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A METHOD OF DETERMINING RESONANCE PARAMETERS BY PADE APPROXIMANTS USING INTENSITIES AND RELATIVE PHASES OF THE PARTIAL WAVES

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Recently, a partial wave analysis of $\pi^+\pi^-\pi^-$ events coherently produced by 40 GeV/c π^- mesons on nuclear targets has been performed [1].

The analysis gives a set of intensities as 3π -mass distributions for each partial wave amplitude (PWA) and a relative phase between them ($J^P L$: 1^+S , 0^-S , 0^-P, etc.).

The PWA can be presented in the following form [2]

$$F_k = |F_k| e^{i\delta_k} \quad . \tag{1}$$

The high coherence of different PWA in the productions of the 3π system (the strong angular interference between waves) makes reliable the determination of the relative phase of two waves.

For two PWA, $k=1,2$ (say, 1^+S and 0^-P are the states of the 3π system, then: $F_1 = F_{1^+S}$ and $F_2 = F_{0^-P}$) the result of the PWA-analysis gives $|F_1|^2$, $|F_2|^2$ and the relative phase:

$$\phi = \delta_1 - \delta_2 \quad . \tag{2}$$

The usual method of searching for resonance parameters is based on the Breit-Wigner (B-W) analysis of the intensity distribution. The energy dependence of the relative phase should show a fast variation in the resonance region. Background contributions and sometimes the presence of resonance in both PWA makes such an analysis quite model-dependent.

The aim of this paper is to present a method of determining resonance parameters using Padé approximants and taking into account the whole set of available experimental data obtained from PWA analysis, i.e., the intensities of the PWA and the relative phase between the PWA. This method contrary to B-W analysis, does not make use of any hypothesis about the number of resonances or their parameters in any PWA.

In order to check our method we used a mathematical model with two resonance poles in each PWA. The method is also checked for the case of a resonance modified by a nonresonance background

$$F_1 = F_R \cdot F_B \,, \tag{3}$$

where F_R is the resonant part of the PWA.

The definition which we adopt for the resonance is given by the existence of the PWA pole situated on the lower-half part of the complex energy plane.

The method is based on Padé approximants (PA) analysis which gives an accurate interpolation and extrapolation (in the complex plane) of the analytic complex functions [3,4]. Because we deal with finite numbers of function values, which are experimentally determinated, the most useful method to analyse them via PA should be the so-called second type PA (PA II). The PA of type I, which are the ratio of two polynomials of degree N and M constructed from the given coefficients of the Taylor series of the complex function to be approximated:

$$\mathrm{PAI:}\ f(z) \simeq \tilde{f}(z) = \Sigma\, a_n z^n \Rightarrow \tilde{\tilde{f}}(z) = \frac{P_N(z,a_n)}{Q_M(z,a_n)}\,. \tag{4}$$

The second type PA uses for its construction a set of function values $f_i(x_i)$ at a given set of points x_i $(z = x+iy)$ on the real axis. The PA II, which has been classically called the Couchy interpolation [3,5] is the pointwise interpolation by rational functions:

$$\mathrm{PAII:}\ \{f_i(x_i)\} \Rightarrow \tilde{f}(z) = \frac{P_N(z, f_i(x_i))}{Q_M(z, f_i(x_i))}\,, \tag{5}$$

which is practically constructed using the so-called ν algorithm [4,5] for continued fractions based on an iteration procedure which gives the P_N from P_{N-1} and P_{N-2} and similar for Q_N. In order to best represent the experimental data (the experimental values of the complex function $f_i(x_i)$) we used a subsequent improvement of the approximant by a least squares minimization where as the free parameters exactly the functions are used (the set $\{f_i(x_i)\}$). This new approximant is PA of the third type (PA III).

So, if the PWA (eq.(1)) are known, the PA can give the valid approximant (VA) of the form:

$$F_k = a_k \prod_{i=1}^{n} \frac{w - w_{z_i}^{(k)}}{w - w_{p_i}^{(k)}}\,, \quad k = 1,2\,, \tag{6}$$

where a_k is a complex constant and $w_{z_i}^{(k)}$, $w_{p_i}^{(k)}$ are the zeros and poles in the complex energy plane (w) of the PWA. The resonance pole is that stable pole (it will have the same value when n is increasing) for which $\mathrm{Im}\, w_{p_i} < 0$.

In order to use the information obtained from the PWA analysis ($|F_1|^2$, $|F_2|^2$ and ϕ) one can construct two combinations of the PWA having exactly the same relative phase:

$$F_1 \times \bar{F}_2 = |F_1|\cdot|F_2|\, e^{i\phi}, \tag{7}$$

$$\frac{F_1}{F_2} = \frac{|F_1|}{|F_2|}\, e^{i\phi}\,, \tag{8}$$

i.e., the product of one amplitude and the complex conjugate of the second one, and the ratio of the two PWA.

The PA analysis of eq. (7) and eq.(8) can give the exact location of the zeros and poles as does the VA. The price paid for the lack of knowledge of each individual phase (δ_1 and δ_2) and the knowledge only of the relative phase ϕ, is the determination of the constant norm of the PWA (a_k from eq. (6)) up to a constant phase.

The approximant of each PWA constructed in such a way and called the evaluated valid approximant (EVA) is given by:

$$\tilde{F}_k = |a_k| \prod_{i=1}^{n} \frac{w - w_{z_i}^{(k)}}{w - w_{p_i}^{(k)}}\,. \tag{9}$$

The undetermined constant phase of a_k is irrelevant for our purpose, i.e., for finding the resonance parameters of the PWA or, in other words, the EVA of PWA contains all the information needed for the resonance determination, as the VA itself.

In our procedure, the experimental data on the product (eq. (7)) and ratio (eq.(8)) are analysed by the PA of the III type. From the zero analysis of the nominator and denominator of the PA III, the product and ratio of two PWA are given by:

$$|F_1| \times |F_2|\, e^{i\phi} = \alpha \prod_{i=1}^{N} \frac{w - w_{z_i}}{w - w_{p_i}}\,, \tag{10}$$

$$\frac{|F_1|}{|F_2|}\, e^{i\phi} = \beta \prod_{i=1}^{N} \frac{w - w'_{z_i}}{w - w'_{p_i}} \tag{11}$$

with $N = n/2$.

The EVA of each PWA given by eq. (9) is obtained by comparison of the two sets of zeros and poles from the PA analysis of eq. (7) and eq. (8) in order to identify the zeros and poles belonging to each PWA (eq.(6)):

$$(w_{z_i};\, w_{p_i} : i = 1 \ldots N) \rightarrow (w_{z_i}^{(1)}, \bar{w}_{z_i}^{(2)};\, w_{p_i}^{(1)}, \bar{w}_{p_i}^{(2)} : i = 1 \ldots N/2)\,,$$

$$(w_{z_i};\, w_{p_i} : i = 1 \ldots N) \rightarrow (w_{z_i}^{(1)}, w_{p_i}^{(2)};\, w_{p_i}^{(1)}, w_{z_i}^{(2)} : i = 1 \ldots N/2)\,.$$

From the knowledge of the complex constants α and β from eq. (10) and eq.(11) one can calculate only the absolute value $|a_k|$ from eq. (9) to establish the EVA of the PWA. The unknown constant phase does not affect the position of the zeros and poles of the PWA approximant. This means that our procedure can give information only on the zeros and poles of the PWA. The EVA of the PWA is capable to give resonance parameters just as the full knowledge of the PWA (or VA). The numerical values of the parameters used in our mathematical model (eq. (6)) are given in table 1 (with N=2). The relevant zeros

Table 1

Parameters for eq. (6)

	a_k	W_Z	W_P
$F_1 e^{i\delta_1}$	-0.03484+i0.2687	1604.35+i72.083	1181.13-i153.999 (R)
$F_2 e^{i\delta_2}$	1.0+i0.0	1663.72+i754.103	1535.07-i91.191 (R)

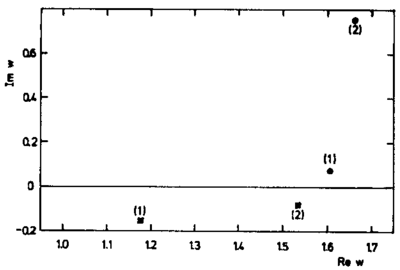

Fig.1. Zeros (•) and poles (x) for PWA model.

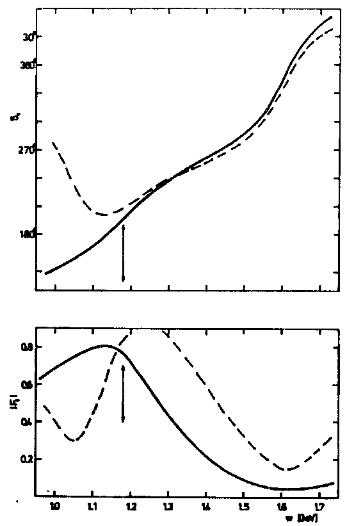

Fig.2. The energy dependence of $|F_1|$ and δ_1 (eq.(11) (by arrows the resonance positions are indicated). The dotted line is F_1 modified by the nonresonant backgrounds (eq.(14)).

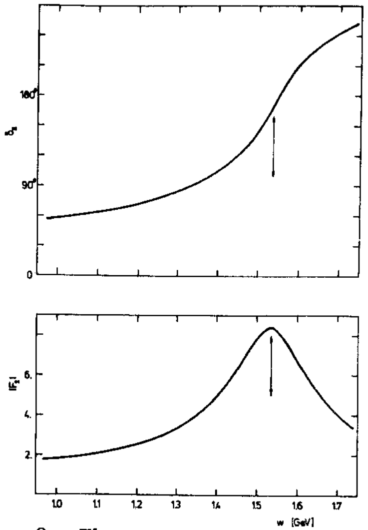

Fig.3. The energy dependence of $|F_2|$ and δ_2 (eq.(1)).

and poles are shown in Fig.1. The example chosen is similar to the real case of 0^-S and 0^-P PWA from the PWA analysis of diffractive 3π production on nuclei /1/. The PA used in our analysis is [2/2]. The energy behaviour of the amplitude modulus and PWA phase are shown in figs.2 and 3. Each PWA has resonance poles and we must find them from the knowledge of the $|F_1|$, $|F_2|$ and only the relative phase $\phi = \delta_1 - \delta_2$ (fig.4).

From inspection of the energy dependence of the PWA modulus we can suspect resonance behaviour in each PWA, but the same energy dependence of $|F_k|$ can be obtained using $\bar{F}_k$ instead of F_k, i.e., without the resonance pole but with a pole on the upper-half part of the complex energy plane. Moreover, if we examine the product and ratio of the two PWA intensities $|F_k|$ (Fig.5), only one resonance can be expected. This means that a simple Breit-Wigner analysis of $|F_k|$ or combinations of $|F_k|$ cannot decide in favour of the existence of a resonance.

A hypothesis, but not always reasonable, which can help in the decision on the existence of a resonance in a given PWA is to consider one PWA phase constant in the analysis of the other, i.e., the main energy dependence of the relative phase ϕ is given by the energy dependence of its PWA phase (at least around the resonance position):

$$F_1(w) = |F_1(w)| e^{i\delta_1(w)} \simeq C_1 |F_1(w)| e^{i\phi(w)} , \qquad (12)$$

$$F_2(w) = |F_2(w)| e^{i\delta_2(w)} \simeq C_2 |F_2(w)| e^{-i\phi(w)} . \qquad (13)$$

The PA analysis of eq. (12) and eq. (13) can give only an indication on the existence of a resonance pole in the lower-half part of the complex energy plane, but the resonance parameters are not correctly determined. The values of the zeros and poles obtained from the PA III analysis of eq. (12) and eq. (13) are given in table 2. Our method of finding the EVA of each PWA by using the PA analysis of the product and ratio (eq.(10) and eq.(11)) is more powerful. It gives the exact values of the resonance poles as well as of the zeros of each PWA. This conclusion becomes clear by examination of

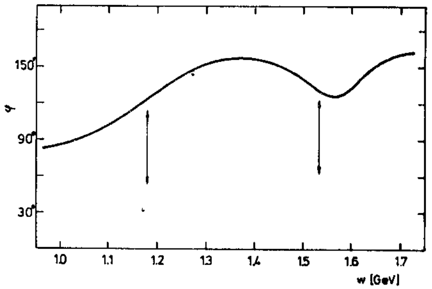

Fig.4. The energy dependence of the relative phase (eq.(2)).

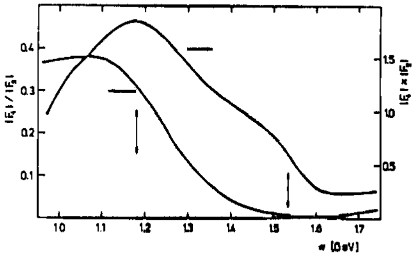

Fig.5. The energy dependence of the product $|F_1| \times |F_2|$ and the ratio $|F_1|/|F_2|$.

Table 2

[2/2] PA III analysis of eqs.(12) and (13)

	a	W_{Z1}	W_{Z2}
$F_1 e^{i\phi}$	0.0423+i0.282	1576.46-i200.88	1616.41+i69.62
$F_2 e^{-i\phi}$	3.547+i0.2043	1162.00-i330.74	1716.56-i139.8
		W_{P1}	W_{P2}
$F_1 e^{i\phi}$		1187.84-i148.18 (R)	1623.32+i213.6
$F_2 e^{-i\phi}$		1539.34-i102.03 (R)	1803.11+i401.23

Table 3

[2/2] PA III analysis of eqs. (10) and (11)

	$\|F_1\| \times \|F_2\| e^{i\phi}$		$\frac{\|F_1\|}{\|F_2\|} e^{i\phi}$	
i	W_{Zi}	$W^{(K)}$	W'_{Zi}	$W^{(K)}$
1	1604.35+i72.09	$W_Z^{(1)}$	1604.34+i72.09	$W_Z^{(1)}$
2	1663.71-i754.18	$W_Z^{(2)}$	1535.07-i.91.20	$W_P^{(2)}$
i	W_{Pi}	$W^{(K)}$	W'_{Pi}	$W^{(K)}$
1	1181.14-i.154.00	$W_P^{(1)}$	1181.13-i154.00	$W_P^{(1)}$
2	1535.08+i91.21	$W_P^{(2)}$	1663.58+i754.05	$W_P^{(2)}$

table 3, where there are presented the values of the zeros and poles obtained in such an analysis.

In order to simulate more accurately the real case, we have complicated the mathematical model of F_1 by introducing a non-resonant background of the type (eq.(3)):

$$F_B = (w - w_{z_2}^{(1)})\, e^{\gamma w}\,, \qquad (14)$$

where γ is a complex constant and F_R is our old example F_1

Table 4

PA III analysis for background case (eqs.3,14)

		$W_P^{(1)}$	$W_P^{(2)}$	$W_{Z_1}^{(1)}$	$W_{Z_2}^{(1)}$	$W_{Z_1}^{(2)}$
exact value		1181.13-i153.999	1535.07-i91.191	1050.-i50.	1604.35+i72.083	1663.72+i754.203
PA [3/2]	×	1178.7 -i158.19	1535.07-i93.84	1049.9-i49.3	1606.41+i72.57	1710.49+i626.65
	/	1177.1-i152.04	1535.61-i90.24	1051.0-i50.0	1604.41+i72.84	1537.35+i748.15
PA [3/3]	×	1180.99-i153.97	1534.99-i91.21	1050.-i49.99	1604.37+i72.17	1673.27+i744.73
	/	1181.17-i153.88	1535.09-i91.15	1050.-i50.	1604.34+i72.084	1663.03+i739.982
PA [4/3]	×	1181.13-i154.0	1535.07-i91.191	1050.-i50.	1604.35+i72.082	1663.08+i754.29
	/	1181.13-i153.999	1535.07-i91.190	1050.-i50.	1604.35+i72.082	1663.18+i754.007

(with the same parameters as in table 1). This new amplitude is shown in Fig.2. Without the exponential part in eq. (14), our method should give the exact results for zero and pole positions using only the [3/2] PA. Due to the exponential part, the resonance poles are shifted from the true position for [3/2] PA analysis, but [3/3] PA gives already a good answer. The [4/3] PA and [4/4] give the exact values of the poles as well as of the zeros of the F_1 and F_2, i.e., the true zeros and poles of the PWA become stable as the order of PA increases. The exponential part of the background is approximated by zeros and poles which have different values when the order of PA increases (these are the unstable zeros and poles). The values of the zeros and poles of the PWA with background are shown in table 4.

In conclusion, we have presented a new method of searching for resonance parameters in PWA, when only the intensity of two PWA and the relative phase between them are known. This method is based on the PA analysis of two combinations of the PWA which preserve the same relative phase or, in other words use the full experimental information obtained in a PWA analysis of coherently produced 3π. From such analysis one can ap proximate the PWA up to a constant phase by the EVA approximant which has the same zeros and poles as the PWA itself.

In practical application of this method to real experimental data we need to make use of the PA III analysis as well as subsequent analysis by a simultaneous fit of the data with eq. (9).

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Никитиу Ф., Займидорога О.А. E2-82-120
Метод определения параметров резонансов
по интенсивности и относительной фазе парциальной волны
с помощью паде-аппроксимации

Представлен метод,основанный на использовании паде-аппроксимации данных парциально-волнового анализа 3π системы, позволяющий найти стабильный полюс в амплитуде на нижней части энергетической комплексной области, отвечающей резонансу в амплитуде. Для совместного описания энергетической зависимости интенсивности и относительных фаз парциальных волн найден вид паде-аппроксимации и осуществлена проверка метода. Исследовано влияние фона на положение полюса. Развитый метод поиска стабильных полюсов в энергетической комплексной плоскости с помощью паде-аппроксимантов позволяет однозначно определить ширину и положение резонансов в парциальных амплитудах.

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Nichitiu F., Zaimidoroga O.A. E2-82-120
A Method of Determining Resonance Parameters
by Padé Approximants Using Intensities and Relative Phases
of the Partial Waves

A new method is presented for resonance parameters searching using Padé approximants and taking into account the intensities of two partial wave amplitudes and only their relative phase. The method is suitable for partial wave analysis of the 3π system produced in diffraction dissociation.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1982