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APPLICATION OF THE MOLIERE MULTIPLE SCATTERING ANGLE-LATERAL DISPLACEMENT FUNCTION IN MONTE-CARLO CALCULATIONS

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1. INTRODUCTION

Several multiple scattering theories [1,2,3] are concerned with the angular distribution of charged particles passing through a thin target. They are based on two general assumptions: small angle approximation and negligible energy losses. As far as a thick target is concerned, the above assumptions are not fulfilled, so the target cannot be approached as a whole, but it has to be divided into sufficiently small steps. In this case, to follow the path of charged particles through the material, taking into account energy losses and scattering processes, the multiple scattering angle-lateral displacement correlation formula has to be used. From the initial position and direction of the particle at the beginning of each step this formula obtains the probability distribution in position and direction at the end of the step. In homogeneous media this distribution is factorized into a product of two equal two-dimensional distributions with respect to two orthogonal directions (x and y) which, in turn, are orthogonal to the initial direction assumed to coincide with the z-axis. In Monte-Carlo calculations so far these two-dimensional probability distributions have been given approximately by the well-known Gaussian multiple scattering correlation function. Experiments, however, show a substantial discrepancy between Gaussian and experimental angular distributions, particularly for large angles. The Moliere multiple scattering correlation formula [4] has been confirmed by experimental results [3,5,6] (as far as the angular distribution is concerned). However, this formula is not suitable for Monte-Carlo calculations. The purpose of this paper is to find a two-dimensional approximation for the multiple scattering correlation formula which satisfies two conditions: first, to be close to the Moliere one (better than 10^{-3}) and, second, to be acceptable for Monte-Carlo calculations.

2. BASIC EQUATIONS

Contemplating the x, z plane for step length ℓ, we call the angle of the particle velocity with respect to the z-axis at the end of the step ϕ' (the initial particle direction has been assumed to coincide with the z-axis) and its displacement from the z-axis at the end of the step ψ'. Figure 1 shows the scattering geometry in plane x, z.

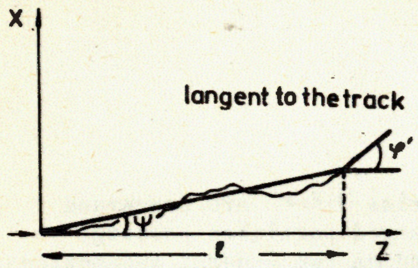

Fig.1. Multiple scattering geometry.

The Moliere angle-lateral displacement correlation formula is given by

$$FM(\phi',\psi') = \frac{1}{(2\pi\Phi_0)^2} \int_{-\eta_1^{max}}^{\eta_1^{max}} \int_{-\eta_2^{max}}^{\eta_2^{max}} d\eta_1 d\eta_2 \cos\left(\frac{\phi'\eta_1+\psi'\eta_2}{\Phi_0}\right)$$

$$\times \exp\left\{-\frac{1}{4}\left(\eta_1^2+\eta_1\eta_2+\frac{\eta_2^2}{3}\right)+\frac{1}{12B}\left[\frac{(\eta_1+\eta_2)^3}{\eta_2}\ln\frac{(\eta_1+\eta_2)^2}{4e^{2/3}} - \frac{\eta_1^3}{\eta_2}\ln\frac{\eta_1^2}{4e^{2/3}}\right]\right\}, \quad (1)$$

where

$$B - \ln B = b - 0.1544, \quad (2)$$

$$b = \ln\frac{8831\ell S z^2}{A\beta^2\left(1.13+3.76\left(\frac{Zz}{137\beta}\right)^2\right)}, \quad (3)$$

$$\Phi_0 = \chi_c\sqrt{B}, \quad (4)$$

$$\chi_c^2 = 0.1572 S_1 z^2\ell/[A(p\beta)^2], \quad (5)$$

in which Z is atomic number of the scattering foil; A, atomic weight of the foil material; ℓ, step length (g/cm^2); z, charge of incident particle (in units of e); p, momentum of incident particle (MeV/c);

$\beta = v/c$ for the incident particle,

$$S = \begin{cases} (Z+1)\cdot Z^{1/3} & \text{for electrons and positrons,} \\ Z^{4/3} & \text{for other particles,} \end{cases}$$

$$S_1 = \begin{cases} Z(Z+1) & \text{for electrons and positrons,} \\ Z^2 & \text{for other particles,} \end{cases}$$

$$\eta_1^{max} = \eta_2^{max} = 2e^{(B-2)/2}.$$

3. TWO-DIMENSIONAL APPROXIMATION

In what follows we consider only the case of relativistic particles ($\beta = 1$).

New variables $\phi = \phi'/\Phi_0$, $\psi = \psi'/\Phi_0$ have to be defined to avoid the dependence of $FM(\phi',\psi')$ on Φ_0. At the end of our Monte-Carlo calculations, the variables are changed back to ϕ' and ψ' in order to obtain the angle and lateral displacement in radians (not in Φ_0 units).

The following calculations have been performed for a specific medium (iron), muons with a specific momentum ($p = 200$ GeV/c) and a given step size (86.46 g/cm^2). However, the results can be used in a more general context. If for the same medium the muon momentum is changed to $p^* = \nu p$, the quantity B remains unchanged (in the relativistic domain) and the quantities χ_c and Φ_0 are transformed according to

$$\chi_c^* = \frac{\chi_c}{\nu}, \quad \Phi_0^* = \frac{\Phi_0}{\nu}. \quad (6)$$

This leads to

$$FM^*(\phi',\psi') = \nu^2 FM(\nu\phi, \nu\psi), \quad (7)$$

i.e., up to a constant normalization factor the correlation function for ν-times lower momentum has the value of the original correlation function at angle $\nu\phi$ and lateral displacement $\nu\psi$.

Similarly, if the medium is changed ($Z \to Z^*$, $A \to A^*$), the step length should be adjusted to ℓ^* so that b remains invariable which leads to unchanged B. This adjustment changes Φ_0 to $\Phi_0^* = \omega\Phi_0$; the latter equation defines ω. Again we have

$$FM^*(\phi',\psi') = \frac{1}{\omega^2} FM\left(\frac{\phi}{\omega}, \frac{\psi}{\omega}\right).$$

$$G(\phi) = \int_{\psi'}^{\psi''} F(\phi,\psi)\, d\psi$$

with ψ' and ψ'' being the limits of the ψ-range where $F(\phi,\psi)$ is different from zero (in practice). The desired number ϕ_t is found as a random number distributed according to the distribution $G(\phi)$. In the next step we should find for any ϕ_t an accompanying random number ψ, distributed according to a distribution function which contains ϕ_t as a parameter. However, this would necessitate the computation of a very large number of such ψ distribution functions. It is more economical to computer a two-dimensional array of cumulative probabilities $H(\phi_r,\psi_\ell)$ defined as

$$H(\phi_r,\psi_\ell) = \int_{\psi'}^{\psi_\ell} \int_{\phi_r}^{\phi_{r+1}} F(\phi,\psi)\, d\phi\, d\psi\,,$$

where the integration interval in ϕ, namely $\phi_{r+1}-\phi_r$, can be chosen rather wide. For any given distribution function and any desired accuracy one can estimate the mesh size of a two-dimensional grid in ϕ and ψ at the knots of which $H(\phi_r,\psi_\ell)$ have to be computed. When $F(\phi,\psi)$ is the Moliere multiple scattering correlation function $FM(\phi,\psi)$ we need a grid with 1125 knots to achieve a 1% accuracy. The problems involved in using directly the original Moliere expression (1) can now be appreciated. Since $FM(\phi,\psi)$ itself is defined as a two-dimensional integral, the same number (1125) of four-dimensional integrals $H(\phi_r,\psi_\ell)$ would have to be computed which is practically impossible. However, using the Chebyshev approximation instead of those four-dimensional integrals, we can compute analytically two-dimensional integrals of a sum of Chebyshev polynomials. In this case, if a 1% accuracy is desired, the interval (ϕ',ϕ'') must be divided into 56 equal parts with points

$$0 = \phi' = \phi_1 < \phi_2 < \dots < \phi_s < \phi_{s+1} < \dots < \phi_{57} = \phi'' = 5.6\,.$$

The step in the ψ-direction must be 0.1. The cumulative probabilities

$$\int_{\psi_s'}^{\psi_\ell} \int_{\phi_s}^{\phi_{s+1}} FMCH(\phi,\psi)\, d\phi\, d\psi \Big/ \int_{\psi_s'}^{\psi_s''} \int_{\phi_s}^{\phi_{s+1}} FMCH(\phi,\psi)\, d\phi\, d\psi$$

have been computed and stored in the two-dimensional array FSPACE (52,56). In the above formula ψ_s',ψ_s'' determine the limits of the ψ-range when $\phi_s \le \phi \le \phi_{s+1}$. ψ_ℓ is the ψ-coordinate of a knot when it satisfies the condition $\psi_s' \le \psi_\ell \le \psi_s''$.

The one-dimensional ϕ distribution function

$$\int_{\psi'}^{\psi''} FM(\phi,\psi)\, d\psi$$

is nothing but the Moliere angular distribution function $fM(\phi)$. The function $fM(\phi)$ has been described by its (one-dimensional) Chebyshev approximation. To minimize the computer space-time product, the range of the variable ϕ has been divided into four equal subintervals. It has been found that Chebyshev polynomials up to the sixth order in ϕ give a fitting precision better than 10^{-4}

$$fMCH(\phi) = \sum_{i=0}^{6} c_i T_i(\phi).$$

The cumulative probabilities

$$\int_{\phi'}^{\phi_s} fMCH(\phi)\, d\phi \Big/ \int_{\phi'}^{\phi''} fMCH(\phi)\, d\phi$$

have been computed analytically and stored in the one-dimensional work space FNORM.

The desired number ϕ is found by calling RNDM and doing a 4-point interpolation on FNORM. Using the formula

$$I = \phi/(\phi_{s+1}-\phi_s) + 1,$$

calling again RNDM and doing a 4-point interpolation on FSPACE(52,I), the corresponding number ψ is found. This ψ is correct only in the case, where ϕ coincides with the centre of the corresponding ϕ interval (number I). This is the reason why a correction is needed to account for the width of the interval $\phi_{s+1}-\phi_s$ and the behaviour of the function $FM(\phi,\psi)$ (the line of the maximum is $\psi = \frac{\phi}{2}$). This correction is

$$\psi = \psi + \Delta\psi,$$

where

$$\Delta\psi = [\phi - (I-1)\, 0.1 - 0.05]/2.$$

The second ϕ, ψ - correction taking into account the quotient of the actual and nominal (200 GeV/c) momentum is applied (see expression 7). Using the equations

$$\phi' = \phi \cdot \Phi_0\,,$$

$$\psi' = \psi \cdot \Phi_0\,,$$

the scattering angle ϕ' and lateral displacement ψ' are obtained in radians.

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Применение корреляционной формулы Мольера для процессов многократного рассеяния частиц в расчетах по методу Монте-Карло

Получена аппроксимация корреляционной угловой-пространственной формулы Мольера для процессов многократного рассеяния релятивистских частиц, удовлетворяющая двум условиям: 1/ она достаточно точная /относительные отклонения меньше, чем 10^{-3} /, и 2/ она применима для расчетов по методу Монте-Карло.

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Todorova G. E1-83-84

Application of the Moliere Multiple Scattering Angle-Lateral Displacement Function in Monte-Carlo Calculations

A two-dimensional approximation is found for the multiple scattering angle-lateral displacement correlation formula which satisfies two conditions: first, to be close to the Moliere one (better than 10^{-3}) and, second, to be suitable for Monte-Carlo calculations. These Monte-Carlo calculations are made for relativistic particles.

The investigation has been performed at the Laboratory of High Energies, JINR.

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