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I.P.Nedyalkov

HIGH-ACCURACY MEASUREMENT OF THE FOUR-MOMENTUM OF TAGGED NEUTRINOS

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In recent years the method of tagged-beam acquires still wider application in various fields, in particular, in neutrino physics[1-3].

In this note we analyse an arrangement for tagging neutrinos the parameters of which are determined with high accuracy.

Lately it has become clear that to determine the density distribution of the Eath by neutrino experiments, it is necessary to measure the neutrino flux with a relative accuracy of the order of 0.1-0.01%[4-6]. The problem of investigation of the Earth by neutrino beams is considered also in some other papers[7-11]. However, in these papers the required relative accuracy either is not defined or is established under nonrealistic conditions. Meanwhile, a ceraful analysis shows that in all cases the geophysical neutrino experiment aimed to determine the Earth's density distribution necessarily requires a very high accuracy of the measurement of the neutrino flux.

In ref.[11] the assumption is made that success along this line can be provided by using tagged neutrino beams. An idea of the method of performing the tagging so as to assure high accuracy of the measurement of the neutrino flux and of the four-momentum of each neutrino was proposed in ref.[4]. The method provides an arrangement which consists of an evacuated tube and several coordinate planes situated within it. In two successive coordinate planes the angle of flight of the parent π-meson is measured before it decays in the decay volume into the μ-meson and neutrino ν. In a similar way the angle of the produced μ-meson is measured*. Besides, the energies of these particles are determined by measuring the angles of rotation in magnetic fields acting in separate sections of the tube. Therefore, the pion and muon four-momenta, p_π and p_μ, are known. From these data the neutrino four-momentum, p_ν, is calculated by the formula: $p_\nu = p_\pi - p_\mu$.

As an illustration we shall consider a system of generation of neutrino beams the flux of which is measured with a relative accuracy Δ_N of the order of 0.01%. A beam of this

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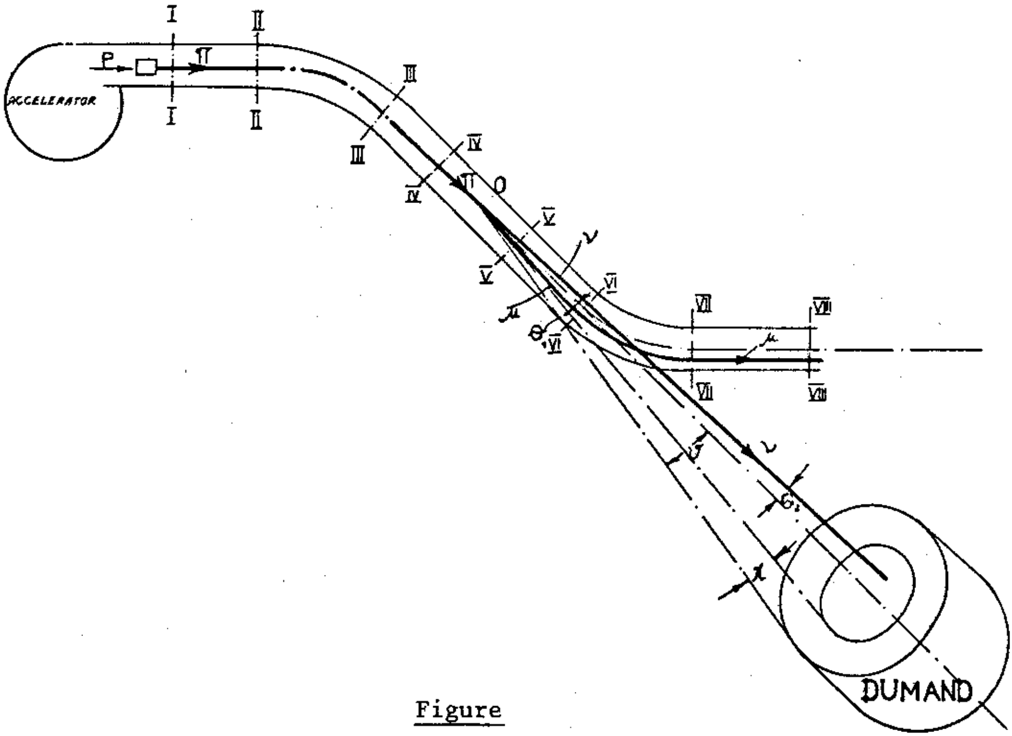
*Here we consider the π-meson beam, however, the idea is applicable for K- and π+ K-meson beams, too.



Figure

type is necessary for the geophysical neutrino experiment if the energy of the proton beam that generates parent π- and K-mesons E_p=1 TeV or E_p = 3 TeV. At E_p=20 TeV Δ_N may be of the order of 0.1% /5/.

The same system can be used also for other, nongeophysical experiments, where Δ_N may be of the order of 1% and more.

Therefore, if the system with Δ_N=0.01% cannot be realized at the present level of experimental technique, then it may be useful for neutrino geophysical experiments where E_p=20 TeV and Δ_N= 0.1% and for nongeophysical experiments with still lower accuracy.

We proceed to describe the system. Let in the coordinate planes I-I,....VII-VII and VIII-VIII, which are placed in the evacuated tube (Figure), one registers coordinates x and y of pions travelling through this tube. Denote by $a^{(1)}, \ldots a^{(7)}$ the distances between planes I-I and II-II,....VII-VII and VIII-VIII, respectively, and by $V^{(1)}, \ldots V^{(7)}$ the regions defined by these planes and inner surfaces of the tube. We shall consider only those processes $\pi \to \mu + \nu$, which occur in $V^{(4)}$, so that despite that π-mesons decay also in other parts of the tube, our "decay-tunnel" will be just $V^{(4)}$.

Let us denote by Q the plane passing through the axis of the decay-tunnel and through the S-axis of the muon beam, and by P- the plane passing through S so that $P \perp Q$.

Straight lines, in which planes V-V and VI-VI cut P and Q, form rectangular systems of coordinates. Let $x^{(5)}$, $y^{(5)}$ be coordinates of a point in plane V-V, through which the trajectory T_μ of a given μ-meson passes, and let $\alpha^{(5)}$ be the angle of intersection of the projection T_μ onto P with S-axis. Let $\sigma_x^{(5)}$ be the standard deviation of the error of measurement of $x^{(5)}$. Then at $y^{(5)} = 0$, $\alpha^{(5)} = \frac{x^{(6)} - x^{(5)}}{a^{(5)}}$ and the standard deviation of its error $\tilde{\sigma}_\alpha^{(5)} = \frac{\sqrt{2}\,\sigma_x^{(5)}}{a^{(5)}}$, as by assumption $\sigma_x^{(6)} = \sigma_x^{(5)}$ and $x^{(5)}$ and $x^{(6)}$ are independent random numbers. If x and y are measured by means of proportional chambers /12,13/ one may put $\sigma_x^{(1)} = \ldots = \sigma_x^{(7)} = \sigma_x = 0.1$ mm. Then, for instance, $a^{(5)} = 300$ m, $\tilde{\sigma}_\alpha^{(5)} \approx 0.3 \cdot 10^{-6}$.

Since the μ-meson is a charged particle, while passing through the proportional chamber it suffers a multiple scattering. Therefore the initial error of measurement $\alpha^{(5)}$ should be added by an order with the standard deviation

$$\tilde{\tilde{\sigma}}_\alpha^{(5)} = 0{,}756\,\frac{\sqrt{b}}{E_\mu}\,\sqrt{\ln(6900b)} \cdot 10^{-6}, \qquad (1)$$

where E_μ is the muon energy in TeV, b is the thickness of the wall of proportional chambers in the beam direction in mm. In this case it is assumed that walls are made of aluminium.

Formula (1) is nothing but formula (1.102) from ref. /14/ written in our notation. In formula (1) it has been taken into account that the μ-meson crosses twice the wall of thickness b, i.e., when it enters and leaves the proportional chamber.

At $b = 0.1$ mm*, $E_p = 1$ TeV and $E_\mu = 0.286$ TeV** and from eq. (1) we obtain $\tilde{\tilde{\sigma}}_\alpha^{(5)} = 1.65 \cdot 10^{-6}$. If we take into account the multiple scattering in the gas when the chamber is filled with A_r under the pressure of 1 atm, then $\tilde{\tilde{\sigma}}_\alpha^{(5)}$ increases by about 1%.

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* This value of b is very much larger than that required from the strength of the material. It is chosen so as to make minimal the risk of the gas leakage from the chamber into the evacuated tube.

** By assumption $E_\pi \approx 0{,}5E_p$ and $E_\mu = \frac{m_\mu^2}{m_\pi^2} E_\pi$, where $m_{\pi(\mu)}$ is the $\pi(\mu)$-meson mass, E_π is the π-meson energy.

Apparently, the pion multiple scattering caused by nuclear forces is considerably smaller then the scattering due to electromagnetic forces. Therefore, (1) may be used to calculate $\tilde{\tilde{\sigma}}_a^{(3)}$. Notation $x^{(3)}$, $y^{(3)}$ for the π-meson is analogous to $x^{(5)}$, $y^{(5)}$ used for μ-mesons.

For $\tilde{\tilde{\sigma}}_a^{(3)}$ we have

$$\tilde{\tilde{\sigma}}_a^{(3)} = \frac{m_\mu^2}{m_\pi^2} \tilde{\tilde{\sigma}}_a^{(5)} = 0{,}945 \cdot 10^{-6} ;$$

and for $\sigma_a^{(3)}$,

$$\sigma_a^{(3)} = \sqrt{[\tilde{\sigma}_a^{(3)}]^2 + [\tilde{\tilde{\sigma}}_a^{(3)}]^2} = 1.3 \cdot 10^{-6} .$$

In a similar way we obtain that $\sigma_a^{(5)} = 1.8 \cdot 10^{-6}$.

It what follows by θ_1 we denote the angle made by the direction T_μ of a given μ-meson and the direction T_π of that π-meson which has produced this μ-meson. By θ_2 we denote the angle made by T_π and T_ν with θ_2, T_ν being the direction of the neutrino which is produced simulatenously with the μ-meson. If T_π and T_μ are in P, then the random quantity θ_1 has the expectation value $\overline{a^{(3)}} - \overline{a^{(5)}}$ and the standard deviation

$$\sigma_1 = \sqrt{[\sigma_a^{(3)}]^2 + [\sigma_a^{(5)}]^2} = 2{,}2 \cdot 10^{-6} .$$

If T_π and T_μ are not in P, then the standard deviation is $\sqrt{2}$ times larger and $\sigma_1 = 3.1 \cdot 10^{-6}$.

When θ_1 fluctuates, so does θ_2. Let σ_2 be a standard deviation of fluctuations of θ_2. If we neglect the fluctuations of E_π and θ_π, then in the ultrarelativistic limit we obtain the following relation between σ_1 and σ_2:

$$\sigma_2 = k\sigma_1 , \qquad (2)$$

where

$$k = (m_\mu^2 m_\pi^{-2} + \theta_2 \psi^{-2})(1 - m_\mu^2 m_\pi^{-2})^{-1}(\theta_2^2 \psi^{-2} - m_\mu^2 m_\pi^{-2})^{-1},$$

$$\psi = E_\pi^{-1} m_\pi .$$

Now let the detector be a circular cylinder with the axis coinciding with the neutrino-beam axis (Figure). Let an observer in the center of the decay volume see the detector radius at angle θ. If $\theta_2 \lesssim \theta$. then, because of the fluctuations, a part of neutrinos in an uncontrolled way fall either inside or outside the detector. To avoid the uncertainty caused by this fact, one should count only those neutrinos, which pass through a circle with radius $\theta-\chi$ with a probability $\geq 1-10^{-4}$ (10^{-4} being the error of the measurement of the neutrino flux). If the distribution of the random quantity θ_2 is Gaussian, then χ is defined by the formula

$$\chi = \sigma_2 \sqrt{2\ln(10^4)} .$$

As has been noticed above, very high accuracies of measurement of the intensity of the neutrino beam are necessary for geophysical neutrino experiments aimed at studying the Earth structure. In this case, if the neutrino beam is produced at a given point on the Earth surface, its detection on the opposite side of the Earth is performed by the gigantic detector DUMAND /4-6/. Since the size of DUMAND is of the order of 1 km, for θ we obtain a value of the order of 10^{-4} rad, which coincides in order of magnitude with the angle ψ at $E_p = 1$ TeV. Let us now make use of data of the numerical experiment from /5/ at $E_p = 1$ TeV. If θ_2 changes in the interval $0 < \theta_2 < \theta$ the coefficient k grows from $k = 1.34$ at $\theta_2 = 0$ up to $k = 1.55$ for $\theta_2 = \theta$. For $k = 1.55$ from (2) we find $\sigma_2 = 4.8 \cdot 10^{-6}$ and $\chi = 2.1 \cdot 10^{-5}$. If G is the physical volume of the detector, its fiducial volume is

$$F = \frac{(\theta-\chi)^2}{\theta^2} G \approx 0.5\, G .$$

This result corresponds to the DUMAND with approximate sizes 1 km x 1 km x 1 km. For the same volume the fiducial volume grows considerably if it will have sizes 2 km x 2 km x x 0.25 km.

The superhigh accuracy $\Delta_N \approx 0.01\%$ of the measurement of the neutrino flux is necessary at energies E_p of the order of 1 TeV. At $E_p = 20$ TeV and for the same sizes of the DUMAND the necessary accuracy is diminished to about $\Delta_N \approx 0.1\%$ and in this case the realization of the above system is very simplified.

As is mentioned above in some nongeophysical experiments Δ_N may be of the order of 1%, which will make the realization of the experiment still more easy.

Problems concerning the intensity of the neutrino beam will be considered elsewhere. Here we only note that if the geophysical experiment with neutrinos is performed at $E_p = 20$ TeV, then due to the high-energy effects one can decrease the intensity $N_{\pi+K}$ of parental mesons to $N_{\pi+K} \approx 10^6$ particles sec^{-1}. At this value of $N_{\pi+K}$ the use of proportional chambers avoids additional complications. Besides, at $E_p = 20$ TeV the decrease of $N_{\pi+K}$ does not make worse the accuracy of determination of the density distribution of the Earth. At $E_p = 1$ TeV and 3 TeV the unfavourable effect of low energies can partly be compensated by the use of a sufficiently great number of proportional chambers.

The described method is noteworthy, in the first place, by a high accuracy of the determination of parameters of the tagged neutrino beam. Besides, in contrast to other methods [1-3] it is applicable both to wide and narrow beams, the accuracy of measurement being not affected. Another advantage consists in that simultaneously with the tagging of a neutrino, the type of reaction, where the neutrino is produced, is established.

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