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## METHOD

OF NUCLEON STRUCTURE FUNCTION RECONSTRUCTION
FROM THE DEEP INELASTIC SCATTERING
DATA

## 1. INTRODUCTION

This work has arisen from processing of the experimental data obtained at NA-4 set-up in CERN.

The NA-4 set-up created for muon-nucleon and nuclear interaction research is a cylindrical-symmetric focussing spectrometer with the toroidal magnetic field ${ }^{1 /}$. The spectrometer is $\approx 55 \mathrm{~m}$ long and has a diameter of 2.75 m . An extended target is inside the torus. A scattered muon oscillates inside the spectrometer due to the field effect. Parameters of secondary particles determined in the experiment differ from true parameters because of multiple scattering of particles in iron of the magnet, the discrete step of registration apparatus, inaccuracy of the analysis programs and other causes. For reconstruction of the true spectra of secondary particles we propose to solve the Fredholm integral equation of the first kind. We have chosen the statistical regularization method for solving this equation. Among known methods ${ }^{2,3 /}$ this one better corresponds to the probabilistic character of the data processing problem.
2. THE PROBLEM

Deep inelastic muon-nucleon cross section can be written as $/ 4 /$
$d^{2} \sigma / d x d Q^{2}=F_{2}\left(x, Q^{2}\right) \cdot K\left(x, Q^{2}, E\right) \cdot \Delta\left(x, Q^{2}, E\right)$.
where $F_{2}$ - the nucleon structure function, $K$ - the kinematical factor, $\Delta^{-1}$ - the radiative correction factor, $E$ - the muon beam energy, $Q^{2}$ - the four-momentum transfer, $\quad x=Q^{2}\left[2 M_{p}\left(E-E^{\prime}\right)\right]$ $M_{p}$ - the proton mass, $E^{\prime}$ - the scattered muon energy.

The measurable cross section $d^{2} \sigma\left(x, Q^{2}\right) / d x d Q_{\text {exp }}^{2}$ is connected with true cross section (1) by the dependence:
$\frac{d^{2} \sigma\left(x, Q^{2}\right)}{d x d Q^{2} e^{2} p}=\int \frac{d^{2} \sigma\left(x^{\prime} ; Q^{2}\right)}{d x^{\prime} d Q^{2}} P\left(x^{\prime}, Q^{2} ; x, Q^{2}\right) \mathrm{dx}^{\prime} \mathrm{d} Q^{2}$,
where $P\left(x^{\prime}, Q^{2 \prime} ; x, Q^{2}\right)$ is the set-up resolution function for the point with kinematical parameters $x^{\prime}$ and $Q^{2}$ '. In (2) we integrate over the kinematically permissible region.

We know the left part of (2) and the functions $K$, $\Delta$, too. A resolution function can be obtained by its direct measure, when it is possible, or by the Monte-Carlo simulation of the experiment.

Our problem is to obtain the structure function $\mathrm{F}_{2}$ from (1), (2).

It should be noted, that an actual incident muon beam is not monoenergetic, therefore eq. (2) must be integrated over the beam energetic distribution $\mathrm{P}(\mathrm{E})$. We do not write this more complicated equation and take into account the energetic distribution just on an average.

## 3. STATISTICAL REGULARIZATION METHOD

After algebraization of equation (2) we obtain a system of linear equations
$\overrightarrow{\boldsymbol{f}}=\mathrm{P} \overrightarrow{\boldsymbol{\phi}}+\vec{\epsilon}$,
where $\vec{f}$ is an $m$-dimensional vector corresponding to the measured cross section, $\vec{\phi}$ is an $n$-dimensional vector corresponding to the true cross section, $P$ is the $m \times n$ matrix corresponding to the resolution function multiplied by $K$ and $\Delta, \vec{\epsilon}$ is an m-dimensional random vector with an average value $E \vec{\epsilon}=0$

$\operatorname{Var} \vec{\epsilon}=\operatorname{diag}\left(\sigma_{1}^{2}, \ldots, \sigma_{\mathrm{m}}^{2}\right)$,
where $\sigma_{i}$ denoted statistical errors of the measured cross section. It is well known/2,3/ that a system of linear equations obtained from integral equation (2) is a poorly determined system, i.e., its solution is in an extraordinary strong dependence on variations of the inhomogeneous terms, also on the errors in matrix elements and computer round-off errors. Therefore, if we solve equation (3) by the least squared method
$\vec{\phi}=\left(P^{T} V^{-1} P\right)^{-1} P^{T} V^{-1} \vec{f}$, where $V=\operatorname{Var} \vec{\epsilon}$,
we can obtain $\vec{\phi}$ with very big erfors or do not solve (3), as there is no inverse matrix for $\mathrm{P}^{\mathrm{T}} \mathrm{V}^{-1} \mathrm{P}$.

To obtain a stable solution for equation (2) we will use the statistical regularization method. The essence of the method is that we use a priori information about the solution. In our case this information is that solution is a smooth function. We can make this information more precise by introducing a functional characterizing the degree of smoothness of the function
$\Omega\left[F_{2}\left(x, Q^{2}\right)\right]=\int\left[a_{1}\left(F_{\& x}^{\prime}\right)^{2}+a_{2}\left(F_{R Q^{2}}^{\prime}\right)^{2}\right] d x d Q^{2}$,
$\mathrm{a}_{1}, \mathrm{a}_{2}$ are constants, $\mathrm{a}_{1}>0, \mathrm{a}_{\Omega}>0$, and by fixing an expected approximate value of this functional $\Omega\left[F_{2}\left(x, Q^{2}\right)\right] \approx \omega$,

After the algebraization is performed the a priori information has the form:
$(\vec{\phi}, \Omega \vec{\phi})=\Sigma_{i, j} \phi_{i} \Omega_{i j} \phi_{j}=\omega$.
Further we choose density of the a priori distribution $\mathbf{P}(\vec{\phi})$. In order to introduce as little arbitrariness as possible we choose $P(\vec{\phi})$ which minimize the quantity of information:
$\int \mathrm{P}(\vec{\phi}) \ln \mathrm{P}(\vec{\phi}) \mathrm{d} \vec{\phi}$
on condition that:
$\int(\vec{\phi}, \Omega \vec{\phi}) \mathrm{P}(\vec{\phi}) \mathrm{d} \vec{\phi}=\omega$.
This function $P(\vec{\phi})$ will be
$P_{a}(\vec{\phi})=c_{1} a^{\frac{\mathrm{n}-\mathrm{r}}{\mathrm{Z}}} \exp \left\{-\frac{a}{2}(\vec{\phi}, \Omega \vec{\phi})\right\}$,
where $a=n / \omega, r$ is a defect of the matrix $\Omega, c_{1}$ is a normalization constant.
 rizes the measurement process is taken in the form of a normal distribution/3/
$P(\vec{f} / \vec{\phi})={ }_{j}^{m}=1 \frac{1}{\sqrt{2 \pi \sigma_{j}^{2}}} \exp \left\{-\frac{\left(f_{j}-\sum_{i=1}^{n} p_{i j} \phi_{i}\right)}{2 \sigma{ }_{j}^{2}}\right\}$.
By the method of statistical regularization the estimator of the vector $\vec{\phi}$ is expectation $\vec{\phi}$ over a posteriori distribution:

$$
\begin{aligned}
& \mathrm{P}(\vec{\phi} / \overrightarrow{\mathrm{f}}, a)=\mathrm{c}_{2} \exp \left\{-\frac{1}{2}\left(\vec{\phi},\left[\mathrm{P}^{\mathrm{T}} \mathrm{~V}^{-1} \mathrm{P}+a \Omega\right] \vec{\phi}\right)+\right. \\
& \left.+\left(\vec{\phi}, \mathrm{P}^{T} V^{-1} \overrightarrow{\mathrm{f}}\right)-\frac{1}{2}\left(\overrightarrow{\mathrm{f}}, \mathrm{~V}^{-1} \overrightarrow{\mathrm{f}}\right)\right],
\end{aligned}
$$

obtained by the Bayes formula.
Calculation of the expectation gives us
$\overrightarrow{\hat{\phi}}=\left(P^{T} V^{-1} P+a \Omega\right)^{-1} P^{T} V^{-1} \vec{f}$.

We can choose a parameter by the maximum likelihood method ${ }^{\prime \prime}$, here $a$ is found from the equation:
$\frac{\partial}{\partial a} \ln P\left(a / \vec{D} \equiv \frac{n-r}{2 a}-\frac{1}{2} \operatorname{Sp}\left\{\Omega\left(P^{T} V^{-1} P+a \Omega\right)^{-1}\right\}+\right.$
$+\frac{1}{2}\left(\overrightarrow{\hat{\phi}}_{a}, \Omega \overrightarrow{\hat{\phi}}_{a}\right)=0$,
where $\overrightarrow{\boldsymbol{\phi}}_{a}$ is the regularized solution with a given $a$.
4. GENERALIZATION OF STATISTICAL REGULARIZATION METHOD FOR THE MATRIX P GIVEN WITH STATISTICAL ERRORS

Either of two mentioned in section 3 methods of obtaining the resolution function, as well as $P$ matrix, allows one to get it with statistical errors. The real matrix element $\tilde{\mathrm{p}}_{1 j}$ equals
$\bar{p}_{i j}=p_{i j}+\xi_{i j}$,
where $p_{i j}$ is the true value of the matrix element and $\xi_{i j}$ is a random value with
$E \xi_{i j}=0, \quad \operatorname{Var} \xi_{i j}=d_{i j}, \operatorname{Cov} \xi_{i j} \xi_{k \ell}=0$.
It is shown in paper $/ 5$ / that for the estimation of $\dot{\Phi}$ by the least squares method with an approximately given matrix $\ddot{\mathbf{P}}$
$\overrightarrow{\hat{\phi}}=\left(\overrightarrow{\mathrm{P}}^{\mathrm{T}} \mathrm{V}^{-1} \overrightarrow{\mathrm{P}}\right)^{-1} \overrightarrow{\mathrm{P}}^{\mathrm{T}} \mathrm{V}^{-1} \overrightarrow{\mathrm{f}}$
there is a bias, which may be significant for ill-conditioned matrix $P$ and large values of $d_{i j}$. For this case a satisfactory estimation of $\phi$ is suggested in ref. ${ }^{/ 6 /}$
$\overrightarrow{\hat{\phi}}=\left(\vec{P}^{T} \vec{V}^{-1} \vec{P}-\right.$ 包 $) \vec{P}^{T} \bar{V}^{-1} \overrightarrow{\mathrm{f}}$,
where $\overline{\mathrm{V}}=\operatorname{diag}\left(\sum_{i=1}^{n} \phi_{i}^{2} \mathrm{~d}_{11}+\sigma_{1}^{2}, \ldots, \sum_{i=1}^{n} \phi_{i}^{2} d_{m i}+\sigma_{m}^{2}\right)$,
$\Xi=\sum_{j=1}^{m} \operatorname{diag}\left(d_{j 1}, \ldots, d_{j n}\right) /\left(\sum_{i=1}^{n} \phi_{i}^{2} d_{j i}+\sigma_{j}^{2}\right)$.
Since the true value of $\phi_{i}$ which enters the right-hand part of (7) is unknown, values of $\vec{\phi}$ from the previous experiment can be employed, or ${ }_{\hat{\phi}}^{\vec{~}}$ can be found by means of iteration which is convergent, as is shown in ref. ${ }^{16 /}$.

Generalization of (7) by the statistical regularization method leads to the final expression for the estimation of $\vec{\phi}$ :

$$
\begin{equation*}
\overrightarrow{\hat{\phi}}=\left(\tilde{\mathbf{P}}^{\mathrm{T}} \vec{V}^{-1} \tilde{\mathbf{P}}-\Xi+a \Omega\right)^{-1} \overrightarrow{\mathbf{P}}^{\mathrm{T}} \overline{\mathrm{~V}}^{-1} \overrightarrow{\mathbf{f}} . \tag{8}
\end{equation*}
$$

Matrix $\left(\widetilde{\mathbf{P}}^{\mathbf{T}} \overline{\mathrm{V}}^{-1} \overline{\mathbf{P}}-\xi+a \Omega\right)^{-1}$ is the estimator of the full matrix of errors for $\overrightarrow{\boldsymbol{\phi}}$.

Parameter a can be obtained from Eq. ${ }_{\overline{\mathrm{F}}}{ }^{(6)}$, if one replaces everywhere $P$ with $\mathbb{P}, V$ with $\vec{V}$ and $\mathbf{P}^{T} V^{-1} P$ with $\overline{\mathbf{P}}^{T} V^{-1} \widetilde{\mathbf{P}}-$ E.

To make the regularizations equivalent in both directions, we shall solve Eq. (2) with weights $a_{1}, a_{2}$ equal to:
$a_{1}=n / \omega_{1}, \quad a_{2}=n / \omega_{2}$,
where $\omega_{1}$ and $\omega_{2}$ mean the same as $\omega$, but for matrices $\Omega_{1}$ and $\Omega_{2}$, respectively. Estimations for $\mathrm{a}_{1}$ and $\mathrm{a}_{2}$ can be obtained in the same way as for a by the maximum likelihood method.

## 5. CHOICE OF BINNING, CALCULATION OF MATRIX ELEMENTS

Without limiting the general discussion, below we shall describe the method as applied to the part of experimental material obtained in 1979 , for the 280 GeV beam. The number of experimental events is 80 K . The number of simulated events is 400 K , among them 130K are under reconstruction.

Figs. 1 and 2 show binnings for the experimental data and the structure function, respectively.

Boundaries of the binning for the experimental data are chosen in such a way that maximum of the available experimental information was nsed. Tho siro of hine 11 nowe gt loget 20 events within them, and good providing with statistics assures for each 4 bins of experimental information one bin for the structure function.

Boundaries of the binning for the structure function are set by kinematic parameters of the events detected in the experimental set-up. In this case the boundaries can be determined through distribution of the reconstructed model events by their original kinematic parameters. The lower boundary in $x$ is replaced by the lower boundary in the scattered muon energy. The boundary $\mathrm{E}_{\min }^{\prime}$ is shown in Fig. 3. When normalizing to the bin areas for bins $1,10,19,28,36,44,52,60,68,75,82,88$, 94,99 , the lower boundary in $x$ is determined by the formula:
$x_{\text {min }}=2 \Delta^{2} M_{p} /\left(E_{A}-E_{\text {min }}^{\prime}\right)$,
where $E_{A}$ is a mean value of the beam energy. Thus, the normalization somehow brings back a number of events which are out of the boundary $x_{\text {min }}$ because in their case the incident muon energy $E$ was higher than the average beam energy $E_{A^{\prime}}$

The value of the structure function in the bin can be ascribed to mean values of kinematic parameters in this bin.




Fig.3. Minimum energy $E$ of the scattered muon registered in NA-4 set-up vs $Q^{2}$.

These mean values are calculated by formulae:
$x_{A}=\Sigma u_{1} x_{1} / \Sigma u_{i}, \quad Q_{A}^{R}=\Sigma u_{1} Q_{i}^{R} / \Sigma u_{i} ; E_{A}=\Sigma v_{i} E_{i} / \Sigma v_{1}$,
here summation covers all simulated events in the bin, and
$u_{1}=\frac{Q_{i}^{2}}{1-x_{i}} \cdot F_{2}\left(x_{i}, Q_{1}^{R}\right), \quad v_{1}=P\left(E_{i}\right) / P\left(E_{i}\right)_{a}$.
where $(1-x) / Q^{2}$ and $P(E) a$ are the distribution of the event simulation. For $F_{\mathcal{L}}\left(x, Q^{2}\right)$ the parameter formula was used
$F_{2}\left(x, Q^{2}\right)=p_{1}\left(1+p_{2}\right)(1-x){ }^{P_{8}}\left(\frac{Q^{2}}{5}\right) P_{4} \ln 4 x$,
where $p_{1}=.52, p_{2}=.9, p_{3}=3.2, p_{4}=-.155$. This parametrization describes satisfactorily the data on the structure functions obtained in other experiments and ensures agreement between the spectrum of the reconstructed simulated events and the experimental spectrum. The dispersion in actual distribution $P(E)$ is $+4 \%$, and simulated distribution $P(E)_{0}$ is close to the actual one.

The matrix elements are calculated by the formula
$p_{i j}=\frac{\Sigma_{g}}{\Sigma_{\underline{W}}} \cdot W$,
summation covers all the events simulated in the $j$-th bin for the structure function, here:
$w_{\ell}=u_{\ell} \cdot v_{\ell} \cdot K\left(x_{\ell}, Q_{\ell}^{\ell}, E_{\ell}\right) \cdot \Delta\left(x_{\ell}, Q_{\ell}^{2}, E_{\ell}\right)$,
$\mathbf{g}_{\ell}=\left\{\begin{array}{l}\text { W}, \text { if the event was reconstructed and after reconst- } \\ \text { ruction got into the } 1-t h \text { bin for the experimental } \\ \text { data; } \\ 0, \text { if any of the two conditions is viola }\end{array}\right.$
$W=R\left(X_{A}, Q_{A}^{2}, E_{A}\right) \cdot \Delta\left(X_{A}, Q_{A}^{2}, E_{A}\right) \cdot s \cdot a_{N}$,
$s$ is the bin area, ${ }^{a_{N}}$ is the normalization constant, $a_{N}=N \cdot \rho \cdot t \cdot N_{A}$, $N$ is the muon flux, $\rho$ is the target density, $t$ is the target length, $N_{A}$ is the Avogadro number.

The error $p_{i j}$ is approximately
$\Delta p_{i j}=\frac{\sqrt{\Sigma g} \boldsymbol{q}}{\Sigma W_{l}} \cdot W$.


Fig.4. $x$ resolution FWHM vs $x$ of NA-4 set-up.


Fig.5. $Q^{2}$ resolution FWHM vs $Q^{2}$ of NA-4 set-up,

The use of weights $w_{p}$ helps to reduce a little.the approximation error caused by the fact that bins are not infinitesimal in their size. The error can be reduced even more by repeating the whole procedure after $\mathrm{F}_{2}$ is calculated.

For a large number events obtained at the set-up ( $s 17 \%$ ) the incident muon energy is not known; all kinematical parameters for these events are calculated in the reconstruction programmes with the energy $E=E$. Therefore, simulated data were prepared as a statistical mixture of the events with known and unknown (after the reconstruction) energy of the incident muon, the ratio being 83:17. x and $Q^{2}$ resolution of the NA-4 set-up FWHM is shown in Figs. 4 and $5 /{ }^{1 /}$.

## 6. TRIAL FUNCTION $\mathbf{F}_{\mathbf{Z}}$

There are two obstacles to the effective use of the method described in sections 3,4 . The first one is the fact that the method is good for comparatively smooth functions, and $F_{2}$ descends rapidly with increasing $x$. This leads to the fact, that the reconstructed function $F_{\mathcal{E}}$ is too smooth at small $x$ and fluctuates much at large $\mathbf{x}$.

The second obstacle is that values of $x$ for which $F_{8}$ is determined in some intervals do not lie on the same straight line, e.g., for bins 2, $11,20,28,36,44,52$, 60 x equal . 253, $.253, .253, .225, .236, .248, .264, .284$, respectively, and the a priori information on smoothness in $\boldsymbol{Q}^{2}$ cannot be applied to this set of points.

These two obstacles can be eliminated; for this purpose let us present (3) in the form
$\vec{f}=P^{\prime} \vec{\phi}^{\prime}+\vec{\epsilon}$,
where
$P^{\prime}=P D, \quad \vec{\phi}^{\prime}=D^{-1} \vec{\phi}, \quad D=\operatorname{diag}\left(F_{2 t}\left(x, Q^{2}\right), \ldots, F_{2 t}\left(x, Q^{2}\right)\right)$,
here $F_{2 t}\left(\mathbf{x}, \mathbb{Q}^{2}\right)$ is a trial function for $F_{2}$ which is given in the parametric form and shows the assumed behaviour of the resu1t. Tn new equation (9) the unknown $\phi^{\prime}$ does not cause so much changes as $\vec{\phi}$ does in (3), therefore both obstacles become insignificant. Eq. (9) being solved, $\vec{\phi}$ is easily obtained from the equation
$\vec{\phi}=D \vec{\phi}^{\prime}$
and if we want to know the value of the structure function in bin centres, we can find it by the approximate formula
$\vec{\phi}_{c}=D_{c} \vec{\phi}^{\prime}$,
where $D_{c}$ is a matrix with the trial function values in bin centres.

## 7. ANALYSIS OF RESIDUALS

Let us find residuals through $r_{i}$
$r_{i}=\frac{\mathbf{f}_{i}-\sum_{j=1}^{n} \bar{p}_{i j} \phi_{j}}{\sqrt{\sigma_{i}^{2}+\sum_{j=1}^{n} d_{i j} \phi_{j}}} \quad$ noteworthy is that $\sum_{i=1}^{m} r_{i}^{2}=x^{2}$.

The analysis of residuals allows:
(i) to reveal sharply distinctive components of $f$, e.g., for which $\left|r_{1}\right|>3$, in order to analyse better the reasons for their origination (a systematic error, resonance, etc.).
(ii) to select $\boldsymbol{F}_{\text {ft }}$ more thoroughly.

It was observed, that if some components became distinguished due to systematic errors, this leads to wrong estimations of $a_{1}, a_{2}, a$. Therefore, when determining $a_{1}, a_{2}, a$, these components can be excluded from the process and introduced again when ${ }^{1} 1^{2}{ }^{2}$, a are calculated.

By the distributions of residuals for various parts of the experimental data binning one can judge about the regularization effect in these parts. Thus, if one does not use the trial function, i.e., $F_{2 t}=1$, one obtains significantly different root-mean-square dispersions in the distribution of residuals for the upper and the lower parts of the binning. In this case a smaller smoothing effect corresponds to the smaller root-mean-square dispersion ${ }^{\prime 7} /^{\prime}$.

By the deviation of the means from zero for the distribution of residuals one can judge about the distortion due to a priori information.

We would like to note, that for the least squares method the residuals may be considered as independent and equally distributed by random values of normal distribution with $E r_{1}=0$ and $\operatorname{Var} \mathrm{r}_{1}=m /(m-n) / 5 \rho^{\prime}$.

The dependence of the result upon $F_{2 t}$ is quite weak. Nevertheless, it depends upon $F_{\mathbf{2 t}}$ for the bins with poor statistics or very bad resolution. Here the best $F_{2 t}$ is the one which gives:
(i) a smaller values of $x^{2}$;
(ii) smaller (in absolute value) means in the distributions of residuals;
(iii) a smaller difference between the values of the root-mean-square dispersion in the distribution of residuals for various parts of binning;
(iv) better agreement with $F_{q}$ reconstructed with the accuracy up to normalization;
(v) smaller errors in the reconstructed $F_{2}$.

## 8. SIMULATIVE PROBLEM

Solution of simulative problems is not unimportant for the reconstruction procedure. In this case pseudodata are obtained as follows:

1. A parametric formula for $F_{\mathcal{R}}$ is taken and the vector $\vec{\phi}$ is formed.
2. The actual matrix $\tilde{\mathbf{P}}$ is multiplied by the vector $\vec{\phi}$ and we obtain a pseudoexperimental vector $\overrightarrow{\mathrm{f}}$.
3. Using the generator of random numbers with normal distribution, fluctuations are added to the vector $\overrightarrow{\mathrm{f}}$. These fluctuations correspond to the actual statistical error $\Delta \vec{f}$,
4. In the same way fluctuations are added to the elements of matrix $\hat{P}$. These fluctuations correspond to the actual statistical error $\Delta P$.

Since we know the true $\mathrm{F}_{\mathrm{g}}$ in the simulative problem, its solution allows us to understand in general the influence of the set-up resolution, errors of experimental data, errors of matrix elements, systematic error, etc., upon the quality of the reconstruction. As a simulative $F_{2}$ and $F_{2}$ in the following form can be chose (the Ganzalez-Arroyo parametrization ${ }^{\text {/8/) }}$ )
$F_{2}\left(\mathrm{x}, \mathrm{Q}^{2}\right)=\mathrm{cx} \mathrm{a}_{0}+\alpha_{1}^{\mathrm{s}}(1-\mathrm{x}) \beta_{\mathrm{o}}+\beta_{1} \mathrm{~B}$,
where $s=-\ln \frac{g^{2}\left(Q^{2}\right)}{g^{2}\left(Q_{0}^{2}\right)}, \mathrm{B}^{2}\left(Q^{2}\right)=\frac{10 \pi^{2}}{\bar{\beta}_{0} \ln \left(Q^{2} / \Lambda^{2}\right)}$,
with the following parameters: $Q_{0}^{2}=110, c=1.46, a_{0}=0.49$, $a_{1}=-0.81, \bar{\beta}_{0}=11-2 / 3 \mathrm{f}, \mathrm{f}=4, \Lambda=0.26, \boldsymbol{\beta}_{1}=1.57, \boldsymbol{\beta}_{0}=$ $=4.08$. This parametrization describes satisfactorily the data obtained in experiment/4A-4 at energies 120 GeV and 200 GeV and published in ref.

For the reconstruction a trial function $F_{\mathcal{Z}}$ in the form of (9) is used with parameters $p_{1}=0.17, p_{2}=10.66, p_{3}=3.52$, $\boldsymbol{p}_{4}=-0.22$. In Fig. 6 one can see the results of the reconstruction. In the intervals $x=0.25 \div 0.65$ the reconstruction is good, in the intervals $0.15,0.75 \div 0.95$ the reconstriction is worse. We would like to command attention to the poorest agreement between $\mathrm{F}_{\mathrm{gt}}$ and the reconstructed $\mathrm{F}_{2}$. For the sake of comparison the reconstruction was carried out with $\boldsymbol{F}_{2}$ in the form of (11) with parameters: $Q_{0}^{\mathbb{R}}=108, c=1.43, a_{0}=0.46, a_{1}=$ $=-0.8, \bar{\beta}_{0}=11-2 / 3 \mathrm{f}, \mathrm{f}=4, \Lambda=0.265, \beta_{1}=1.53, \beta_{0}=4.085$. In Fig. 7 one can see the results of this reconstruction.

The Table lists values of $x^{2}$, mean values (M.V.) and root-mean-square values (R.M.S.) for all residuals and the residuals in various parts of the binning for both cases.

According to the criteria from section 7 the test function of the second form is more preferable, as a matter of fact, in the second case the reconstructed structure function is closer to the true $\boldsymbol{F}_{\mathbf{q}}$.


| N | $\chi^{2}$ | All |  | :0 $\leq x \leq 0.6$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | M.V. | R.M.S. | M.V. |  | R.M.S. |
| 1 | 236.0 | $\begin{aligned} & 0.016 \\ & 0.017 \end{aligned}$ | 0.92 | $\begin{aligned} & 0.086 \\ & 0.031 \end{aligned}$ |  | $\begin{aligned} & 0.86 \\ & 0.87 \end{aligned}$ |
| 2 | 233.7 |  | 0.92 |  |  |  |
| N | $0.6 \leq x<2.0$ |  | $Q^{2} \leq 120$ |  | $0^{2} \geq 120$ |  |
|  | M.V. | . R.M.S. | M.V. | R.M.S. | M.v. | R.M.S. |
| 1 | $\begin{aligned} & -0.087 \\ & -0.011 \end{aligned}$ | $\begin{aligned} & 0.99 \\ & 0.99 \end{aligned}$ | 0.007 | 0.91 | 0.039 | 0.94 |
| 2 |  |  | 0.013 | 0.90 | 0.027 | 0.97 |

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Для восстановления структурной функции нуклона из непосредственно измеренного зкспериментальной установкой сечения глубоконеупругого рассеяния мюонов рассматривается интегральное уравнение фредгольма 1 -го рода в двумерном варианте. Свободный член уравнения /зкспериментальное сечение/ и ядро имеют ошибки статистического характера. Решение уравнения /структурная Функция/ находится методом статистической регуляризации с вмбором парамет ров регуляризации из принципа максимума правдоподобия в слоистом ансамбле гладких функций. Предложено обобщение метода, в которсм учитываются ощибки ядра. Показано, как использовать такую априорную информацию, как рост струкядра. Показано, как использовать такую апр
турной функции по одному из направлений.

Работа выполнена в Лаборатории ядерных проблем оияи.

Сообщение 0бъедмненного института ядерных нсследований. Дубна 1983
Gagunashvili N.D. E1-83-703
Method of Nucleon Structure Function Reconstruction from the Deep Inelastic Scattering Data

To reconstruct the nucleon structure function from the directly measured experimental cross section of the deep inelastic scattering of muons, Fredholm integral equation of the first kind is considered in its twodimensional version. The free term of the equation (experimental cross section) and the kernel have statistical errors. The equation"s solution (structure function) is found by the statistical regularization method, the regularization parameters being chosen on the basis of the maximum likelihood in a laminar ensemble of smooth functions. Generalization of the method is suggested, where the kernel errors are taken into account. The use of this a priori information as growth of the structure function in one of the directions is shown.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

