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QCD ANALYSIS OF MUON-NUCLEON DEEP-INELASTIC SCATTERING DATA

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Recent measurements [1] of a muon-carbon deep-inelastic scattering provide new data on the $F_2^{\mu N}(x,Q^2)$ structure function as well as on its moments. This data allows one to extend the confrontation of the asymptotically free gauge theory with the experiment.

The QCD analysis performed in paper [1] in a valence quark (nonsinglet) approximation has produced a small value of Λ both from the F_2 using the evolution equation technique [2,3] and from the moments of F_2. The nonsinglet approximation is justified if the contributions to the structure function from sea quarks and gluons are negligibly small. The latter is not clear from the present experiments [4]. Thus, it is interesting to see what would be the results of a more sophisticated QCD analysis.

As is known [5] the F_2 structure function measured from muon interactions with isoscalar target is an almost pure singlet. In the Q^2-region above 30 GeV^2 the effects of higher twists [6] are expected to be negligibly small and one can use the singlet formulae for the moments, $M_2(n,Q^2)$, of $F_2^{\mu N}(x,Q^2)$ [7].

The even moments are given in ref. [1] for $n=4$ and 6 only. This restriction is due to the fact that the structure functions were measured in the interval $x = 0.3$-0.7. The 4th and 6th moments calculated in the interval $Q^2 = 30 \div 110$ GeV^2 have more than 65% contribution from the experimental data and the rest from data extrapolation. The determination of higher moments would lead to a large errors due to the unseen x region. For the same reasons there is a little advantage in usage for the $F_2^{\mu N}(x,Q^2)$ analysis of the integro-differential equations [3] which in principle require also a knowledge of the structure function behaviour from a fixed x up to $x=1$. While extrapolation to $x=0$ is less important for the moments, because $x^{n-2}F_2$ approaches zero, the extrapolation to $x=1$ is complicated by nuclear effects [8].

The Q^2 evolution of the n-th moment of the singlet combination up to the second order in a running coupling constant is given by the formulae [9]:

$$M_{2,s}(n,Q^2)=$$

$$=Q_{2,s}(n,Q_0^2)\{A\,(n,Q_0^2,Q^2)[\frac{\bar\alpha(Q^2)}{\alpha(Q\;)}]^{\gamma_-^n/2\beta_0}+A_2(n,Q_0^2,Q^2)[\frac{\bar\alpha(Q^2)}{\bar\alpha(Q_0^2)}]^{\gamma_+^n/2\beta_0}\}- \tag{1}$$

$$-G_2(n,Q_0^2)\{B_1(n,Q_0^2,Q^2)[\frac{\bar\alpha(Q^2)}{\bar\alpha(Q_0^2)}]^{\gamma_-^n/2\beta_0}+B_2(n,Q_0^2,Q^2)[\frac{\bar\alpha(Q^2)}{\bar\alpha(Q_0^2)}]^{\gamma_+^n/2\beta_0}\},$$

where

$$\bar\alpha(Q^2)=\bar\alpha_0(Q^2)[1-\frac{\beta_1}{4\pi\beta_0}\bar\alpha_0(Q^2)\ln\ln Q^2/\Lambda^2], \tag{2}$$

$$\bar\alpha_0(Q^2)=4\pi/\beta_0\ln(Q^2/\Lambda^2);\quad \beta_0=11-\frac{2}{3}N_f\;;\;\beta_1=102-\frac{38}{3}N_f \tag{3}$$

and $(i=1;2)$

$$A_i(n,Q_0^2,Q^2)=A_{i1}^n+A_{i2}^{(2),n}\frac{\bar\alpha(Q^2)}{4\pi}+A_{i3}^{(2),n}\frac{\bar\alpha(Q_0^2)}{4\pi},$$

$$B_i(n,Q_0^2,Q^2)=B_{i1}^n+B_{i2}^{(2),n}\frac{\bar\alpha(Q^2)}{4\pi}+B_{i3}^{(2),n}\frac{\bar\alpha(Q_0^2)}{4\pi}.$$

The numerical values of A_{ij}, B_{ij} and $\gamma_\pm^n$ coefficients are taken for $N_f=4$ from refs. /9/ and /10/. These coefficients were calculated using the dimensional regularization scheme with a minimal subtraction (MS) renormalization prescription*.

The parameters $Q_{2s}(n,Q_0^2)$ and $G_2(n,Q_0^2)$ have the meaning of the n-th moment of quark singlet and gluon distributions taken at a reference point $Q^2=Q_0^2$ /12/ Their values are not directly calculated from the basic QCD renorm-group equations. To find them one has to know the wave functions of quarks and gluons inside the hadrons. The solution of this problem requires the extension of QCD calculus beyond the framework of perturbative theory. So at a present stage of the theory they have to be taken from experiments.

The unconstrained fit of data /1/ by formula (1) is unreasonable due to too many parameters ($Q_{2s}(n,Q_0^2)$, $G(n,Q_0^2)$ for $n=4,6$ and Λ) that are strongly correlated in a case of small scaling violation (small Λ). Moreover in this case the factors $[\bar\alpha(Q^2)/\bar\alpha(Q_0^2)]^{\gamma_+/2\beta_0}$ and $[\bar\alpha(Q^2)/\bar\alpha(Q_0^2)]^{\gamma_-/2\beta_0}$ would not be much different. From /9,10/ one can find that $A_{11}^n+A_{21}^n=1$ but $B_{11}^n+B_{21}^n=0$. So the gluon moment $G_2(n,Q_0^2)$ is multiplied

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* The values of A_{ij} and B_{ij} recalculated in paper /11/ differ from the used ones by less than 1%.

by a small factor and its determination from the fit is very uncertain* . For these reasons we have fixed the ratio

$$G_2(n,Q_0^2)/Q_{2s}(n,Q_0^2)=k$$

and performed the fitting of the data by expr. (1) with Λ and $Q_{2,s}(n,Q_0^2)$ taken as free parameters and k being independent of the moment number. The latter assumption does not contradict to muon data/4/.

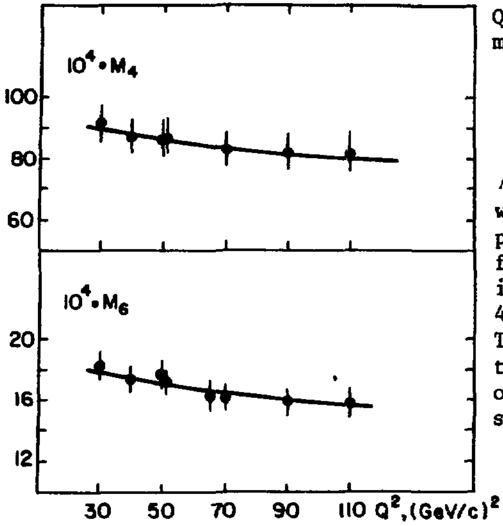
The fit is performed for various k from 0 to 1 and the reference point Q_0^2 is taken at 5 GeV^2. Although the value of Λ is meaningless while using the leading order approximation/13,14/ to see the effect of the next order we present in the Table the best fit values of Λ found both for the case of the second order corrections included (SOC ON) and not included (SOC OFF). The comparison of the fit ($k=0.5$ SOC ON case) with the experiment is shown in the Figure. The errors of Λ contain statistical ($\Delta\Lambda$ stat) and systematical uncertainties ($\Delta\Lambda$ syst.) (see/1/ for details). The values of $Q_{2s}(n,Q^2)$ found from the fit are: $Q_{2s}(4,Q_0^2=5)=0.0105\pm0.009$ and $Q_{2s}(6,Q_0^2=5)=0.0022\pm0.002$. They are constant within $\pm4\%$ when k varies from 0 to 1. Note that the parameters of the best fit values are obtained with the values of χ^2 per degree of freedom $\chi^2_{d.f.}\leq 0.3$.

Table

The best fit values of Λ_{MS} found from QCD analysis of BCDMS data in two cases: SOC ON - when the second order corrections are taken into account; SOC OFF - in the leading order of the running coupling constant

$k=\dfrac{G_2(n,Q_0^2)}{Q_{2s}(n,Q_0^2)}$	$\Lambda_{MS}\pm\Delta\Lambda_{stat}\pm\Delta\Lambda_{syst}$ (MeV) SOC ON	SOC OFF
0.00	$12^{+32}_{-12}\,{}^{+34}_{-8}$	$41^{+79}_{-41}\,{}^{+71}_{-35}$
0.05	$15^{+34}_{-15}\,{}^{+39}_{-11}$	$39^{+81}_{-39}\,{}^{+78}_{-31}$
0.20	$18^{+39}_{-18}\,{}^{+34}_{-13}$	$44^{+88}_{-44}\,{}^{+79}_{-35}$
0.50	$27^{+48}_{-27}\,{}^{+44}_{-20}$	$60^{+104}_{-60}\,{}^{+86}_{-47}$
1.00	$49^{+72}_{-49}\,{}^{+57}_{-35}$	$80^{+128}_{-80}\,{}^{+110}_{-55}$

* The same problem is discussed in paper/18/.



QCD fit of 4th and 6th moments.

To see which values of Λ are still consistent with the new data we have performed a fit with the fixed value of Λ_{MS} varying it from 1 MeV up to 450 MeV in steps of 6 MeV. The results have shown that a good description of the data is still possible with

$$0 < \Lambda_{MS} \lesssim 100 \text{ MeV}.$$

As is seen from the Table, the central values of Λ, all smaller than those found from other deep inelastic experiments[4,5], are varied by a factor 4 within the accepted limits of the gluon contributions. Comparing with the nonsinglet analysis[1] ($\Lambda = 32$ MeV) we see that the singlet formulae produce even smaller Λ ($\Lambda = 12$ MeV) if one neglects the gluons. Due to the energy-momentum conservation the 2nd moments of quarks and gluons do not differ much from one another even at $Q^2 \to \infty$ [15]. The analysis of μ(e)N data [4,16] has shown that for higher moments up to $n = 6$ the ratio of gluon and quark moments is between 0.9 and 0.6 at $Q^2 = 5$ GeV^2 which is our reference value. So with the higher probability the true value of Λ is that between the two last lines of the Table, i.e., $\Lambda = 40$ MeV. This value is in good agreement with the result obtained from the QCD analysis of e^+e^- annihilation into hadrons[17] although the higher value of Λ is still possible within the statistic and systematic errors. More precise data extended to higher energies which are expected from CERN muon experiments could reduce this uncertainty.

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