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ON EVIDENCE FOR THE RADIAL-EXCITED STATE OF π' PION AND AXIAL-VECTOR A1-RESONANCE COHERENTLY PRODUCED ON NUCLEI BY PI-MESONS AT 40 GeV/c

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In this paper the results of the analysis are presented which is based on the Pade approximation of partial amplitudes in the complex plane in search for the stable poles for $\mathrm{Im}E < 0$ responsible for existence of resonances.

Input data for this analysis are the results of the partial wave analysis (PWA) of $\pi^+\pi^-\pi^-$ system coherently produced on nuclei by pions at 40 GeV/c, which have been obtained by Bologna-Dubna-Milano collaboration /1/.

$$\pi^- + A \to \pi^+ + \pi^- + \pi^- + A,$$

where A → Be , C , Al , Si , Ti , Cu , Ag , Ta , Pb.

Density of events is 4000 events per 40 MeV in 0.8-1.4 GeV/c^2 mass region of 3π-system. Incoherent contribution in the diffraction peak is 2% for Be and 0.4% for Pb.

The PWA analysis results in a set of intensities $|F_i|^2$ and relative phases δ_i between waves dependent upon 3π-mass (JPL: 0^-S , 0^-P , 1^+S , etc., JP - spin and parity of 3π-state, L - orbital moment of a bachelor pion with respect to a dipion).

The high coherence of different partial waves in the production of the 3π-system (the strong angular interference between waves) allows the relative phases of two waves to be determined reliably. In PWA programe the 3π-system is assumed to decay into an intermediate dipion and a pion. While $1^-(\rho)$ and $2^+(f)$ dipions are good resonances, there is a difficulty in the description of 0^+ dipion. Parametrization of the dipion amplitude is made both with ϵ-resonance and with the phase of elastic $\pi\pi$-scattering.

Figure 1 a presents the intensities of $1^+S(\rho\pi)$, fig.1b presents the relative phase $\delta(1^+S-0^-P)$ and fig. 1c. presents the interference term 1^+SP for ϵ and $\pi\pi$-parametrization. The relative phase $\delta(1^+S-0^-P)$ varies by 130° and 100° for two parametrizations in 0.8-1.4 GeV/c^2 region of 3π-mass. The number of events assigned to the interference term 1^+SP in the ϵ-parametrization is negative at the threshold and large and positive above 1 GeV/c^2, while for the elastic $\pi\pi$ phase shifts it is negative everywhere and compatible with intensities. The shape of the 1^+S mass distribution depends upon the parametrization mode of the dipion amplitude.

Figure 2 shows the intensity of 0^-S, its relative phase $\delta(0^-S-0^-P)$ and interference term 0^-SP for two para-

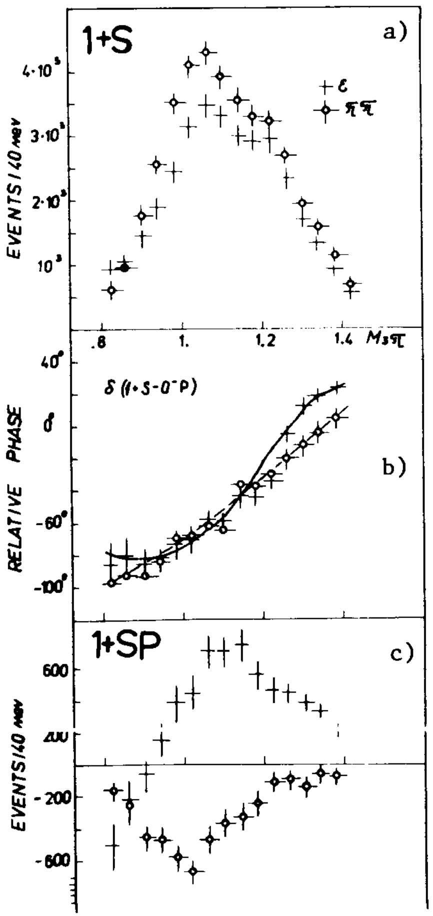

Fig.1. a) is the intensity of the wave 1^+S; b) is the relative phase (1^+S-0^-P); c) is the interference of the waves 1^+S and 1^+P for ϵ and $\pi\pi$ parametrizations of the dipion amplitude.

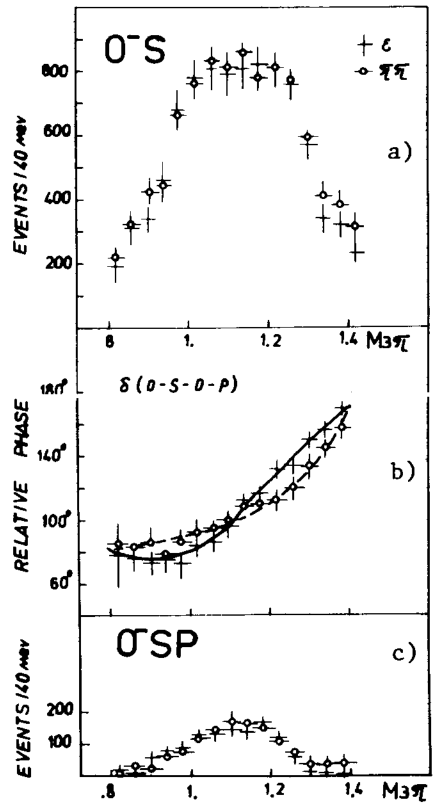

Fig.2. a) is the intensity of the wave 0^-S; b) is the relative phase (0^-S-0^-P); c) is the interference of the waves 0^-S and 0^-P.

metrizations. The mass spectra of 0^-S and interference term do not change in two parametrizations. The relative phase $\delta(0^-S-0^-P)$ varies by 85° and 75° in each parametrization, respectively.

The usual method of determining the resonance parameters is based on the Breit-Wigner analysis of the intensity and the relative phase and depends upon the apriori hypothesis on the number of resonances, upon the assumption about the constancy of the basic wave and upon the parametrization of dipion. It can be applied to each partial wave separately. Besides, when inspecting the energy dependence of the partial wave's modulus, one can suspect resonance-like behaviour of the partial wave, but the same energy dependence of $|F_i|^2$ can be obtained through $|\bar{F}_i|^2$, i.e., without the resonance pole in the complex energy plane.

The method of determining the resonance parameters through Pade approximants [3] does not employ any hypothesis on the number of resonances, or assumption about the basic wave, but takes into account the whole set of the available data obtained from PWA (intensities and relative phases of $1^+S, 0^-S, 0^-P$ ...).

A PW approximant can be constructed in the following way:

$$\{f_i(x_i)\} \Rightarrow \tilde{f}(z) = \frac{P_N(z, f_i(x_i))}{Q_M(z, f_i(x_i))} .$$

Investigations of zeros of the polynomial P_N and Q_M allow one to find the zeros and poles of a partial wave. For the best representation of the data we have employed a subsequent improvement of the approximant by the least squares minimization, where functions $f(x_i)$ are the free parameters of the approximant.

Pade approximant of the partial wave has a form:

$$F_{J^P} = |a_{J^P}| \prod_{i=1}^{n} \frac{W - W^{J^P}_{zi}}{W - W^{J^P}_{pi}} , \qquad (1)$$

where a_{J^P} is a complex constant which can be determined up to a phase due to the lack of knowledge of individual phases; W_z and W_p are zeros and poles in the complex energy plane.

Pade approximant of the partial wave converges to the Jost function with zeros for physical energies associated with bound states and resonances. The analysis of the convergence of the diagonal Pade series can provide information on the existence of stable poles in the lower-half complex energy plane. So the true resonance is a stable pole for which $\mathrm{Im}\, W_p < 0$ and is constant while the power of the approximant increases.

In order to use the full experimental information of PWA (intensities and relative phases) an adequate method has been found to construct the Pade approximant through the product and ratio of partial waves with the same relative phase:

$$F_\alpha \bar{F}_\beta = |F_\alpha| \times |\bar{F}_\beta| e^{i\delta} = A \prod_{i=1}^{N} \frac{W - W_z^\alpha}{W - W_p^\alpha} \times \frac{W - \bar{W}_z^\beta}{W - \bar{W}_p^\beta} .$$

and

$$\frac{F_\alpha}{F_\beta} = \frac{|F_\alpha|}{|F_\beta|} e^{i\delta} = B \prod_{i=1}^{N} \frac{W - W_z^\alpha}{W - W_p^\alpha} \times \frac{W - W_p^\beta}{W - W_z^\beta} , \qquad (2)$$

A and B are complex constants which determine a_{J^P} from eq. (1).

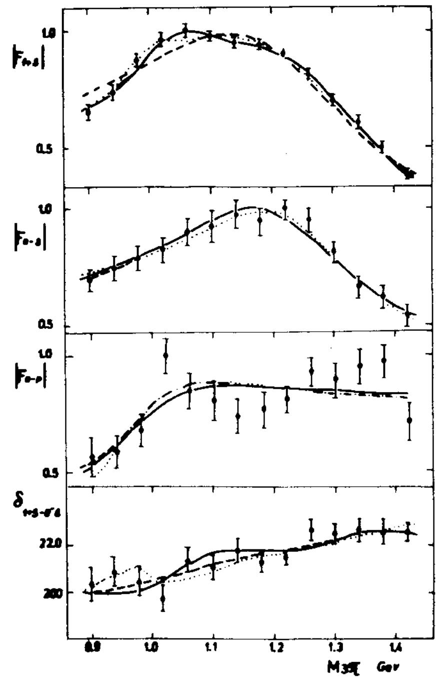

Fig.3. Amplitudes of the waves $1^+S, 0^-S$ and 0^-P and their relative phases. The continuous curve shows the result of fit by Pade approximant.

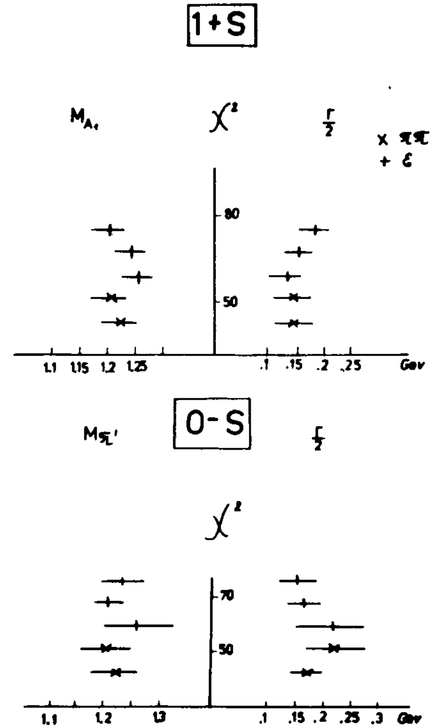

Fig.4. χ^2-dependence of the position and halfwidth of A1 and π' resonances for ϵ and $\pi\pi$ parametrization and different types of Pade approximation.

The mass spectra of partial waves 1^+S, 0^-S and 0^-P and relative phases have been used to build the Pade approximants following equation (2) for ϵ and $\pi\pi$ parametrization.

To find the initial parameters of Pade approximants for each partial wave the independent fit of two equations (2) was done for wave pairs 1^+S-0^-P, 0^-S-0^-P, 1^+S-0^-S. From this fit one can get only an indication of the existence of a resonance pole in partial waves, analysing zeros and poles of the amplitude.

At a second stage the already established parameters of Pade approximant of each partial wave have been employed as input data for searching for the stable poles in the combined fit of equations (2) using the intensities of 1^+S, 0^-S, 0^-P and their relative phases simultaneously.

The amplitudes of 1^+S, 0^-S, 0^-P and their relative phases as well as the results of fit for three types of Pade approximants are presented in Fig.3.

The exponential non-resonance background contribution in 0^-S wave is approximated by zeros and poles which have different values when the order of Pade approximant increases while the true zeros and poles of the partial wave are still stable.

1^+S wave has some sensitivity to the contribution of the nonresonance background. The exponential background improved the fit and shifted the location of pole at two standard deviations.

Results of this fit which determined the position and halfwidth of the stable poles in $1^+S(A1)$ and $0^-S(\pi')$ waves are shown in Fig.4 for different types of Pade approximants and for two types of parametrization. The position and width of the resonance in the 1^+S wave is not sensitive to the parametrization of the dipion, but is sensitive to the background term while the 0^-S resonance width increases for $\pi\pi$ parametrization.

The mean weighted values of the position and width of the axial-vector A1 resonance are $M_{A1}=(1255 \pm 23)$ MeV, $\Gamma_{A1}=(292 \pm 40)$ MeV, which is in good agreement with the recent observations /4,5/.

As far as the $\pi'(0^-S)$ resonance is concerned we have obtained $M_{\pi'}=(1208 \pm 21)$ MeV, $\Gamma_{\pi'}=(336 \pm 46)$ MeV. The obtained position and width of the radial-excited state of the pion are in good agreement with papers /4,6/.

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Беллини Д. и др. О доказательстве существования радиально-возбужденного состояния пиона π' и аксиально-векторного A1-резонанса в процессе когерентного образования 3π-системы на ядрах пи-мезонами с энергией 40 ГэВ — E1-82-488

Результаты парциально-волнового анализа 3π-системы в процессе когерентного образования на ядрах при 40 ГэВ/с использованы для определения стабильных полюсов в нижней части комплексной энергетической плоскости с помощью метода паде-аппроксимации парциальных амплитуд. Установлены стабильные полюса в волнах 1^+S, 0^-S, определяющие положение и ширины π'- и A1-мезонов с $M_{A1}=/1255 \pm 23/$ МэВ, $\Gamma_{A1}=/292 \pm 40/$ МэВ, $M_{\pi'}=/1208 \pm 21/$ МэВ, $\Gamma_{\pi'}=/336 \pm 46/$ МэВ.

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Bellini G. et al. On Evidence for the Radial-Excited State of π' Pion and Axial-Vector A1-Resonance Coherently Produced on Nuclei by pi-Mesons at 40 GeV/c — E1-82-488

Results of the partial wave analysis of 3π-system coherently produced on nuclei at 40 GeV have been used for determining stable poles in the lower half of the complex energy plane within the help of the Pade-approximation method. The stable poles in waves 1^+S, 0^-S determine the position and width of axial-vector A1 resonance $M_{A1}=(1255 \pm 23)$ MeV, $\Gamma_{A1}=(292 \pm 40)$ MeV and radial-excited state of π' pion $M_{\pi'}=(1208 \pm 21)$ MeV, $\Gamma_{\pi'}=(336 \pm 46)$ MeV.

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