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ABOUT THE POSSIBILITY OF APPARENT MEASURING THE ELECTRIC AND MAGNETIC POLARIZABILITIES OF CHARGED PION

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1. Polarizability of the hadron is a fundamental constant, characterizing - together with the effective radius - its electromagnetic structure. In classical electrodynamics electric (α) and magnetic (β) polarizabilities of a particle define induced electric and magnetic dipole moments, respectively*

$$\vec{d} = \alpha\vec{E}, \quad \vec{\mu} = \beta\vec{H}. \tag{1}$$

In the quantum theory polarizabilities of the hadron are defined as parameters in correction to the Born term in low-energy expansion of the Compton-effect amplitude[1]. By now the proton's electric and magnetic polarizabilities ($\alpha_p = (10.7 \pm 1.1)\cdot 10^{-43}\,cm^3$, $\beta_p = (0.7 \pm 1.6)\cdot 10^{-43}\,cm^3$ [2,3]) and polarizability of the pion ($\alpha_\pi = (6.8 \pm 1.4)\cdot 10^{-43}\,cm^3$ [4] have been experimentally defined (under the assumption that the electric polarizability is equal to the magnetic one with the opposite sign). Limits for electric polarizabilities of the kaon ($|\alpha_k| < 200\cdot 10^{-43}\,cm^3$) and the neutron ($|\alpha_n| < 60\cdot 10^{-43}\,cm^3$) have also been obtained.

The polarizability of the pion was measured [4] in the reaction

$$\pi^- + A \to \pi^- + \gamma + A. \tag{2}$$

Reaction (2) was investigated in the region of small momentum transfer to the nucleus, corresponding to the Coulomb-scattering (see Fig.1). In this experiment the energy of the initial pion was 40 GeV. While studying process (2), information about the Compton-effect on the π-meson was obtained, because four-momentum squared of the virtual photon in the experiment was less than $6\cdot 10^{-4}$ $(GeV/c)^2$, that is much smaller than the pion mass squared. Such an experiment was proposed in the paper [7]. Data of the experiment [4] have been analysed under the assumption that $\alpha_\pi + \beta_\pi = 0$**.

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*Here and further the Gauss system is used $\alpha = e^2 = 1/137$.

**In papers [8,9,10] it was shown that $\alpha_\pi + \beta_\pi = 0$ with an accuracy $10^{-43}\,cm^3$ ([8] - current algebra predictions, [9] - calculations in the chiral model, [10] - predictions of the non-local quark model). In ref. [11] in the framework of dispersion model it has been obtained that $(\alpha_\pi + \beta_\pi) = (0.35 \pm 0.05)\cdot 10^{-43}\,cm^3$, see also [16] $(\alpha_\pi + \beta_\pi) = 0.2.10^{-43}\,cm^3$.

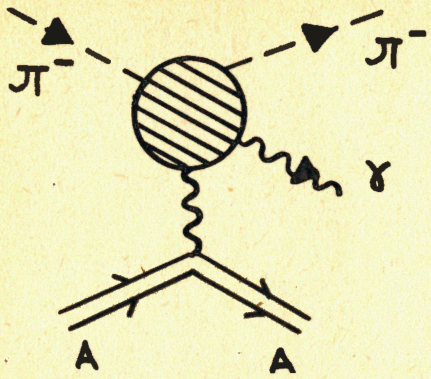

Fig.1. Diagram of the radiative scattering of the pion in the Coulomb field of the nucleus.

The aim of the present paper is to consider the possibility of measuring the magnetic polarizability together with the sum of electric and magnetic polarizabilities of the charged pion in radiative scattering of the pion in the Coulomb field of the nucleus. Such an experiment would allow one to define the values of the pion electric and magnetic polarizabilities.

2. In this section we will obtain expressions for the differential cross section of the Compton-effect and for the cross section of radiative scattering in the Coulomb field of the nucleus, taking into account the electric and magnetic polarizabilities of the charged pion*.

Fig.2. Diagrams of the Compton-effect.

In general, four diagrams (see Fig.2) correspond to the Compton-effect amplitude. The first three diagrams describe the Compton effect for a point-like pion, the fourth one takes into account strong interactions.

A contribution of the last diagram to the amplitude can be written in the following form:

$$M_{\mu\nu}(p_1, k_1, k_2)\epsilon^{\mu}(k_1)\,\epsilon^{\nu}(k_2) . \qquad (3)$$

Here p_1 is the initial pion momentum, k_1, k_2 and $\epsilon(k_1), \epsilon(k_2)$ are momenta and polarizations of the initial and final photons. From the crossing-symmetry and the gauge invariance one obtains the following conditions

$$M_{\mu\nu}\, k_1^{\mu} = M_{\mu\nu}\, k_2^{\nu} = 0, \quad M_{\mu\nu}(p_1, k_1, k_2) = M_{\nu\mu}(p_1, -k_2, -k_1) . \qquad (4)$$

Taking into account this requirement, we obtain a general expression for the tensor $M_{\mu\nu}$ [8]:

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*Analogous calculations under the assumption that $\alpha_\pi + \beta_\pi = 0$ were made in paper [13].

$$M_{\mu\nu} = F_1(s,t)(k_1 k_2 g_{\mu\nu} - k_{2\mu} k_{1\nu}) +$$
$$+ F_2(s,t)((k_1 p_1)(k_2 p_1) g_{\mu\nu} + k_1 k_2 p_{1\mu} p_{1\nu} - k_2 p_1 k_{1\nu} p_{1\mu} - k_1 p_1 k_{2\mu} p_{1\nu}) , \qquad (5)$$

where F_1 and F_2 are functions of kinematical invariants $s = (p_1 + k_1)^2$, $t = -(k_1 - k_2)^2$. In the low-energy approximation in expression (5) one should replace functions F_1 and F_2 by their values at $s = m_\pi^2$, $t = 0$:

$$F_1(s,t) \to F_1(m_\pi^2, 0) , \quad F_2(s,t) \to F_2(m_\pi^2, 0) . \qquad (6)$$

To determine structure corrections, it is necessary to carry out measurements in the region where invariants $p_1 k_1$, $p_1 k_2$, $k_1 k_2$ are of order of m_π^2*. For the invariants much smaller than m_π^2, the structure corrections are too small; for such larger values of the invariants ($\sim 1\ \mathrm{GeV}^2$) the low-energy approximation is not valid. Further we will impose restriction $s, u < 10 m_\pi^2$ ($u = (p_1 + k_2)^2$) for all numerical calculations.

Taking into account (5) and (6), the total Compton-effect amplitude can be written in the following form:

$$T = 2i(4\pi a)\epsilon_{\mu}(k_1)\epsilon_{\nu}(k_2)\Big(g^{\mu\nu} - \frac{p_1^{\mu} p_2^{\nu}}{p_1 k_1} + \frac{p_1^{\nu} p_2^{\mu}}{p_1 k_2} +$$
$$+ F_1(m_\pi^2, 0)(g^{\mu\nu} k_1 k_2 - k_2^{\mu} k_1^{\nu}) + \qquad (7)$$
$$+ F_2(m_\pi, 0)(g^{\mu\nu}(k_1 p_1)(k_2 p_1) + k_1 k_2 p_1^{\mu} p_1^{\nu} - k_1^{\nu} p_1^{\mu} k_2 p_1 - k_2^{\mu} p_1^{\nu} k_1 p_1) .$$

Total correction to the Born-term in the amplitude corresponds to an additional term in the initial Hamiltonian. Its classical analogy has the following form

$$H_{eff} = -\Big(\frac{1}{2}\alpha_\pi \vec{E}^2 + \frac{1}{2}\beta_\pi \vec{H}^2\Big) , \qquad (8)$$

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*For process (2) this condition is fulfilled, because bremsstrahlung of the relativistic pion is concentrated at small angles $\theta = m_\pi / E_1$ (E_1 is the pion energy). Consider, e.g., the invariant $p_1 k_2$

$$p_1 k_2 = E_1 \omega_2 - p_1 \omega_2 \cos\theta =$$
$$= E_1 \omega_2 - E_1 \Big(1 - \frac{1}{2}\frac{m_\pi^2}{E_1^2} + \ldots\Big)\Big(1 - \frac{1}{2}\theta^2 + \ldots\Big)\omega_2 = \frac{\omega_2}{2}\Big(\frac{m_\pi^2}{E_1} + \theta^2 E_1\Big) \simeq m_\pi^2 .$$

where

$$a_\pi = \frac{(F_1(m_\pi^2,0) - F_2(m_\pi^2,0))}{m_\pi e^2}, \quad \beta_\pi = \frac{F_2(m_\pi^2,0)}{m_\pi e^2}.$$

So a_π and β_π can be interpreted as electric and magnetic polarizabilities of the charged pion.

The Compton-effect amplitude is given by the expression:

$$d\sigma_{C.eff.} = \frac{1}{2}\sum_{pol}|T| \frac{\delta^4(p_1+k_1-p_2-k_2)}{4\omega_1' m_\pi} \frac{d\vec{p}_2' d\vec{k}_2'}{2E_2' 2\omega_2'}. \quad (9)$$

(Primed variables refer to the pion rest frame, unprimed variables refer to the laboratory frame).

From expressions (7) and (9), taking into account the Compton equation

$$\frac{1}{\omega_2'} - \frac{1}{\omega_1'} = \frac{1}{m_\pi}(1-\cos\theta'), \quad (10)$$

(where θ' is the angle of photon scattering), we obtain

$$\frac{d\sigma_{C.eff}(\omega_1',\omega_2')}{d\omega_2'} = \frac{\pi a^2}{m_\pi}\frac{1}{\omega'^2}\{(2+(\frac{m_\pi}{\omega_2'}-\frac{m_\pi}{\omega_1'})^2 - 2(\frac{m_\pi}{\omega_2'}-\frac{m_\pi}{\omega_1'})) \times$$
$$\times (1 - \frac{2(a_\pi+\beta_\pi)}{a} m_\pi\omega_1'\omega_2') + \frac{2\beta_\pi m_\pi}{a}(\frac{m_\pi}{\omega_2'} - \frac{m_\pi}{\omega_1'})^2 \omega_1'\omega_2'\}. \quad (11)$$

The cross section of bremsstrahlung which is differential over the photon energy and over 4-momentum transferred to nucleus is connected with the Compton-effect cross section by the following equations

$$d\sigma_{Brem}(\omega_2, k_{1\perp}) = \int d\sigma_{C.eff}(\omega_2', \omega_1') n(\omega_1', k_{1\perp}) d\omega_1' \; dk_{1\perp}^2, \quad (12)$$

where ω_2 is the energy of the emitted photon and $k_{1\perp} = |\vec{k}_{1\perp}|$ is the momentum transferred to the nucleus in the laboratory frame, $n(\omega, k_\perp) = \frac{Z^2 a k_\perp^2}{\pi\omega(k_\perp^2+\omega^2 m_\pi^2/E_1^2)^2}$ is the density of equivalent photons. We have also the relations $\omega_2' = \omega_1'(E_2/E_1)$ and $d\omega_2' = (\omega_1'/E_1)\cdot d\omega_2$. The integration limits can be obtained from the equation[14]

$$k_1^2 = q^2 = k_{1\perp}^2 + \frac{\omega_1'^2 m_\pi^2}{E_1^2}. \quad (13)$$

The minimal value of q^2 is defined by the threshold of reaction (2) with the fixed energy of the emitted photon $q^2_{min} = (m_\pi^2\omega_2/2E_1E_2)^2$. Let us choose $q^2_{max} = 6\cdot10^{-4}$ (GeV/c)^2. This value is given by a really achieved accuracy in momentum transfer measurement and ensures dominance of the electromagnetic interaction over the strong one. Calculating integral (12) and making simple transformations, we obtain the equation:

$$\frac{d\sigma_{Brem}}{dxdT} = \frac{8\pi Z^2}{m_\pi^2}\frac{\sqrt{y-1}}{y^2}\frac{x}{1-x}(\ln\frac{A^2}{y^2} - 1 + \frac{y^2}{A^2}) \times$$
$$\times\{(1-\frac{2}{y}+\frac{2}{y^2})(1-\frac{1}{2}\frac{(a_\pi+\beta_\pi)}{a}m_\pi^3 y^2\frac{x^2}{1-x}) + \frac{\beta_\pi m_\pi^3}{a}\frac{x^2}{1-x}\}. \quad (14)$$

This turns out to be useful in analysing the experimental data. Here $T = \theta(E_1/m_\pi)$, where $\theta = (\vec{k}_2, \vec{p}_1)$, m_π/E_1 is a characteristic angle of bremsstrahlung, $y = 1+T^2$, $x = \omega_2/E_1$, Z is the electric charge of the nucleus, $A = q_{max}/q_{min}$.

3. Let us rewrite expression (14) in the following form

$$\frac{d\sigma_{Brem}}{dxdT} = f(x,T) = f_{Born} - (a_\pi+\beta_\pi)f_{POL1} + \beta_\pi f_{POL2}. \quad (15)$$

From expression (14) it is seen that the correction to the Born-term given by $(a_\pi+\beta_\pi)f_{POL1}$ grows strongly (as $\sim T^4$) with T. It follows from expression (11) that in the pion rest frame the main contribution to the correction is made by Compton events with forward photons. Such events correspond to photons scattered at large angles in the laboratory frame. From expression (14) it is also seen that a relative contribution of $(a_\pi+\beta_\pi)f_{POL1}$ to the cross section depends weakly on the energy of the photon emitted. Fig.4 shows the T-dependence for the relative contribution of the $(a_\pi+\beta_\pi)f_{POL1}$ term (integrated over x)*.

To determine the parameter $(a_\pi+\beta_\pi)$ one should investigate the T-distribution of experimental data. From Fig.4 it is seen that the value of the normalization factor can be obtained from the $0 < T < 1$ region, because the cross section depends very weakly on the value of $(a_\pi+\beta_\pi)$ in this interval. Note that the term $\beta_\pi f_{POL2}$ does not contribute to the T-distribution. However, this term makes an essential contribution to the x-distribution[4,7].

* Kinematic variables vary in the region defined by conditions $u,s < 10m_\pi^2$. This region is shown in Fig.3.

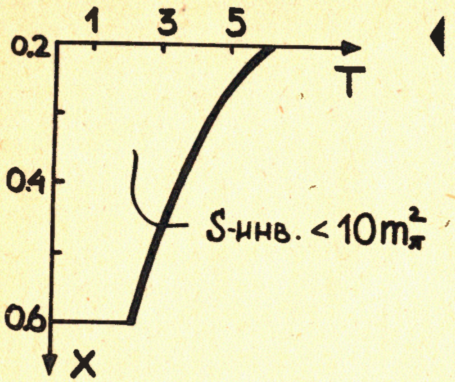


Fig.3. The region of the variation of x and T variables.

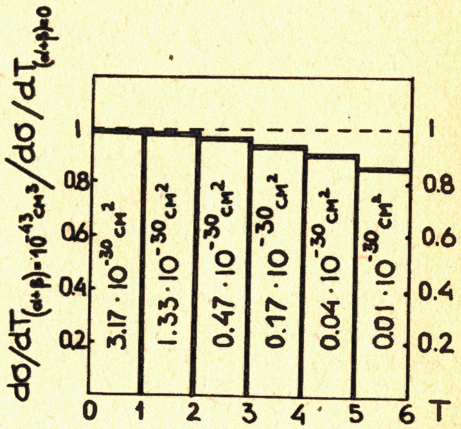


Fig.4. Cross section of the radiative scattering vs the emitted photon angle (under the assumption that $\alpha_\pi + \beta_\pi$ = $= 10^{-43}cm^3$).

Now we define the optimal kinematic region for the experiment considered and estimate the statistical accuracy of the parameter determination. The accuracy of the parameter $\alpha_\pi + \beta_\pi$ -determination can be written in the following form (the detection efficiency is neglected)

$$\Delta(\alpha_\pi + \beta_\pi) = \frac{1}{\sqrt{B}} \left(\int_{T_{min}}^{T_{max}} I(T)\, dT \right)^{-1/2} . \qquad (16)$$

Here we used the function of informativity I(T) *, characterizing the dependence of the accuracy on the measured variables, B is the integral luminocity. If f(x), the differential cross section of the process, depends on parameter ξ, than informativity is defined by

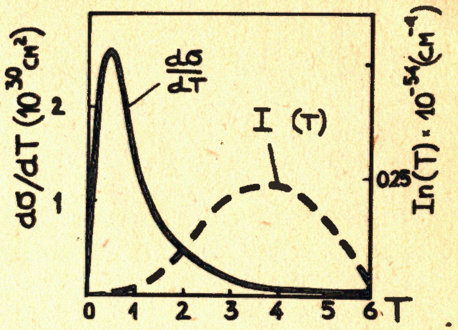
$$I_\xi(T) = \left(\frac{\partial f(x)}{\partial \xi}\right)^2 / f(x) . \qquad (17)$$

From expressions (15), (17), taking into account the fact that terms with the polarizabilities are small corrections to the Born term, we obtain

$$I(T) = \frac{f^2_{POL1}(T)}{f_{Born}} . \qquad (18)$$

*This method of the accuracy evaluation will be described in the special paper.

Fig.5. The function of informativity for $\alpha_\pi + \beta_\pi$ and the cross section of the process (2) on carbon nucleus vs the emitted photon angle.



The best accuracy can be achieved if the experiment is carried out in a region of maximal informativity. Calculations have shown that the following interval is optimal for $(\alpha_\pi + \beta_\pi)$ measurement:

$$0.2 \le x \le 0.6 . \qquad (19)$$

Fig.5. presents the dependence of informativity I(T) on the T -variable (the spectrum is integrated over x from 0.2 to 0.6). From Fig.5 it is seen that the optimal interval to determine $(\alpha_\pi + \beta_\pi)$ is

$$0 < T \le 5 . \qquad (20)$$

Note, that for determination of the parameter β_π the optimal region is /4,7/

$$0.5 \le x \le 0.9, \quad 0 < T \le 2 . \qquad (21)$$

So one should make the experiment measurement β_π together with $(\alpha_\pi + \beta_\pi)$ in the kinematical region (19), (20), (21). From (16), (14), (19), (20) assuming that the detection efficiency equals one in the whole region of interest one expects the following accuracy for $(\alpha_\pi + \beta_\pi)$*

$$\Delta(\alpha_\pi + \beta_\pi) \simeq 0.1 \cdot 10^{-43} cm^3 . \qquad (22)$$

The real accuracy of $(\alpha_\pi + \beta_\pi)$ determination will be somewhat worse (about 1.5-2 times). This is because in (16) we did not take into account the fact that experimental data are fitted with two parameters. The real detection efficiency also worsens the accuracy.

To measure parameter $(\alpha_\pi + \beta_\pi)$ in process (2) it is necessary to ensure high efficiency of the experimental set-up for detection of the final pion and photon in the region of kinematic variables (19,20). For this purpose the experimental setup co-

*Calculations were made for the carbon nucleus Z = 6, with a reasonable value of integral luminocity $B = 10^{34} cm^{-2}$.

vers the angular interval 2.5-3 times larger than in the experiment [4]. It is more useful to carry out the discussed experiment with larger energies (100-200 GeV), because angular sizes of detectors are decreasing with energy. For example, we estimate detection efficiency for process (2) of "FRAMM" setup in geometry of the experiment for investigations of the reaction $\pi^- + A \to \pi^- + \pi^0 + A$ with the pion beam energy 200 GeV [16]. Calculations show that detection efficiency is high enough and one expects the accuracy for the parameter $(\alpha_\pi + \beta_\pi)$ (integral luminocity 10^{34} cm^{-2}) $\Delta(\alpha_\pi + \beta_\pi) \simeq 0.2 \cdot 10^{-43}$ cm^3. At the end we note that during investigations of the process (2) it is necessary to suppress the background from beam K^--mesons decays $K^- \to \pi\, \pi^0$ with sharp asymmetric decay of $\pi^0 \to 2\gamma$.

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О возможности раздельного определения электрической и магнитной поляризуемостей заряженного пиона

Рассмотрена возможность экспериментального определения как магнитной поляризуемости, так и суммы электрической и магнитной поляризуемостей заряженного пиона в реакции радиационного рассеяния пиона в кулоновском поле ядра. Такой эксперимент позволит раздельно определить электрическую и магнитную поляризуемости заряженного пиона.

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About the Possibility of Apparent Measuring the Electric and Magnetic Polarizabilities of Charged Pion

The possibility of measuring the magnetic polarizability together with the sum of electric and magnetic polarizabilities of the charged pion in radiative scattering of the pion in the Coulomb field of the nucleus is considered. Such an experiment would allow one to determine the electric and magnetic polarizabilities of the pion independently.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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