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PHENOMENOLOGICAL DESCRIPTION OF THE SPATIAL DISTRIBUTION OF IONIZATION LOSSES IN ELECTROMAGNETIC SHOWERS PRODUCED BY GAMMA-QUANTA WITH THE ENERGY $E_\gamma = 60-3000$ MeV

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I. INTRODUCTION

In spite of the fact that all elementary phenomena making up a cascade electromagnetic process (CEP) produced by high energy ($E \gg 2m_ec^2$) gamma-quanta or electrons are well known long ago[1], there is no hitherto sufficiently compact and convenient, from the practical point of view, description of this process. The analytical representation of the cascade theory existing nowadays[2] is valid to some purposes in light media, in which a multiple scattering of shower electrons is negligible, and fails entirely in heavy absorbents. In one's turn the method of computer simulation of a CEP by means of the Monte-Carlo technique has no shortcomings mentioned above but gives possibility of obtaining fragmentary numerical results only useless for generalization and prognosis and requires a lot of time of big computers. Meanwhile in different domains of physics and other fields, it is needed and even necessary to have at least estimative information about electromagnetic showers (ES). In particular it is of great practical interest to know the dependence on the primary gamma-quanta energy E_γ of a base diameter and a hight of a cylinder (or a cone) within which in average some part of shower electron ionization losses (SEIL) is released. The reasons mentioned above have inspired the authors to elaborate a phenomenological approach to the investigation of ionization losses of electrons and positrons (later: electrons) in ES produced by gamma-quanta in liquid xenon[4-6]. This approach consists in statistical description of a spatial distribution and fluctuations of experimentally measured SEIL. For this purpose the pictures from the xenon bubble chambers (XBC) of JINR (Dubna) and ITEP (Moscow) have been used as an empirical material. Such a choice is stimulated by the following reasons. Firstly, XBC gives possibilities of observing and measuring the ranges of practically all shower electrons with the energy $E \gtrsim 0$-1.5 MeV in ES generated by gamma-quanta within sufficiently wide energy interval, from some tens of MeV to some GeV's. Secondly, liquid xenon is enough heavy media with a radiation unit $X_0 \approx 4$ cm[7] close to corresponding values of gamma-quanta absorbents most frequently used in practice.

In this report basic results concerning the energy interval $E_\gamma = 60 - 3000$ MeV and two cut-off shower electron energy values: $E_0 = 0 - 1.5$ MeV and 3 MeV are presented.

2. METHOD AND EXPERIMENTAL DATA

The formulae presented in this report, describing longitudinal and transversal SEIL distributions, have been obtained as a result of statistical analysis of the sample consisting of 2045 events of ES registered on pictures of the XBC: the XBC of JINR (Dubna) exposed in the beam of π^+ mesons at 2.34 GeV/c and the XBC of IETP (Moscow) irradiated in the π^- meson beam at 3.5 GeV/c. In each selected event total shower electron ranges, $\Delta \sum_i r_i(E_\gamma, E_0, t, p) / \Delta t \Delta p$, observed inside a square of the surface $\Delta t \Delta p$ have been measured. Vertices of such a square have coordinates: $(t,\ t-\Delta t)$ - along the shower development axis (DSA), counting out from a primary gamma-quanta conversion point, and $(p,\ p-\Delta p)$ - in the direction perpendicular to a DSA. Here $\Delta t = 2 \cdot \Delta p = 0.588 \cdot X_0$, where $X_0 = 40.5 \pm 1.7$ mm[7] is a radiation unit of liquid xenon. Later in the text all lengths are expressed in X_0 units. Events of the sample have been devided among 23 energy intervals (Table 1).

Table 1

Numbers N_γ of events of ES and corresponding values of the energy E_γ of gamma-quanta producing ES

N_γ	E_γ (MeV)	N_γ	E_γ (MeV)
23	65 ± 5	75	555 ± 55
23	75 ± 5	82	680 ± 70
46	85 ± 5	129	875 ± 125
130	100 ± 10	108	1125 ± 125
143	120 ± 10	92	1375 ± 125
208	145 ± 15	61	1625 ± 125
190	175 ± 15	51	1875 ± 125
191	210 ± 20	31	2125 ± 125
136	255 ± 25	27	2375 ± 125
121	310 ± 30	17	2625 ± 125
82	375 ± 35	7	2875 ± 125
72	455 ± 45		

As has been pointed out theoretically[8], experimentally[9] and by means of a computer simulation[10], the total range of shower electrons, $\sum_i R_i$, is proportional to the energy E_γ of a gamma-quantum generating shower, e.g.,

$$E_\gamma = \alpha \sum_i R_i, \qquad /1/$$

where $\alpha = (0.59 \pm 0.02)$ MeV/mm for liquid xenon[9,10]. It is shown also that with a good accuracy more general relation is true:

$$\frac{\Delta e(E_\gamma, E_0, t, p)}{\Delta t \Delta p} = \eta \cdot \frac{\Delta \sum r_i(E_\gamma, E_0, t, p)}{\Delta t \Delta p}, \qquad /2/$$

where $f(E_\gamma,E_o,t,p) = \frac{\Delta e(E_\gamma,E_o,t,p)}{\Delta t\,\Delta p}$ are SEIL inside a square $\Delta t \Delta p$. The coefficient η is constant to within about 3% and equal to the constant α multiplied by the average coefficient $\bar{K}$ of shower electron range projection enlargement ($\bar{K} = \sum R_i / \sum r_i$)/11/.

The function $f(E_\gamma,E_o,t,p)$ may be defined in the form of a product of conditional distributions:

$$f(E_\gamma,E_o,t,p) = f_1(E_\gamma,E_o,t)\cdot f_2(E_\gamma,E_o,p\,|\,t), \qquad /3/$$

where functions f_1 and f_2 describe a longitudinal (f_1) and transversal (f_2) development of ES in a picture projection plane. The corresponding distributions in a space are related by the analogous formula:

$$F(E_\gamma,E_o,t,\rho) = F_1(E_\gamma,E_o,t)\cdot F_2(E_\gamma,E_o,\rho\,|\,t), \qquad /4/$$

where ρ is a distance from a DSA and t has the same meaning as earlier.

Naturally enough to admit that

$$F_1(E_\gamma,E_o,t) = f_1(E_\gamma,E_o,t), \qquad /5/$$

and the normalization condition:

$$\int_0^\infty F_1(E_\gamma,E_o,t)\cdot dt = E_\gamma. \qquad /6/$$

Then owing to axis symmetry (in average) of ESs developing inside a uniform medium and taking into account of the photographing conditions of ES in XBCs we have/12/:

$$f_2(E_\gamma,E_o,p\,|\,t) = 2\cdot\int_0^\infty F_2(E_\gamma,E_o,\rho\,|\,t)\cdot\frac{d\rho}{\sqrt{1-(p/\rho)^2}}. \qquad /7/$$

From /3/ - /7/ it follows that

$$2\int_0^\infty f_2(E_\gamma,E_o,p\,|\,t)\cdot dp = 1, \qquad /8/$$

$$2\pi\int_0^\infty F_2(E_\gamma,E_o,\rho\,|\,t)\rho\,d\rho = 1. \qquad /9/$$

Hence

$$F(E_\gamma,E_o,t,\rho) = \frac{\Delta E(E_\gamma,E_o,t,\rho)}{2\pi\rho\,\Delta\rho\,\Delta t} \qquad /10/$$

is an average value of SEIL inside a ring of the thickness $\Delta\rho$ along the radious ρ, measured from a DSA; Δt is the thickness of a ring along a DSA at the point t.

3. LONGITUDINAL DEVELOPMENT OF AN ES

The distribution of SEIL along a DSA has been approximated by the function:

$$F_1(E_\gamma,E_o,t) = \alpha t^{\gamma} \exp/-\tfrac{1}{2}(\tfrac{t}{t_o})^2/, \qquad /11/$$

where the parameters α, γ, and t_o depend on E_γ and E_o. It turned out to describe the dependence on E_γ by the following simple functions:

$$\gamma(E_\gamma,E_o) = a_1\cdot e^{-a_2\cdot E_\gamma} + a_3\cdot E_\gamma + a_4, \qquad /12/$$

$$t_o(E_\gamma,E_o) = b_1\cdot \ln E_\gamma - b_2. \qquad /13/$$

Because of the normalization condition /6/ we have:

$$\alpha = \frac{2^{(1-\gamma)/2}\cdot 0.588^{-\gamma}}{\Gamma(\frac{1+\gamma}{2})\cdot t_o^{1+\gamma}}\cdot E_\gamma. \qquad /14/$$

Numerical values of constants a_i ($i=1,\dots,4$) and b_i ($i=1,2$) are presented in table 2.

Table 2

Numerical values of constants a_i ($i=1,\dots,4$) and b_i ($i=1,2$) of the formulae /12/ and /13/, and corresponding values of probability P according to a χ^2 criterion

i	$E_o \lesssim 1.5$ MeV		$E_o = 3$ MeV	
	a_i	b_i	a_i	b_i
1	0.4 ± 0.2	0.87 ± 0.04	5.6 ± 11.8	0.89 ± 0.04
2	$(5 \pm 4)\cdot 10^{-3}$	2.95 ± 0.16	$(36 \pm 26)\cdot 10^{-3}$	3.05 ± 0.16
3	$(29 \pm 2)\cdot 10^{-5}$		$(30 \pm 1)\cdot 10^{-5}$	
4	0.58 ± 0.05		0.53 ± 0.03	
	P = 0.46	P = 0.99	P = 0.45	P = 0.99

Note should be taken that at $E_\gamma \gtrsim 500$ MeV the dependence of the parameter γ on E_γ reaches practically its asymptotic behaviour, i.e.,

$$\gamma(E_\gamma,E_o) \cong a_3\cdot E_\gamma + a_4. \qquad /12'/$$

The formula /11/ describes experimental data on the confidence level of 0.3. From /11/ it follows, in particular, that if a ES develops in an absorbent of the thickness t, then in average it is possible to register the part $\bar{E}(t)$ of the energy E_γ of a gamma-quantum generating an ES. This part is:

$$\bar{E}(t) = \int_0^t F_1(E_\gamma, E_0, \tau)\, d\tau = E_\gamma \cdot \frac{2^{(1-\gamma)/2}}{\Gamma\left(\frac{1+\gamma}{2}\right)} \cdot \left(\frac{t}{t_0}\right)^{1+\gamma} \sum_{k=0}^{\infty} \frac{(-1)^k}{\gamma+1+2k} \cdot \frac{\left(\frac{t}{t_0}\right)^{2k}}{2^k k!} . \qquad /15/$$

When the ratio

$$\bar{A}(t) = \bar{E}(t)/E_\gamma \qquad /16/$$

is sufficiently close to unity ($\bar{A} \gtrsim 0.6-0.7$), the total energy E_γ of an ES may be evaluated reliably enough by iteration method inserting into right-hand part of /15/ a measured value of energy $E_{meas}(t)$ in place of E_γ as a first approximation.

4. TRANSVERSAL ES DEVELOPMENT

As a result of experimental material analysis it has been pointed out that the Gaussian function for SEIL transversal distribution may be taken at the energy interval $E_\gamma \lesssim 700$ MeV, i.e.,

$$f_2(E_\gamma, E_0, \rho | t) = N(0; \sigma^2). \qquad /17/$$

At higher energy values some part of ionization losses is collimated with a low dispersion near by a DSA. The rest part has a significantly greater dispertion, $\sigma_1 \gtrsim 2.5 \cdot \sigma_2$. Both parts may be approximated at the confidence level of 0.6 by normal function so that at $E_\gamma \gtrsim 700$ MeV we have:

$$f_2(E_\gamma, E_0, \rho | t) = \frac{1}{\sqrt{2\pi}(\sigma_1 + a\sigma_2)} \cdot (N(0; \sigma_1^2) + a \cdot N(0; \sigma_2^2)). \qquad /18/$$

Here a, σ_1, and σ_2 are the parameters depending on t, E_γ, and E_0 and being determined by experimental data statistical analysis. The dependence of these parameters on t and E_γ may be possibly represented by means of simple following functions:

$$\sigma_1 = m_1 \cdot t + b_1, \qquad /19/$$

$$\sigma_2 = m_2 \cdot t + b_2, \qquad /20/$$

$$a = A \cdot t^{-B}, \qquad /21/$$

$$m_1 = (0.05 \pm 0.02) + (3.6 \pm 1.1) \cdot 10^{-5} E_\gamma . \qquad /22/$$

Here $b_1 = 0.59 \pm 0.18$, $m_2 = 0.04 \pm 0.01$, $b_2 = 0.27 \pm 0.06$,

$A = 39 \pm 24$, $B = 1.42 \pm 0.10$, the energy E_γ is expressed in MeV; the length t, in rad. units. The dependence of the values m_i and b_i (i = 1, 2) on E_0 at experimental accuracy achieved is not observed.

Taking into account the transformation /7/ we get at the energy interval $E_\gamma \gtrsim 700$ MeV the following expression for the probability distribution function $F_2(E_\gamma, E_0, t|\rho)$ of average SEIL in a plane perpendicular to a DSA intersecting this axis at a distance t from a primary gamma-quantum conversion point:

$$F_2(E_\gamma, E_0, \rho | t) = \frac{1}{2\pi(\sigma_1 + a\sigma_2)} \left(\frac{1}{\sigma_1} \cdot e^{-\rho^2/2\sigma_1^2} + \frac{a}{\sigma_2} \cdot e^{-\rho^2/2\sigma_2^2}\right). \qquad /23/$$

Corresponding relation at $E_\gamma \lesssim 700$ MeV may be simply obtained from /23/ when a = 0.

5. GENERAL FORM OF SPATIAL DISTRIBUTION OF AVERAGE SEIL IN ES

It arises from the expressions /10/, /4/, /11/, and /23/ that a distribution of average SEIL released within a ring of a volume $\Delta V = 2\pi\rho\Delta\rho\Delta t$ may be described at $E_\gamma \gtrsim 700$ MeV using the following function:

$$\frac{\Delta E(E_\gamma, E_0, t, \rho)}{2\pi\rho\Delta\rho\Delta t} = \frac{E_\gamma l^\gamma e^{-l^2/2}}{2^z \pi t_0 \Gamma(z)} \cdot \frac{\frac{1}{\sigma_1} \cdot e^{-r_1^2/2} + \frac{a}{\sigma_2} \cdot e^{-r_2^2/2}}{\sigma_1 + a \cdot \sigma_2} . \qquad /24/$$

Here $z = (1 + \gamma)/2$; $l = t/t_0$, $r_1 = \rho/\sigma_1$, and $r_2 = \rho/\sigma_2$ are normalized lengths. The expressions for γ, t_0, σ_1, and σ_2 as well as numerical values of corresponding constants are given in sections 3 and 4. At $E_\gamma \lesssim 700$ MeV the function /24/ simplifies considerably because in this case a = 0. Then the numerical values for the coefficients may be taken from /5/. Such a form as at a = 0 has the distribution of average SEIL far from shower maximum since an effect of a collimation near by a DSA vanishes quickly.

6. LONGITUDINAL FLUCTUATIONS OF IONIZATION LOSSES

The essential source of gamma-quanta energy measurement errors, when ionization effect of ESs produced by these gammas is detected, are fluctuations in longitudinal development of electromagnetic showers. To evaluate quantitatively such fluctuations it is suitable to use a distribution of the part $A(t)$ of the shower energy $E(t)$ released inside an absorbent of the thickness t. Once assumed this thickness coincide with a cascade development length we have:

$$A(t) = E(t)/E_\gamma \quad . \qquad /25/$$

Fluctuations due to a gamma-quanta conversion length distribution may be also taken into account as a problem of two known stochastic processes superposition. This problem is to be solved easily using a computer. The expression for an average value of $A(t)$, averaged upon longitudinal SEIL fluctuations, is given by the formulae /16/ and /15/.

It has been ascertained that the random variable $A(t)$ has approximately normal distribution, $A(t) \sim N(\bar{A}(t); \sigma_A^2)$, at the interval $\bar{A}(t) \gtrsim 0.5$ which is of most interest from practical point of view. The dependence of the relative dispersion $\sigma_A/\bar{A}(t)$ on $\bar{A}(t)$ is described by the relation

$$\sigma_A/\bar{A}(t) = \left(\frac{1}{b}\cdot \ln \frac{a}{\bar{A}(t)}\right)^{1/2} , \qquad /26/$$

which is valid within the limits $\bar{A} \in /0.1;\ 0.95/$ at the confidence level 0.3. Here

$$a = \alpha - \beta\cdot E_\gamma , \qquad /27/$$

and $\alpha = 1.02$, $\beta = 1.3\cdot 10^{-5} MeV^{-1}$, $b = 13.6 \pm 1.3$.
Therefore the value $E(t)$ is submitted to a normal distribution at $\bar{A}(t) \gtrsim 0.5$ also, so that

$$E(t) \sim N(\bar{E}(t);\ (\sigma_A \bar{E}(t))^2), \qquad /28/$$

where $\bar{E}(t)$ is determined by the expression /15/; σ_A, by the formulae /25/ and /16/.

The relation /28/ describes in average the distribution of the part $E(t)$ of the shower energy E_γ released as SEIL along the absorbent thickness t (measured from a gamma-quantum conversion point) within the shower energy $\bar{E}(t)$ is observed.

7. CONCLUSIONS

The results of investigation of shower electron ionization losses in electromagnetic cascades produced in liquid xenon by

gamma-quanta with the energy E_γ = 60 - 3000 MeV may be summarized as follows:

1. The distribution of average shower electron ionization losses along a cascade development axis is described by the formula /11/ together with /12/, /13/, and /14/. In particular if some part $E(t)$ of shower energy is registered in an absorbent of the thickness t, then the primary gamma-quantum energy E_γ is to be evaluated using the relation /15/. Such an estimation is the more reliable the greater is the energy E_γ and the closer to E_γ is the measured energy part $E(t)$, i.e., the higher is the thickness t of an absorbent.
2. Transversal shower dimensions may be evaluated using the formulae /22/, /18/-/21/. At $E_\gamma \gtrsim 700$ MeV ionization losses collimation near by a shower axis has been observed which diminishes with increasing the distance t from a primary gamma-quantum conversion point.
3. Longitudinal ionization losses fluctuations which are commonly an essential source of errors, when an energy of gamma-quanta generating showers is to be determined, describes the normal distribution /28/. A dependence on the length t and the energy E_γ of a dispersion of this distribution is approximated by the formulae /25/, /16/, and /15/.

The distributions quoted here may be used also in the case of others sufficiently dense absorbents as well as when not only ionization but other effects are registered (for example, Čerenkov radiation). Finally it follows to note that so far as the experimental material used in this work has been obtained by means of the detectors of dimensions limited (the length of the xenon bubble chamber of ITEP does not exceed about 25 rad. units) longitudinal shower dimension determined using our formulae may turn out to be a little shorter than the real one.

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Феноменологическое описание пространственного распределения ионизационных потерь в электронно-фотонных ливнях, вызванных гамма-квантами с энергией E_γ=60-3000 МэВ

Приведены результаты анализа экспериментально измеренных ионизационных потерь ливневых электронов в электронно-фотонных ливнях, вызванных гамма-квантами с энергией E_γ=60-3000 МэВ в жидком ксеноне. Получены формулы, описывающие продольное и поперечное распределение средних ионизационных потерь, а также флуктуации потерь электронов на ионизацию вдоль оси развития ливней. Экспериментальный материал получен на снимках с ксеноновых пузырьковых камер ОИЯИ и ИТЭФ /Москва/.

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Phenomenological Description of the Spatial Distribution of Ionization Losses in Electromagnetic Showers Produced by Gamma-Quanta with the Energy E_γ= 60-3000 MeV

The results of analysis of experimentally measured shower electron ionization losses in electromagnetic showers produced by gamma-quanta with the energy E_γ=60-3000 MeV in liquid xenon are presented. The formulae have been obtained describing longitudinal and transversal average ionization losses distributions as well as fluctuations of energy ionization losses of electrons along the shower development axes. The experimental material is obtained using pictures from xenon bubble chambers of JINR and ITEP (Moscow).

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR and at the Physical Institute of the Warsaw Technical University.

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