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CYCLE-BY-CYCLE ANALYSIS OF MUON CATALYSED FUSION IN A ONE-COMPONENT MEDIUM

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1. INTRODUCTION

The arrival of the experimental results [1] confirming the theoretical predictions [2] of the resonance character of $d\mu d$- and $d\mu t$ -molecule formation has given rise to considerable growth of interest in the investigation of the muon catalysis of nuclear synthesis of hydrogen isotopes. One of the reasons behind the renewed excitement in this field is the emerging possibility of practical application of muon catalysis in energy production. A crucial point in this connection is understanding the kinetics of the processes leading to the muon catalysed synthesis.

In ref. [3] general formulae describing the time distributions of fusion events have been obtained for the deuterium-tritium mixture by solving the "all-cycles" (AC) kinetic equations. In this approach the appropriate time distributions are sums corresponding to all cycles initiated consecutively by a single muon as it reenters the chain of processes after being released when nuclear synthesis in the μ -molecule takes place. In ref. [4] it has been argued that investigating the muon catalysis in one-component media (pure deuterium or tritium) is particularly useful as it can provide necessary input information for the analysis of the more complicated two-component (deuterium-tritium) case. In particular the corresponding AC-solution for the time distribution of neutrons produced in the $(tt)\mu \to \to (^4He2n)\mu$ muon catalysed fusion has been analysed in detail with the aim of establishing the experimental conditions for determining the parameters of the $(tt)\mu$ fusion chain. The conclusions of ref. [4] apply also to fusion in pure deuterium. Although it has been shown that the AC-approach is sufficient for the determination of the sought for parameters, it is interesting, both from the theoretical and experimental point of view, to have a picture of the kinetics in which the successive cycles in the chain are described separately.

The first attempt at the cycle-by-cycle description of the muon catalysis chain has been made in ref. [5] in an effort to avoid outside evaluation of registration efficiency in the analysis of the data. The method proposed by the authors is based on measuring the time distribution of the first registered cycle and the total yields of the first two registered cycles which are attributed to one muon entering the target.

In the present paper we derive for a one-component medium the formulae describing the time distributions of fusion events ending an arbitrary k-th cycle and the corresponding fusion-event yields per one muon.

First, real physical cycles in the chain are considered and then the obtained formulae are applied to cycles actually registered in the experiment (registration efficiency $\epsilon < 1$). The ensuing experimental implications are briefly discussed.

2. REAL CYCLES

The processes forming the muon-catalysis chain are presented in fig.1.

The (μ)-nodes denote the muon; (μa)-nodes, the μ-atoms ($d\mu$ or $t\mu$); (μm) ones, the μ-molecules ($d\mu d$ or $t\mu t$); and [s], the consecutive acts of nuclear synthesis. The first (μ)-node can be considered as a beam muon entering the target, the other ones represent muons released from the μ-molecules with probability $(1-\omega)$ when fusion takes place. λ_a and λ_m are the rates of μ-atom and μ-molecule formation, respectively; and λ_f is fusion rate in the μ-molecule.

Instead of writing down the differential equations for separate cycles we shall proceed straightforward to construct the corresponding time-distributions of fusion events. Since $\lambda_a \gg \lambda_m$ /6/ we shall neglect the time gaps between (μ) and (μa) and treat them as one object. According to the notation of fig.1, we shall increase the number of cycles by adding one cycle at a time at the beginning of the chain. The probability for a μ-atom which appears at time t_i to form a μ-molecule in dt'_i at t'_i is then

$$dP_m = \lambda_m \exp[-(\lambda_0 + \lambda_m)(t'_i - t_i)]\, dt'_i \, , \qquad (1)$$

where λ_0 is the muon decay rate ($\lambda_0 = 0.455 \cdot 10^6\ s^{-1}$). Analogously, the probability for nuclear synthesis to take place at t_{i-1} in the μ-molecule created at t'_i is

$$dP_f = \lambda_f \exp[-(\lambda_0 + \lambda_f)(t_{i-1} - t'_i)]\, dt_{i-1} \, . \qquad (2)$$

It is easy now to write down the time distributions of fusion events ending the consecutive cycles.

$$F_1(t,t_1) = \frac{dN_1^{[s]}}{dt} = \int_{t_1}^{t} dt'_1\, \lambda_m e^{-\Lambda_m (t'_1 - t_1)} \lambda_f e^{-\Lambda_f (t - t'_1)} \, , \qquad (3.1)$$

$$F_2(t,t_2) = \int_{t_2}^{t} dt'_2\, \lambda_m e^{-\Lambda_m (t'_2 - t_2)} \int_{t'_2}^{t} dt_1\, \lambda_f e^{-\Lambda_f (t_1 - t'_2)} (1-\omega) F_1(t,t_1) , \qquad (3.2)$$

Higher registered cycles can be now generated using eq. (11) or (12). Extending eq. (11) to the description of the registered cycles is justified by its natural physical interpretation of adding one more cycle (or registered cycle) with probability $(1-\omega)$ at the end of the $k-1$ preceding cycles. Since eq.(11) is equivalent to eq. (5) it is seen that $\bar{F}_k(t)$ can be constructed from $\bar{F}'_0(t,t_0) = \delta(t - t_0)$ using the same procedure as before if the following replacements are made

$$\lambda_f \lambda_m \longrightarrow \epsilon \lambda_f \lambda_m , \quad (\Lambda_f , \Lambda_m) \longrightarrow (S_+, S_-), \quad (1-\omega) \longrightarrow (1-\omega). \qquad (28)$$

Therefore, the formulae obtained before for real cycles hold also for the registered cycles under replacements (28). In particular the total yield per one muon of the registered cycles attributed to a single muon is now

$$\bar{Y}_k = (1-\omega)^{k-1} \left[\frac{\epsilon \lambda_f \lambda_m}{\Lambda_f \Lambda_m - \lambda_f \lambda_m (1-\omega)(1-\epsilon)} \right]^k . \qquad (29)$$

Analogously to eqs. (13), (14) and (22) one can check that

$$\bar{F}(t) = \sum_{k=1}^{\infty} \bar{F}_k(t) = \epsilon F^{(AC)}(t) \qquad (30)$$

and

$$\bar{Y}_{tot} = \sum_{k=1}^{\infty} \bar{Y}_k = \epsilon Y^{(AC)} . \qquad (31)$$

Figs. 2b-c show the contributions of separate registered cycles to the "all-cycles" registered time-distribution $\epsilon F^{(AC)}(t)$. It is seen that the slope of $\bar{F}_k(t)$ at large t decreases with decreasing ϵ. For $\lambda_m \gtrsim 10^6\ s^{-1}$ and small ϵ this may render the method proposed in ref./5/ difficult for practical application due to uncertainities in extrapolating the measured total cycle yields $\bar{Y}_1$ and $\bar{Y}_2$ to infinity. On the other hand, the formulae derived in this paper enable one to obtain parameters λ_m, λ_f and ω from direct fits of the data in a finite time-interval for any number of registered cycles. If such procedure is adopted, the conclusions of ref./4/ remain valid and apply to each cycle multiplicity separately. Hence, all these parameters can be obtained by measuring the time-distribution corresponding to a k-th cycle at two target densities. However, the simultaneous analysis of several registered cycles may provide a useful test for the assumptions made in constructing the kinetic equations. Again, fusion in one-component medium suits better for this purpose than fusion in the $D_2 - T_2$ mixture.

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Анализ последовательных циклов мюонного катализа ядерных реакций синтеза в однокомпонентной среде

В работе получены и обсуждены формулы, описывающие последовательные циклы мюонного катализа ядерных реакций синтеза изотопов водорода в однокомпонентной среде /чистый дейтерий либо тритий/.

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Cycle-by-Cycle Analysis of Muon Catalysed Fusion in a One-Component Medium

The formulae decribing the separate cycles of muon catalysed nuclear synthesis of hydrogen isotopes in a one-component medium (pure deuterium or tritium) are derived and discussed.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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