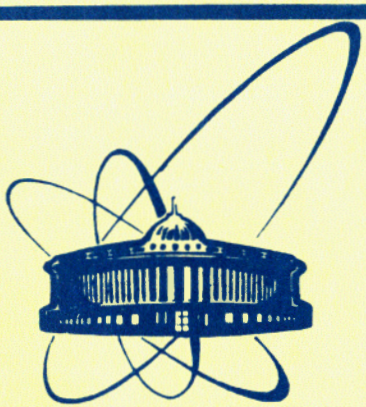

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APPLICATION OF CYCLE-BY-CYCLE KINETIC FORMULAE TO THE DESCRIPTION OF MUON-CATALYZED FUSION IN DEUTERIUM

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1. INTRODUCTION

The interest in the muon-catalysis of nuclear fusion has grown considerably over the past year [1-4]. One of the important problems arising in this connection is understanding of the kinetics of the associated processes. Recently, kinetic formulae have been derived giving the time distributions of fusion events ending the consecutive cycles of the muon-catalyzed fusion (MCF) chain [5-8]. In this note we confront these formulae with the first data on the cycle-by-cycle evolution in time of the MCF-chain which have become available for the deuterium target [9]. The time distributions of fusion events were measured in [9] for three initial cycles using an ionization chamber filled with deuterium at room temperature and 91.6 atm pressure. The chamber served as a target and a detector of charged fusion products at the same time. This guaranteed a 4π geometry and a hundred per cent detection efficiency ($\epsilon = 1$) [9].

The data are shown in Fig. 1a-c (circles) together with the curves calculated using the kinetic formulae of [7-8] within the framework developed in Ref. 10. Figure 2 illustrates the processes taken into account in the calculations. The muon entering the target, or the muon released in a fusion act, forms an excited muonic deuterium atom ($n \approx 14$) [1,11]. After deexcitation to the ground state the relative populations of the two possible $n = 1$ spin states, $d\mu$ ($s = 1/2$) and $d\mu$ ($s = 3/2$), are W_1 and W_3, respectively. Since both the $d\mu$-atom formation and cascade to the ground state are very fast processes [1,11] as compared to the competing transitions one can expect that:

$$N^{(k)}_{d\mu(1/2)}(t) \approx W_1 N^{(k)}_{\mu}(t), \qquad N^{(k)}_{d\mu(3/2)}(t) \approx W_3 N^{(k)}_{\mu}(t), \tag{1}$$

where $N^{(k)}_{d\mu}(t)$, $N^{(k)}_{\mu}(t)$ are numbers of $d\mu$-atoms (in either spin state) and muons at the beginning of the k-th cycle, respectively.

Transitions between the $s = 1/2$ and $s = 3/2$ states occur in collisions of the $d\mu$ atoms with the target centers. Since at room temperature the average kinetic energy of the thermalized $d\mu$-atoms is still slightly smaller than $\Delta E_{hfs} = E_{d\mu(3/2)} - E_{d\mu(1/2)} = 0.049$ eV [12], the transitions from the lower to higher hfs state, (1,3), can be expected to have little influence on the time distributions obtained in [9].

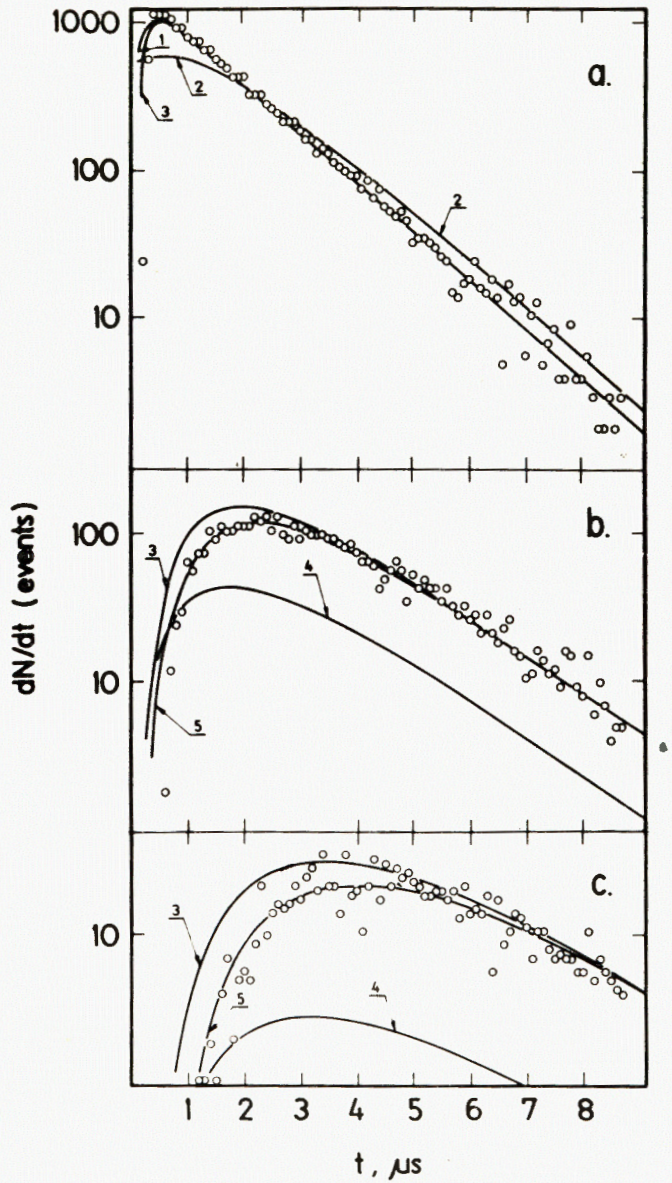

Fig. 1. Time distributions of fusion events ending three initial cycles catalyzed by a single muon in deuterium (sections a-c, respectively). Circles represent the data of Ref. 9. The curves are described in the text.

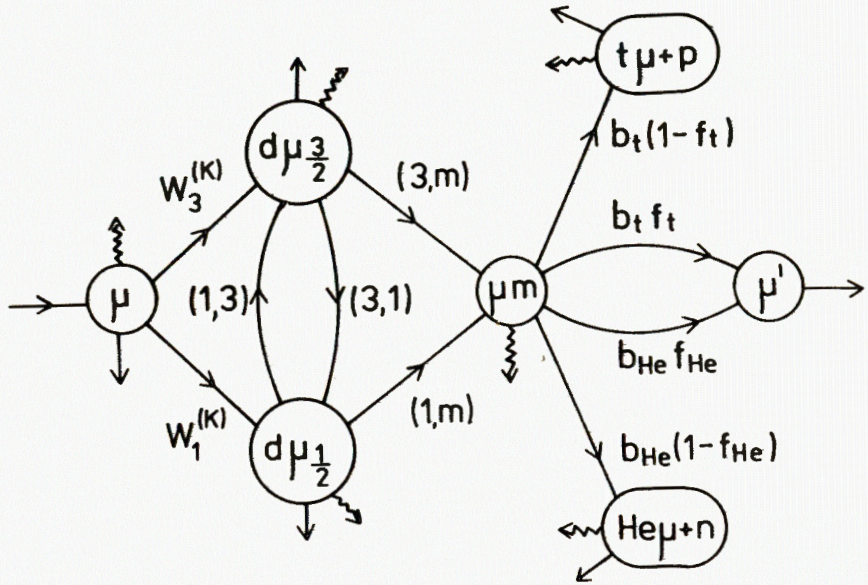

Fig. 2. Graph presenting the processes taken into account in the calculations, including dμ-atom formation in the two hyperfine (ground) states, and transitions between them, ddμ -formation from either dμ -state, nuclear synthesis into two fusion channels and muon release in the act of fusion.

Formation of the ddμ-molecule (μm) can proceed from either $d\mu_{(3/2)}$ - (3, m) or $d\mu_{(1/2)}$ - (1, m) state. However, according to present understanding [1,13], the (3,m) transition is slow at room temperature and can be practically neglected in presence of the much faster [14,15] competing transition (3,1). On the other hand, the (1,m) transition is a resonant transition [1,13] giving the main contribution to the (μm) = (ddμ) vertex.

There are two dominant channels in d+d nuclear synthesis:

$$d + d \rightarrow {}^3He + n, \qquad d + d \rightarrow t + p, \tag{2}$$

characterized by the corresponding branching ratios b_{He} and b_t. In the muon-catalyzed fusion taking place in the ddμ -molecule they are both split into two subchannels:

i) (μ'), in which the muon is freed, nuclear synthesis has taken place, and

ii) the channel in which the muon is captured by the charged fusion product forming a Heμ- or tμ -atom (muon sticking to the proton can be neglected).

Let us denote the probabilities of the first subchannel by f_{He} and f_t, respectively *. Then, the probability for the muon to enter again the MCF chain (muon feedback) is:

$$f = b_{He} f_{He} + b_t f_t \qquad (3)$$

with $b_{He} + b_t \approx 1$. Since $f_t \approx 1.0$[16] $f \approx 1 - b_{He}(1 - f_{He})$.

The wiggled arrows in the graph of Fig.2 indicate muon decay with the rate $\lambda_0 = 0.455 \cdot 10^6\ s^{-1}$, and the solid ones show transitions to the elements with $Z > 1$ contaminating the target ($\mu \to Z\mu$, $d\mu \to Z\mu$).

According to Ref.[10] the time distribution of fusion events ending the k-th cycle is a sum

$$F_k(t) = \sum_m C_m \cdot \frac{(k!)^{n'}}{[(k-1)!]^{n-1}} \sum_{i=1}^{n} e^{-R_i \cdot t} \times \sum_{j=1}^{k} t^{k-j} \times$$

$$\times \left\{ \frac{1}{(k-j)!} \cdot \sum_{\ell=0}^{j-1} (-1)^{\ell} \cdot \sum_{\{\rho_\alpha\}}{}' \prod_{\alpha=1}^{n'} \frac{(\Lambda_\alpha - R_i)^{k-\rho_\alpha}}{\rho_\alpha!\,(k-\rho_\alpha)!} \cdot \sum_{\{q_\beta\}} \prod_{\substack{\beta=1 \\ \beta \neq i}}^{n} \frac{(k-1+q_\beta)!}{q_\beta!\,(R_\beta - R_i)^{k+q_\beta}} \right\} \qquad (4)$$

$$\Sigma \rho_\alpha = j-\ell-1 \qquad \Sigma_{\beta \neq i} q_\beta = \ell$$

running over all possible forward paths (m) leading from the initial (beam) muon to the (μm) vertex in the k-th cycle, through k identical graphs of Fig.2. The notation used is the following: C_m - the m-th path weight equal to the product of all transition rates along the path, n - the number of vertices in the m-th path, n' - the number of vertices omitted by the m-th path, $\Lambda_\alpha = \Sigma \lambda_{j\alpha}$, where $\lambda_{j\alpha}$ are $(\alpha) \to (j)$ transition rates, the sum running over all lines leaving vertex α, $R_\mu = \Lambda_\mu$, $R_{\mu'} = \Lambda_{\mu'}$, $R_{\mu m} = \Lambda_{\mu m}$, and

$$R_{d\mu(1/2)} = \frac{1}{2} \cdot \left[\Lambda_{1/2} + \Lambda_{3/2} + \sqrt{(\Lambda_{1/2} - \Lambda_{3/2})^2 + 4 \cdot \lambda_{(1,3)} \cdot \lambda_{(3,1)}}\right],$$

$$R_{d\mu(3/2)} = \frac{1}{2} \cdot \left[\Lambda_{1/2} + \Lambda_{3/2} - \sqrt{(\Lambda_{1/2} - \Lambda_{3/2})^2 + 4 \cdot \lambda_{(3,1)} \cdot \lambda_{(1,3)}}\right].$$

There are four forward paths through the graph in Fig.2:

$$m = 1: \quad \mu \to d\mu(3/2) \to \mu m \to \mu'$$

$$m = 2: \quad \mu \to d\mu(3/2) \to d\mu(1/2) \to \mu m \to \mu' \qquad (5)$$

*In the traditional notation: $f = 1 - \omega$, where ω is the sticking coefficient.

$$m = 3: \quad \mu \to d\mu(1/2) \to d\mu(3/2) \to \mu m \to \mu'$$

$$m = 4: \quad \mu \to d\mu(1/2) \to \mu m \to \mu'.$$

Analogously, for the second and third cycles there are 16 and 64 forward paths, respectively, which are obtained by combining paths (5).

In the next Section we discuss the time distributions corresponding to the graph in Fig.2 calculated for three initial cycles for different values of parameters, and compare them with the experimental data of Ref.[9]. Since the data are not corrected for dead time, which can influence the shapes of the time distributions below the respective maxima, our conclusions concerning the rates of fusion and hfs transitions should be considered as indications of possible effects rather than firm results. It is worth mentioning them, however, as these are, at least, not excluded by the data in their present shape and, certainly, deserve a more scrutinized analysis.

2. COMPARISON WITH THE EXPERIMENTAL DATA

Unless specifically mentioned the values of the parameters characterizing these processes were fixed in our calculations at:

i) muon feedback probabilities $f_t = 1.0$, $f_{He} = 0.874$ (the latter value has been determined in[9] in an independent measurement),

ii) fusion channel branching ratio $b_{He}/b_t = 1.39$ (also determined independently in[9]),

iii) $dd\mu$-formation rates: $\lambda^\circ_{(1,m)} = 2.51\,\mu s^{-1}$ (resonant) and $\lambda^\circ_{(3,m)} = 0.04\,\mu s^{-1}$ (nonresonant[13]),

iv) $d\mu$ ($s = 1/2$) $\to$ $d\mu$ ($s = 3/2$) hfs transition rate, $\lambda^\circ_{(1,3)} = 0.0$,

v) $d\mu$-formation rate (including cascade within $d\mu$) $\lambda^\circ_a = 5 \cdot 10^4\,\mu s^{-1}$[1] (upper index 0 denotes the rates at liquid hydrogen density) and

vi) statistical weights: $W_1 = 1/3$, $W_3 = 2/3$.

Normalization, common to all three cycles, was a fitted parameter. The registration efficiency of Ref.[9], $\epsilon = 1$, was used.

Let us start by considering the first cycle in more detail. The large time slope of dN_1/dt is determined by the $dd\mu$-formation rate and the rate of transitions from $d\mu$-atoms to $Z > 1$ elements contaminating the target[8,9]. Once the value of $\lambda^\circ_{dd\mu} = \lambda^\circ_{(1,m)}$ is fixed, the latter can be determined from the data for the first cycle and used to describe the subsequent cycles. We shall return to this point below.

Curve (1) has been calculated using $\lambda^{o}_{(3,1)} = 37\,\mu s^{-1}$ (the value obtained recently in [15] and the theoretically predicted value of the rate of fusion in the $dd\mu$-molecule, $\lambda_f = 10^3\,\mu s^{-1}$ [17]). To emphasize the importance of including the hfs transitions, let us remark that if $W_1 = 1.0$ (only the resonant branch open, hfs effects excluded) and the same value of λ_f is used, the curve (not shown) is a continuation of the straight line from large t practically down to $t = 0$. As is seen, curve (1) overestimates the experimental values at small t. Since smaller values of $\lambda^{o}_{(3,1)}$ have been also reported in the literature [18] the most natural way to make the first cycle slower is to decrease $\lambda^{o}_{(3,1)}$. This leads, however, to a rather dramatic change of the shape of dN_1/dt illustrated in the figure for $\lambda^{o}_{(3,1)} = 7\,\mu s^{-1}$ (2). In fact, according to our fits the values below $\lambda^{o}_{(3,1)} \approx 20\,\mu s^{-1}$ seem to be excluded by these data.

An unexpected result follows when λ_f and $\lambda^{o}_{(3,1)}$ are left free in the fits: $\lambda^{o}_{(3,1)}$ tends to increase and λ_f falls much below the values predicted theoretically*. For the statistical weight ($W_1 = 1/3$) their best fit values are: $\lambda_f = 9.8 \pm 2.6\ \mu s^{-1}$ and $\lambda^{o}_{(3,1)} = 52.2 \pm 13.4\ \mu s^{-1}$. Variation of W_1 does not change the situation and λ_f remains small, with $\lambda_f \cdot W_1 \approx 3\,\mu s^{-1}$ giving equally good fits for $0.3 \leq W_1 \leq 0.5$. The corresponding curves (3) are shown for all three cycles. It is seen that both the large time slopes and absolute values of dN/dt at large t are reasonably well reproduced. This indicates that the $dd\mu$-formation rate, feedback probabilities and branching ratios used in the fits have not been grossly missed. (Our best fit to all cycles at t above the respective maxima, assuming the quoted values of $f_{He}(f_t)$ and $b_{He}(b_t)$, gives $\lambda^{o}_{(1,m)} = 2.51 \pm 0.22$ vs $2.76 \pm 0.08\ \mu s^{-1}$ obtained in [9]).

On the other hand if the value $\lambda^{o}_{(1,m)} = 0.76 \pm 0.11\ \mu s^{-1}$ [20] is used instead, and concentration of $Z > 1$ admixtures is chosen similarly to describe the RHS-slope of the first cycle, the resulting time distributions (4) for the remaining cycles fall much below the experimental points. Thus, the result of Ref.[20] seems to be incompatible with the data analysed above.

To remove the discrepancy between the best fit curves (3) for cycles 2 and 3 and the experimental data at small t yet one more slow process has to be included in the MCF-chain in the second and third cycle, which is absent in the first cycle. Curves (5) correspond to inclusion of such a process with $\lambda = 2.6\ \mu s^{-1}$. This puzzling result can be attributed [21] to the fact that dead time after the muon signal is not reflected in the data in contrast to the subsequent signals [23]. We mention this result to emphasize the sensitivity of the method. The time distributions can be equally well reproduced, if one assumes that in the first cycle the $d\mu$ (1/2) state is populated dominantly ($W_1 \approx 1$, $W_3 \approx 0$) and that in the subsequent cycles $W_1 \approx 0$ ($W_3 \approx 1$). In particular, for $W_1 = 1$ (1-st cycle) and $W_1 = 0$ (2nd and 3rd cycles) $\lambda_f = 6.28 \pm 1.91\ \mu s^{-1}$, $\lambda^{o}_{(3,1)} = 21.4 \pm 6.5\ \mu s^{-1}$ and the corresponding time distributions coincide practically with curves (5). However, this is rather an unrealistic assumption. We have mentioned this result to indicate that the effect of varying population of the hfs states can be naturally accounted for within the kinetic framework of Ref.[10]. An alternative explanation is the presence of the detector dead time after the registration of a fusion act. Taking into account dead time of the order of about 300 ns produces a similar effect [21]. This would indicate that using only the integrated neutron yields as proposed in Ref.[5,6] without a careful analysis of the shapes of the separate cycle time distributions can lead to very uncertain results.

Finally, let us remark that no choice of parameters for the graph shown in Fig.2 can reproduce the fast decrease of dN_1/dt at $t < 200$ ns. The fits can be possibly improved by admitting more parameters, e.g., by taking into account transitions within the μ-molecular node between states characterized by different fusion rates [21]. Alternatively, one can attribute this discrepancy to smaller registration efficiency at $t < 200$ ns. The discussion presented above indicates that a careful assessment of such systematic errors and dead time effects deserve effort as very interesting information about the MCF parameters is at stake.

Additionally, it would be greatly desirable to repeat the analysis presented above for analogous data taken at different target density and temperature. Indeed, as shown in [8,22], unambiguous assignment of the fitted values of transition rates to the particular processes present in the chain requires such a verification.

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*It should be noticed that in Ref.[19] it has been suggested that back decay of the μ-molecular resonance should also lead to a significant decrease of the efficiency of the muon catalyzed fusion in deuterium.

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Применение формализма кинетики последовательных циклов для описания мюонного катализа ядерной реакции синтеза в дейтерии

Первые экспериментальные данные по циклам цепочки процессов мюонного катализа ядерного синтеза проанализированы при помощи формул, описывающих временные распределения актов синтеза, завершающих последовательные циклы мю-катализа. Эти данные, приведенные для комнатной температуры и давления 91,6 атм., имеют характерные особенности, показывающие, что анализ по циклам мюонного катализа ядерного синтеза представляет значительный интерес и является богатым источником информации о параметрах этого процесса.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1984

Gula A., Bubak M., Niwicki J. E1-84-560

Application of Cycle-by-Cycle Kinetic Formulae to the Description of Muon-Catalyzed Fusion in Deuterium

The kinetic formulae developed recently for the cycle-by-cycle description of muon catalyzed fusion are confronted with the first data on the time distributions of fusion events ending the consecutive cycles which have become available for deuterium at room temperature and 91.6 atm pressure. It is shown that the formulae provide an adequate description of the data. The data reveal interesting features which indicate that a cycle-by-cycle analysis of the muon-catalysis fusion chain can be a rich source of information about the parameters of the processes involved.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1984