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дубна

E1-83-854

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DETERMINATION OF THE MULTIPLE MUON CATALYSIS PROCESS PARAMETERS

Submitted to "Атомная энергия"

1983

A characteristic feature of the muon catalysis process of the nuclear fusion reaction is that the same muon can cause a series of such reactions/1,2/. It allows us to suggest a full cycle of the measurements of the parameters characterizing the kinetics of the muon catalysis process.

The theoretical study of the muon catalysis kinetics has been made in ref./3/, where the expressions for the yield and time distribution of the fusion reaction products have been obtained. So, the time distribution of the acts of reaction

$$t\mu + t \to tt\mu \to {}^4He + 2n + \mu \qquad (1)$$

has the form*

$$dn/dt = \lambda_{tt\mu} \exp[-(\lambda_0 + \omega_{tt}\lambda_{tt\mu})t] \qquad (2)$$

and their mean multiplicity is

$$\bar{n} = \lambda_{tt\mu} / (\lambda_0 + \omega_{tt}\lambda_{tt\mu}). \qquad (3)$$

Expressions (2-3) are normalized to the number of muons which case reaction (1) and they were derived under the condition/4/ that the fusion reaction rate $\lambda_f^{tt} \gg \lambda_0$, $\lambda_{tt\mu}$.

In real experimental conditions the fusion reaction products are detected with an efficiency $\epsilon < 1$. It can be easy shown that the time distribution of all (without any selection) detected acts of reaction (1) is

$$\frac{dn^{exp}}{dt} = \epsilon\lambda_{tt\mu} \exp[-(\lambda_0 + \omega_{tt}\lambda_{tt\mu})t] \qquad (4)$$

and their yield (experimental multiplicity) is

$$\bar{n}^{exp} = \epsilon\lambda_{tt\mu}/(\lambda_0 + \omega_{tt}\lambda_{tt\mu}). \qquad (5)$$

As is seen from formulae (4)-(5) their use in the experimental data analysis gives the possibility of finding only $\omega_{tt}\lambda_{tt\mu}$ and $\epsilon\lambda_{tt\mu}$, thus, to obtain independently the values of $\lambda_{tt\mu}$ and ω_{tt}, we have to know the detection efficiency ϵ. The determination of the efficiency is quite difficult. Usually the efficiency is calculated by the Monte-Carlo method taking into account the parameters of the investigated process and geometry of the experimental set-up. For reaction (1) such calculations are additionally complicated because of the unambiguous information about the neutron energy distribution.

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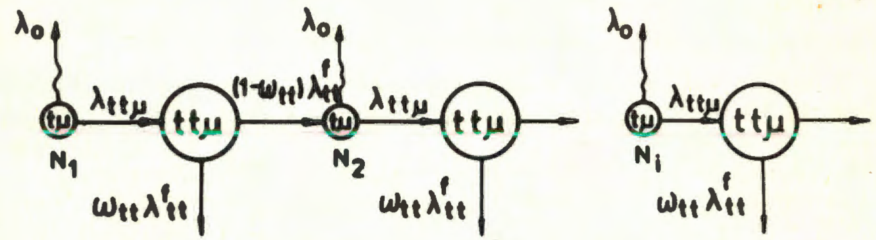
*We use standard notations: $\lambda_0 = 4.55\cdot10^5 s^{-1}$ - the free muon decay rate, $\lambda_{tt\mu}$ - the $tt\mu$ -molecule formation rate, ω_{tt} - the probability of muon sticking to ^4He nucleus.

It should be mentioned out, however, that the yield and time distribution of all reaction acts do not give the full information about the process of successive muon catalysis. We can additionally measure the time distributions of the first, second, etc., detected acts and their yields or the yields of single, double, etc., detected acts. As is shown below, if such additional information is used, it is not necessary to know the detection efficiency apriori. We shall show that the measurements of the yield and time distribution of the first detected acts and the yield of the second detected acts would be enough to independently determine the values of ω_{tt} and $\lambda_{tt\mu}$ (also the efficiency ϵ).



The scheme of the successive muon catalysis of reaction (1) is shown in the figure. The muon released in (1) forms a $t\mu$ -atom and then a $tt\mu$ -molecule. We denote the number of $t\mu$ -atoms conserved to the i-th act of the reaction as N_i. The functions of $N_i(t)$ can be obtained by solving the set of equations:

$$dN_1/dt = -\lambda N_1,$$

$$dN_2/dt = -\lambda N_2 + (1-\omega_{tt})\lambda_{tt\mu} N_1,$$

. . . .

$$dN_i/dt = -\lambda N_i + (1-\omega_{tt})\lambda_{tt\mu} N_{i-1},$$

where we denote $\lambda \equiv \lambda_0 + \lambda_{tt\mu}$.

The solution of this system for the boundary condition $N_1(0) = 1$ has the form

$$N_i(t) = [(1-\omega_{tt})\lambda_{t t\mu}]^{i-1} t^{i-1} [\exp(-\lambda t)]/(i-1)!$$

Since the fusion reaction rate $\lambda_f^{tt} \gg \lambda_0, \lambda_{tt\mu}$ the neutron (reaction products) time distribution from the i-th act of the reaction is

$$f_i(t) = dn_i/dt = \lambda_{tt\mu} N_i(t) = \lambda^i_{tt\mu} (1-\omega_{tt})^{i-1} t^{i-1} e^{-\lambda t}/(i-1)! \tag{6}$$

and the neutron yield from the i-th act

$$n_i = \int_0^\infty f_i(t)\, dt = (1-\omega_{tt})^{i-1} (\lambda_{tt\mu}/\lambda)^i . \tag{7}$$

From expression (7) one can obtain the formula for the yield of single, etc., reactions:

$$n(i) = n_{i+1} - n_i = (1-\omega_{tt})^{i-1} \lambda^i_{tt\mu} (\lambda_0 + \omega_{tt}\lambda_{tt\mu})/\lambda^{i+1} . \tag{8}$$

It is easy to show that the time distribution of all acts $n(t) = \Sigma n_i(t)$ has the form of (2) and the yield of all acts $\bar{n} = \Sigma n_i = \int_0^\infty n(t)\, dt$ has the form of (1).

Now we shall take into consideration the detection efficiency ϵ. Since $\epsilon < 1$, the possibility arises that the neutron from the i-th act is the first to be detected (if neutrons from preceding $i-1$ acts were not detected). Thus the time distribution of the first detected acts can be presented in the form

$$f_1^{exp}(t) = \epsilon f_1(t) + (1-\epsilon)\{\epsilon f_2(t) + (1-\epsilon)[\epsilon f_3(t) + \ldots]\} =$$
$$= \epsilon[f_1(t) + (1-\epsilon) f_2(t) + (1-\epsilon)^2 f_3(t) + \ldots] = \epsilon \sum_{i=1}^{\infty} (1-\epsilon)^{i-1} f_i(t) . \tag{9}$$

Substituting expression (6) for $f_i(t)$ in (9) we obtain

$$f_1^{exp}(t) = \epsilon\lambda_{tt\mu} \exp\{-[\lambda_0 + (\epsilon + \omega_{tt} - \epsilon\omega_{tt})\lambda_{tt\mu}]t\} . \tag{10}$$

The yield of the first detected acts

$$n_1^{exp} = \int_0^\infty f_1^{exp}(t)\, dt = \epsilon\lambda_{tt\mu} / [\lambda_0 + \lambda_{tt\mu} (\epsilon + \omega_{tt} - \epsilon\omega_{tt})] . \tag{11}$$

The same formula can be obtained after substituting expression (7) for n_i in (9).

When deriving the formula the yield of the second detected acts one takes into account the possibility of detecting the next act pairs:

(1, 2), (1, 3), (1, 4) ... (1, i); (2,3), ... (2, i); ...

Thus the yield of the second detected acts is

$$n^{exp} = \epsilon^2 n_2 + \epsilon^2 (1-\epsilon) n_3 + \ldots$$
$$= \epsilon^2 [n_2 + 2(1-\epsilon) n_3 + 3(1-\epsilon)^3 n_4 + \ldots + (i-1)(i-\epsilon)^{i-2} n_i + \ldots] =$$
$$= \epsilon^2 (1-\omega_{tt}) \left(\frac{\lambda_{tt\mu}}{\lambda}\right)^2 \sum_{i=2}^{\infty} [(1-\epsilon)(1-\omega_{tt})\lambda_{tt\mu}/\lambda]^{i-2} (i-1) = \tag{12}$$
$$= \epsilon^2 \lambda^2_{tt\mu} (1-\omega_{tt}) / [\lambda_0 + (\epsilon + \omega_{tt} - \epsilon\omega_{tt})\lambda_{tt\mu}]^2 .$$

It should be pointed out that formula (12) can be obtained by using expression (8) for $i = 1$:

$$n_2^{exp} - n_1^{exp} = n^{exp}(1) , \tag{13}$$

where $n^{exp}(1)$ is the yield of the single detected events. Indeed, the expression for the yield of the m detected events may be presented in the form

$$n^{exp}(m) = \Sigma\, n(i)\, P_i^m , \tag{14}$$

where P_i^m is the binomial probability to detect m events from i:

$$P_i^m = C_i^m \epsilon^m (1-\epsilon)^{i-m} . \tag{15}$$

After substituting formulae (11), (17) to (13) we obtain expression (12).

The use of the expressions (10-12,16) is enough to independently determine the value of ω_{tt} and $\lambda_{tt\mu}$. We are of the opinion that the proper algorithm would be as follows:

1. The value of ω_{tt} is determined by using the experimental values of n_1^{exp}, n_2^{exp} and the formula $(1-\omega_{tt}) = n_2^{exp}/(n_1^{exp})^2$.
2. The value of $a \equiv \lambda_0 + (\omega_{tt} + \epsilon - \epsilon\omega_{tt})$ is obtained from the time distribution of the first detected events.
3. The value of $b \equiv \lambda_0 + \omega_t + \lambda_{tt\mu}$ is found from formula $n_1^{exp}(\lambda_0 + \omega_{tt}\lambda_{tt\mu}) - n_1^{exp} - n_2^{exp}$.
4. Substituting the value of ω_{tt} to the expression for b we obtain the value of $\lambda_{tt\mu}$.

Thus, the values of ω_{tt} and $\lambda_{tt\mu}$ can be found without knowing the detection efficiency. It is obvious, that the efficiency itself can be determined from the experimental data if one uses the known values of ω_{tt} and $\lambda_{tt\mu}$, and it is interesting by itself. The comparison, of the experimental value of ϵ with the results of calculations in which different assumptions on the character of the neutron energy distribution are used would give the possibility of obtaining the information about the (n, n) and (n, α) interactions in the final state of reaction (1).

It should be pointed out that the necessary condition for correct determination of the values of ω_{tt} and $\lambda_{tt\mu}$ is as follows: the neutron energy threshold must be less than one half of the minimum possible summary energy of two neutrons.

Expressions (10-12), (16) can be easily summarized to the case of muon catalysis in deuterium. The common remark is that the effective use of these formulae in the experimental data analysis is possible only when the experimental multiplicity $\bar{n}^{exp} \geq 1$, or for tritium when $\epsilon\lambda_{tt\mu}/\lambda_0 \geq 1$ and for deuterium when $\epsilon\lambda_{dd\mu}/\lambda_0 \geq 1$. For the muon catalysis of reaction (1) this condition is satisfied in liquid tritium or in gaseous tritium at $P \gtrsim 100$ atm.

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Received by Publishing Department
on December 12,1983.

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Методика определения параметров процесса множественного мюонного катализа

Рассмотрен процесс множественного мюонного катализа реакции $t+t \to {}^4He+2n$ и намечен полный цикл измерений констант, характеризующих кинетику процесса. Получены выражения для выхода первых, вторых и т.д. зарегистрированных актов последовательного мюонного катализа реакции и временное распределение для первых зарегистрированных актов этой реакции. Использование этих выражений при обработке экспериментальных данных позволяет обойтись без привлечения эффективности регистрации исследуемой реакции, что существенно повышает точность и надежность определения констант мюонного катализа. Полученные результаты легко обобщаются и для катализа реакции синтеза $d+d$ для условий, когда средняя множественность этой реакции $M \gtrsim 1$.

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1983

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Determination of the Multiple Muon Catalysis Process Parameters

The process of successive multiple muon catalysis of the $t+t \to {}^4He+2n$ reaction is studied. In order to determine the constants characterizing the kinetics of this process the full cycle of the measurements is suggested. The expressions for yield of the first, second, etc., detected acts of successive muon catalysis are derived. The use of these expressions makes it possible to exclude the detection efficiency of the studied reaction from the analysis of experimental data and to improve the accuracy and confidence in determining the muon catalysis constants. The obtained results may be applied to muon catalysis of $d+d$ reaction of the mean multiplicity of this reaction $M \gtrsim 1$.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1983