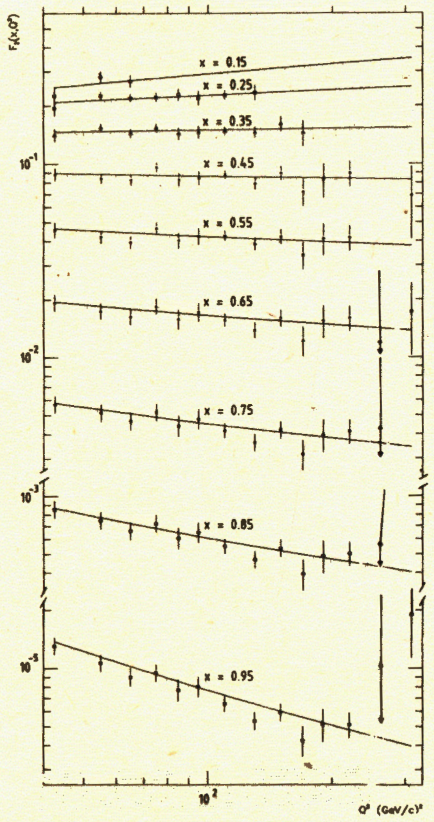


Fig.16. The reconstructed F_2 with trial function equal to the true function (solid line), $\alpha = 0.42$, $\beta = 1$.

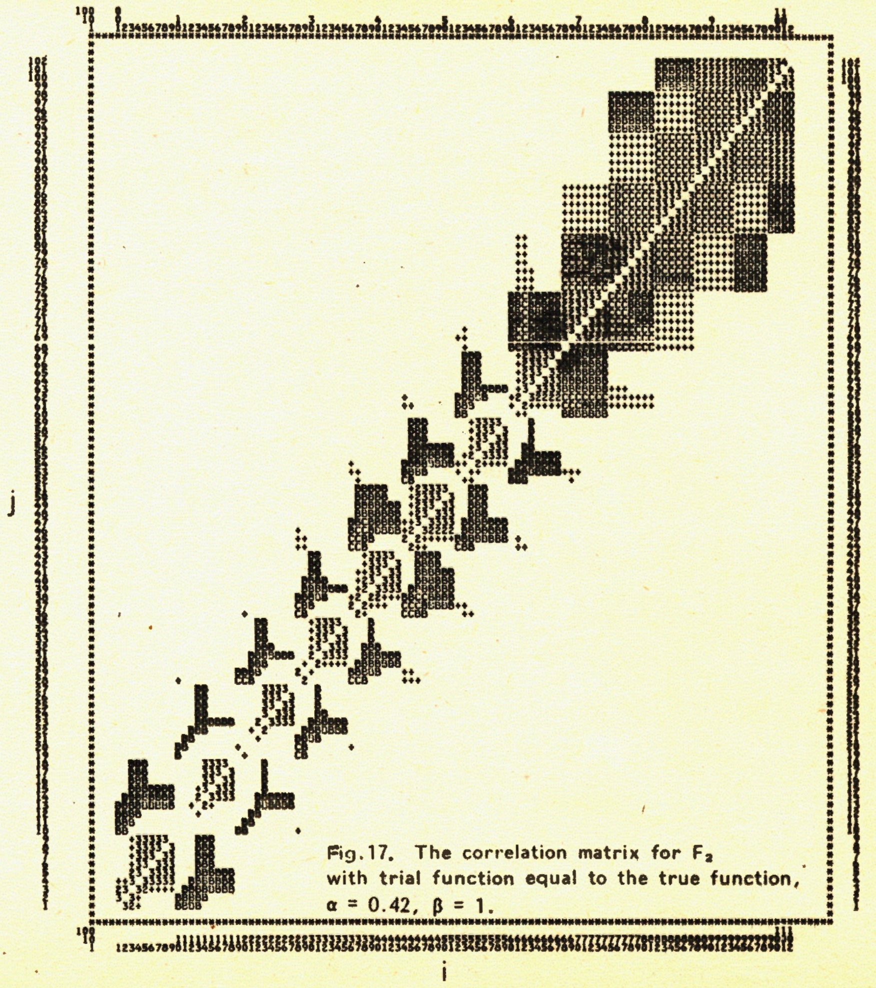


Fig.17. The correlation matrix for F_2 with trial function equal to the true function, $\alpha = 0.42$, $\beta = 1$.

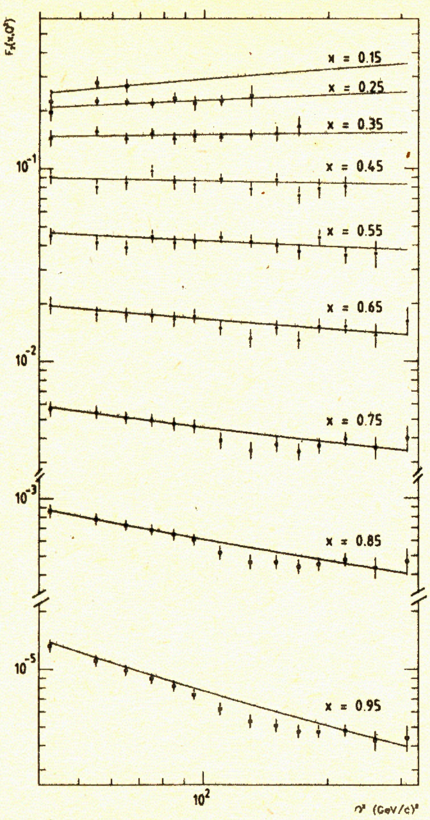


Fig.18. The reconstructed F_2 with trial function equal to the true function (solid line), α = 0.07, β = 1/60.

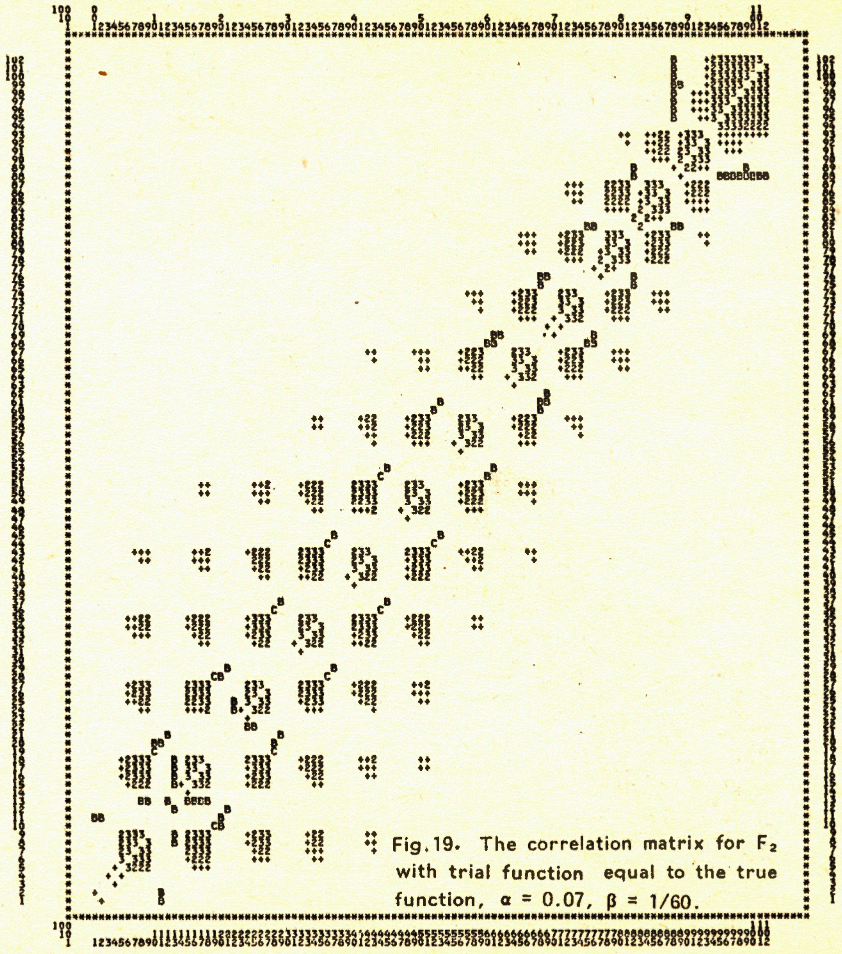


Fig.19. The correlation matrix for F_2 with trial function equal to the true function, α = 0.07, β = 1/60.

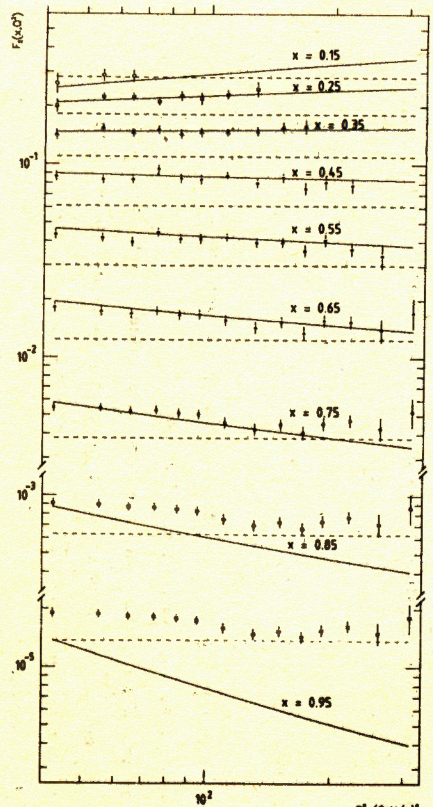


Fig.20. The reconstructed F_2 with trial function in the form (17). Solid line denotes the true F_2; the dashed one the trial function.

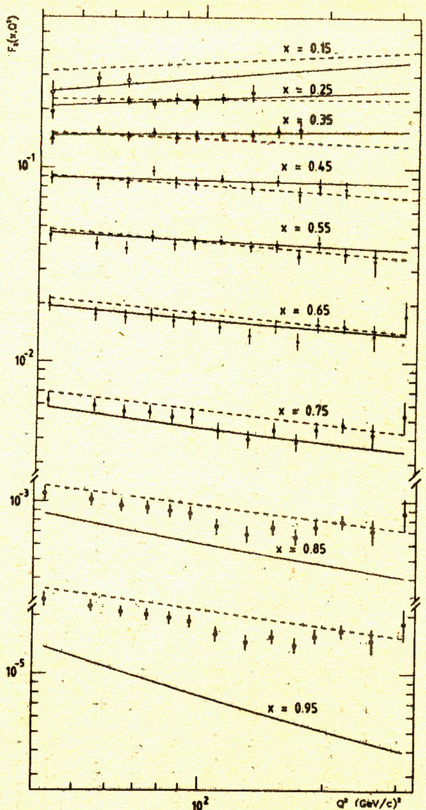


Fig.21. The reconstructed F_2 with trial function in the form (4). Solid line denotes the true F_2; the dashed one the trial function.

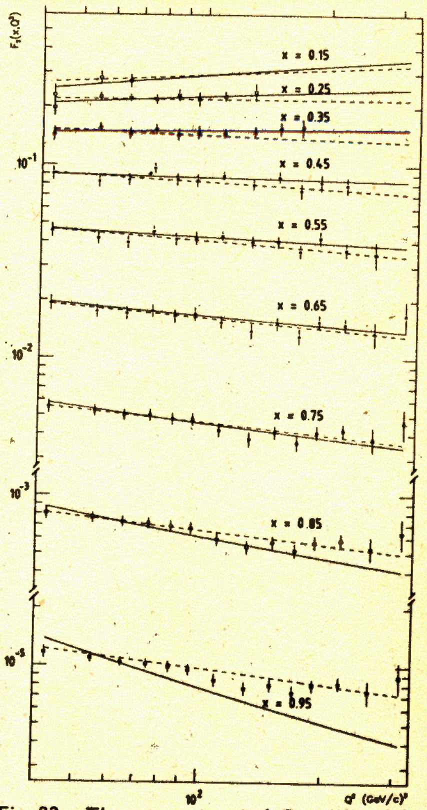


Fig.22. The reconstructed F_2 with trial function in the form (4), 10th iteration. Solid line denotes the true F_2; the dashed one the trial function.

Figs.20 and 21 show the reconstructed structure function for two trial functions not equal to the true F_2: in the form (4) with parameters $p_1 = 0.7$, $p_2 = 10.66$, $p_3 = 3.52$, $p_4 = -0.22$ and in the form:

$$F_2(x,Q^2) = p_1(1 - x)^{p_2} \quad (17)$$

with $p_1 = 0.5$ and $p_2 = 3.52$.

We see that, the reconstructed F_2 depends strongly on the trial function in the region of large x, due to insufficient statistics of the experimental data and the poor resolution of the experimental set-up in this region.

The bias can be diminished by choosing the most probable parameters for the given parametrization, using an iteration procedure. This procedure uses in each step the trial function parameters obtained from a fit to the structure function reconstructed in the preceding step. Fig.22 shows a reconstructed structure function at the 10th step of an iteration procedure with a trial function in the form (4) (first step as shown in fig. 21).

The proposed procedure of choosing the regularization parameters gives a value close to the minimal root-mean-square error of the structure function for the given trial function, or for the respective parametrization of the trial function when an iteration procedure is used. The root-mean-square error of a $\hat{\Phi}_i$ is defined as

$\Delta\hat{\Phi}_{i\ m.s.} = (\Delta\hat{\Phi}^2_{i\ stat} + (\mathrm{Bias}\ \hat{\Phi}_i)^2)^{-2}$.

In fig.5, this is the distance from the origin of the coordinate system to the curve $(\Delta\hat{\Phi}_{i\ stat}(\alpha),\ \mathrm{Bias}\ \hat{\Phi}_i(\alpha))$.

Bias and statistical errors decrease if the statistics of the experimental data is increased, increasing the number of bins for the experimental data accordingly, and if the statistical errors of the matrix elements are reduced [1, 4, 10].

7. Conclusion

We may summarise the main results of this paper as follows:

1. We have derived the basic integral equation.
2. We have transformed the basic equation to an algebraic form suitable for computer calculations, and proposed an algorithm giving expressions for the estimator of the structure function, for the bias of the estimator, and the full error matrix.

3. We have shown how to choose the basic regularization parameters and how to use a priori information to reduce the bias of the estimator.

The proposed method is very general and can be used in any scattering experiment to correct measured distributions for effects of experimental resolution.

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Гагунашвили Н.Д. E1-84-792
Гребневой анализ и данные по глубоконеупругому рассеянию

Для восстановления структурной функции нуклона из непосредственно измеряемого в эксперименте сечения глубоконеупругого рассеяния мюонов, рассматривается интегральное уравнение Фредгольма 1-го рода в его двумерном варианте. Свободный член уравнения /экспериментальное сечение/ и ядро уравнения имеют ошибки статистического характера. Решение уравнения /структурная функция/ находится методом регуляризации, параметры регуляризации выбираются с помощью гребневого анализа данных. Предлагается обобщение метода, в котором принимаются во внимание ошибки матричных элементов. Показано, как использовать априорную информацию для улучшения оценки структурной функции.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1984

Gagunashvili N.D. E1-84-792
Ridge Analysis and Deep Inelastic Scattering Data

To reconstruct the nucleon structure function from the directly measured experimental cross section of deep inelastic scattering of muons, a Fredholm integral equation of the first kind is considered in its two-dimensional version. The free term of the equation /the experimental cross section/ and the kernel have statistical errors. The equation's solution /structure function/ is found by the regularization method, the regularization parameters being chosen by ridge analysis data. A generalization of the method is suggested, where the kernel errors are taken into account. The use of a priori information for improving the structure function estimator is demonstrated.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1984