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DETERMINATION OF THE DEGREE OF THE $\Delta I = 1/2$ RULE VIOLATION AND THE ESTIMATION OF THE S-WAVE PION-PION SCATTERING LENGTHS FROM THE DATA ON $K \to 3\pi$ DECAYS

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DETERMINATION OF THE DEGREE OF THE Δ I = 1/2 RULE VIOLATION AND THE ESTIMATION OF THE S-WAVE PION-PION SCATTERING LENGTHS FROM THE DATA ON $K \to 3\pi$ DECAYS

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Определение степени нарушения правила $\Delta T=1/2$ и оценка s-волновых длин пион-пионного рассеяния из данных по $K\to3\pi$ распадам

Из полуфеноменологического анализа данных по $K\to3\pi$ распадам определен вклад перехода $\Delta T=3/2$ в амплитуды, парциальные ширины и спектры π-мезонов различных каналов $K\to3\pi$. Определены допустимые значения для s-волновых длин $\pi\pi$-рассеяния a_0 и a_2 в изотопических состояниях $T=0$ и $T=2$.

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Determination of the Degree of the $\Delta I=1/2$ Rule Violation and the Estimation of the s-Wave Pion-Pion Scattering Lengths from the Data on $K\to3\pi$ Decays

The analysis is given of experimental data on $K\to3\pi$ decays which is based on the semi-phenomenological theory of three strong-interacting particle production near threshold.

The contribution of $\Delta I=3/2$ transition to the amplitudes partial widths and pion spectra of various $K\to3\pi$ decay modes has been determined.

The allowed values for the s-wave pion-pion scattering lengths a_0 and a_2 in the isotopic states $I=0$ and $I=2$ have been obtained.

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1. Introduction

Since the appearance of Gribov's paper /1/ where the possibility of the experimental observation of the $\pi\pi$-scattering effects in $K\to3\pi$ decays has been indicated some attempts /2-5/ have been made to determine the pion-pion scattering lengths from the data on these decays. However, the insufficiency of experimental data has resulted in rather approximate evaluations of scattering lengths. New data on ~1.5 million $\tau^{\pm}$-decays /6/ (Dalitz plot distribution) and ~0.5 million $K_L^\circ\to\pi^+\pi^-\pi^\circ$ decays /7/ (the π°-meson energy spectrum) are much more accurate than all the experimental data available up to now. These data /6,7/ have been analyzed separately in refs. /8,9/ from the point of view of the theory of three strong-interacting particle production near threshold /1,10/. The analysis performed in /8,9/ has shown that the observed deviation from the $|\Delta I|=1/2$ rule, which predicts the definite relationship between the inclination parameters in pion spectra of the $K^{\pm}\to\pi^{\pm}\pi^{\pm}\pi^{\mp}$ and $K_L^\circ\to\pi^+\pi^-\pi^0$ decays cannot be explained by taking into account particle interaction in the final state and that in the combined analysis of data on the $K\to3\pi$ decays one must take into consideration the effects of the $|\Delta I|=1/2$ rule violation.

Here the combined analysis of the available experimental data on

$$K^{\pm}\to\pi^{\pm}\pi^{\pm}\pi^{\mp} \quad (1)$$

$$K_L^\circ\to\pi^+\pi^-\pi^\circ \quad (2)$$

$$K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ} \qquad (3)$$

$$K^{\circ}_{L} \to \pi^{\circ}\pi^{\circ}\pi^{\circ} \qquad (4)$$

decays has been performed in order to determine the s-wave $\pi\pi$-scattering lengths a_0 and a_2 in the isotopic states $I=0$ and $I=2$ and the degree of the $|\Delta I|=1/2$ rule violation.

II. Determination of the Degree of the $|\Delta I|=1/2$ Rule Violation

The relation between the amplitudes and, hence, between the probabilities and the spectra of final particles in various decay schemes is determined by the isotopic rules valud in the $K\to 3\pi$ decays. The rule $|\Delta I|=1/2$ holds well in the majority of nonlepton decays of strange particles. This rule is noticeably violated in $K\to 3\pi$ decays.

The analysis is made under the assumption that the $|\Delta I|=1/2$ rule is violated in $K\to 3\pi$ decays due to the $|\Delta I|=3/2$ transition to the $I=1$ state (I is the total isotopic spin of final pions). The qualitative estimations show that the contribution of other transition compared to that of the indicated transition is small /11-13/. The analysis neglects the effects of CP-invariance violation. Their contribution to the amplitudes of $K\to 3\pi$ decays is negligible. It has been studied in ref. /14/

Denote the isotopic amplitudes of $K\to 3\pi$ decays as the functions of the pion relative momenta k_{ij} (taken as kinematic variables) by $A_{1,2\Delta I}(k_{12},k_{13},k_{23})$. The index "3" of k_{ij} momenta will refer to the "odd" pion in decays (1), (3) and to the π°-meson in decay (2).

In the above approximation the isotopic structure /13/, of the amplitudes of (1)-(4) is as follows:

$$A_1 = -[A_{11}(k_{13};k_{12},k_{23}) + A_{11}(k_{23};k_{12},k_{13})] + \frac{1}{2}[A_{13}(k_{13};k_{12},k_{23}) + A_{13}(k_{23};k_{12},k_{13})]$$

$$A_2 = -A_{11}(k_{12};k_{13},k_{23}) - A_{13}(k_{12};k_{13},k_{23})$$

$$A_3 = A_{11}(k_{12};k_{13},k_{23}) - \frac{1}{2}A_{13}(k_{12};k_{13},k_{23}) \qquad (5)$$

$$A_4 = A_{11}(k_{12};k_{13},k_{23}) + A_{11}(k_{13};k_{12},k_{23}) + A_{11}(k_{23};k_{12},k_{13}) + A_{13}(k_{12};k_{13},k_{23}) + A_{13}(k_{13};k_{12},k_{23}) + A_{13}(k_{23};k_{12},k_{13}).$$

The amplitudes $A_{1,2\Delta I}(k_{12};k_{13},k_{23})$ are symmetric on two last variables and to an accuracy of cubic terms on k_{ij} have the following form:

$$A_{1,2\Delta I}(k_{12};k_{13},k_{23}) = \lambda_{1,2\Delta I}\{1+\delta_{1,2\Delta I}k_{12}^2 + \mathcal{F}(a_0,a_2;k_{12},k_{13},k_{23})\} \qquad (5a)$$

($\Delta I = 1/2, 3/2$); $\lambda_{11}, \lambda_{13}, \delta_{11}, \delta_{13}$ are real constants.

The last term in expression (5a) is a nonanalytical function of the relative momentum square k_{ij}^2. It arises due to the account of pion rescattering in the final state and depends upon the s-wave $\pi\pi$-scattering lengths a_0 and a_2. The function $\mathcal{F}(a_0,a_2;k_{12},k_{13},k_{23})$ with the account of linear, quadratic and cubic terms on k_{ij} has been calculated in /1,10/.

The matrix elements squared of (1)-(4) are as follows (to an accuracy of cubic terms):

$$|M_1|^2 = 2(\lambda_{11} - \frac{1}{2}\lambda_{13})^2\{1 + C_1(k_{13}^2 + k_{23}^2) + f_1(a_0,a_2;k_{12},k_{13},k_{23})\}$$

$$|M_2|^2 = (\lambda_{11} + \lambda_{13})^2\{1 + 2C_2k_{12}^2 + f_2(a_0,a_2;k_{12},k_{13},k_{23})\}$$

$$|M_3|^2 = \frac{1}{2}(\lambda_{11} - \frac{1}{2}\lambda_{13})^2\{1 + 2C_1k_{12}^2 + f_1(a_0,a_2;k_{12},k_{13},k_{23})\}$$

$$|M_4|^2 = \frac{3}{2}(\lambda_{11} + \lambda_{13})^2\{1 + 2C_2k_0^2 + f_0(a_0,a_2;k_{12},k_{13},k_{23})\}. \qquad (6)$$

Formulas (6) have taken into account the factors arising due to the presence of identical pions in channels (1), (2) and (4). The nonanalytical functions f_1, f_2 (symmetric with respect to the replacement of the k_{ij} indices "1" and "2") are obtained with squared modules of amplitudes (5); they are shown in [2,5]. Similarly the function f_0 (symmetric with respect to the replacement of all indices k_{ij}) is found; k_0^2 is the square of the relative momentum in the centre of the Dalitz plot for reaction (4).

The coefficients C_1 and C_2 giving a constant inclination to the spectrum of the "odd" pion in decay modes (1) and (3) and of the π°-meson in (2) are expressed by the λ_{11}, λ_{13} amplitudes and the δ_{11}, δ_{13} coefficients as follows:

$$C_1 = \frac{\lambda_{11}\delta_{11} - \frac{1}{2}\lambda_{13}\delta_{13}}{\lambda_{11} - \frac{1}{2}\lambda_{13}} \tag{7}$$

$$C_2 = \frac{\lambda_{11}\delta_{11} + \lambda_{13}\delta_{13}}{\lambda_{11} + \lambda_{13}}$$

If only the transition $|\Delta I| = 1/2 (\lambda_{13} = 0)$ takes places, then

$$C_2/C_1 = 1. \tag{8}$$

Note that relation (8) is a necessary but not sufficient condition for the transition $\Delta I = 3/2$ to be absent. Thus, with $\lambda_{13} \neq 0$, but $\delta_{13} = \delta_{11}$, again $C_2/C_1 = 1$.

The following experimental data have been used for our analysis:

a) Ford's et al. [6] data on 1.5 million $\tau^\pm$-decays: the distribution of probability density on the Dalitz plot (the number of bins $n_1 = 145$),

b) Messner's et al. data [7] on ~0.5 million $K^\circ_L \to \pi^+\pi^-\pi^\circ$ decays: π°-meson energy distribution (the number of bins $n_2 = 17$),

c) the partial widths of (1)-(4) decay modes [11].

When comparing formulas (6) with experimental data the least squares method has been applied. By the parameters of expression (6) a_0, a_2, λ_{11}, λ_{13}, C_1, C_2 the functional

$$F = \sum_{m=1}^{2} \sum_{i=1}^{n_m} \left[\frac{\rho^e_{mi} - N_m \rho^t_{mi}(a_0, a_2, C_1, C_2)}{\Delta\rho^e_{mi}} \right]^2 + \tag{9}$$

$$+ \sum_{m=1}^{4} \left[\frac{\Gamma^e_m - \Gamma^t_m(\lambda_{11}, \lambda_{13}, C_1, C_2, a_0, a_2)}{\Delta\Gamma^e_m} \right]^2$$

has been minimized.

The first term of expression (9) refers to the distributions a) and b) for reactions (1) and (2) $(m=1,2)$. The second term refers to the partial widths of decays (1)-(4) $(m=1,2,3,4)$; ρ^e_{mi} and $\Delta\rho^e_{mi}$ $(i=1,\ldots,n_m)$ are experimental values of the distribution function and their errors for the m-th channel. The theoretical values of $\rho^t_{mi}(a_0, a_2, C_1, C_2)$ are the integrals over the corresponding bins of the matrix element $|M_m|^2$. N_m are normalizing factors. Γ^e_m and $\Delta\Gamma^e_m$ $(m=1,\ldots,4)$ are experimental values of the partial widths normalized to the proper phase volumes (the phase volume of the τ-decay was taken as a phase volume unit) and their errors. The theoretical values $\Gamma^t_m(\lambda_{11}, \lambda_{13}, C_1, C_2, a_0\, a_2)$ are

$$\Gamma^t_m = \frac{\int |M_m|^2 d\gamma_m}{\int d\gamma_m}, \quad (m=1,2,3,4).$$

Functional (9) was minimized by the linearization of the quadratic functional [16] be means of the FUMILI standard subroutine. The result of fitting giving two solutions of satisfactory coincidence is presented in Table 1. α_1 and α_2 are the combinations of the λ_{11}

and λ_{13} amplitudes taken as fitted parameters: $a_1 = (\lambda_{11} - \frac{1}{2}\lambda_{13})^2$; $a_2 = (\lambda_{11} + \lambda_{13})^2$.

Table 1 presents the values characterizing the contribution of the $\Delta I = 3/2$ transition to the $K \to 3\pi$ amplitudes: $\lambda_{13}/\lambda_{11}$, δ_{13}/δ_{11}, $\lambda_{13}\delta_{13}/\lambda_{11}\delta_{11}$ as well as the contribution of the $\Delta I = 3/2$ transition to partial widths and to the pion spectra slopes for (1) and (2) (in the last lines of Table 1).

Table 2 gives a comparison of the values of (1)-(4) mode widths obtained by fitting with experimental data /11/.

As is seen from Table 1, the transition $\Delta I = 3/2$ increases the probability of τ -decays by 6-7% and reduces that of the $K^\circ \to \pi^+\pi^-\pi^\circ$ decays by 13-15%. The coefficient of the slope of the "odd" meson spectrum in τ -decays is reduced by 13-14%, the slope of the spectrum of π°-meson in the $K^\circ_L \to \pi^+\pi^-\pi^\circ$ decay is increased by 21-22%.

Another variant of the analysis included in the fit both data on a) - c) and experimental data on the energy spectrum of π^+-meson in the τ' -decay. Note that the data of various studies have a bad statistical accuracy and do not agree with each other. We have included in our analysis the data obtained by Davison et al. /17/, whose statistics is somewhat greater than in other experiments (3406 events with $T_{\pi^+} > 8$ MeV). The obtained parameters do not practically differ from those given in Table 1.

III. s -Wave Pion-Pion Scattering Lengths

As is seen from Table 1, there are two allowed regions (having sufficient confidence levels) for $\pi\pi$ - scattering lengths. Figs. 1 and 2 show the theoretical spectra of pions in (1) and (2) decays calculated for the parameters from solution 1. The contribution of nonanalytical terms causing the deviation of the spectra from linearuty for τ -decays does not exceed 1%, while for the $K^\circ_L \to \pi^+\pi^-\pi^\circ$ decay this contribution reaches 10% at the edge of the spectrum.

Table 1

$\bar{\chi}^2 = 158$	I solution $\chi^2 = 172.6$ (CL=20%)	II solution $\chi^2 = 164.7$ (CL=35%)
a_0	$(0.59\pm0.07)\,\lambda_\pi$	$-(0.43\pm0.04)\,\lambda_\pi$
a_2	$-(0.20\pm0.03)\,\lambda_\pi$	$(0.10\pm0.05)\,\lambda_\pi$
C_1	0.47 ± 0.09	0.56 ± 0.07
C_2	0.86 ± 0.09	0.94 ± 0.07
α_1	1.59 ± 0.02	1.59 ± 0.01
α_2	1.13 ± 0.02	1.10 ± 0.02
C_2/C_1	1.83 ± 0.17	1.69 ± 0.09
$\lambda_{13}/\lambda_{11}$	-0.111 ± 0.007	-0.118 ± 0.006
δ_{13}/δ_{11}	-2.77 ± 0.74	-2.00 ± 0.31
$\lambda_{13}\delta_{13}/\lambda_{11}\delta_{11}$	0.31 ± 0.07	0.24 ± 0.04
$1-\Gamma_1(\lvert\Delta I\rvert=\frac{1}{2})/\Gamma_1$	0.06 ± 0.002	0.07 ± 0.002
$1-\Gamma_2(\lvert\Delta I\rvert=\frac{1}{2})/\Gamma_2$	-0.13 ± 0.005	-0.15 ± 0.005
$1-b_1(\lvert\Delta I\rvert=\frac{1}{2})/b_1$	-0.14 ± 0.01	-0.13 ± 0.01
$1-b_2(\lvert\Delta I\rvert=\frac{1}{2})/b_2$	0.22 ± 0.01	0.21 ± 0.01

Table 2

Decay mode	Γ^e (10^6 sec^{-1})	I solution	II solution
$K^\pm \to \pi^\pm\pi^\pm\pi^\mp$	4.52 ± 0.02	4.51	4.51
$K^\circ_L \to \pi^+\pi^-\pi^\circ$	2.43 ± 0.05	2.45	2.35
$K^\pm \to \pi^\pm\pi^\circ\pi^\circ$	1.40 ± 0.04	1.48	1.48
$K^\circ_L \to \pi^\circ\pi^\circ\pi^\circ$	4.15 ± 0.16	4.31	4.59

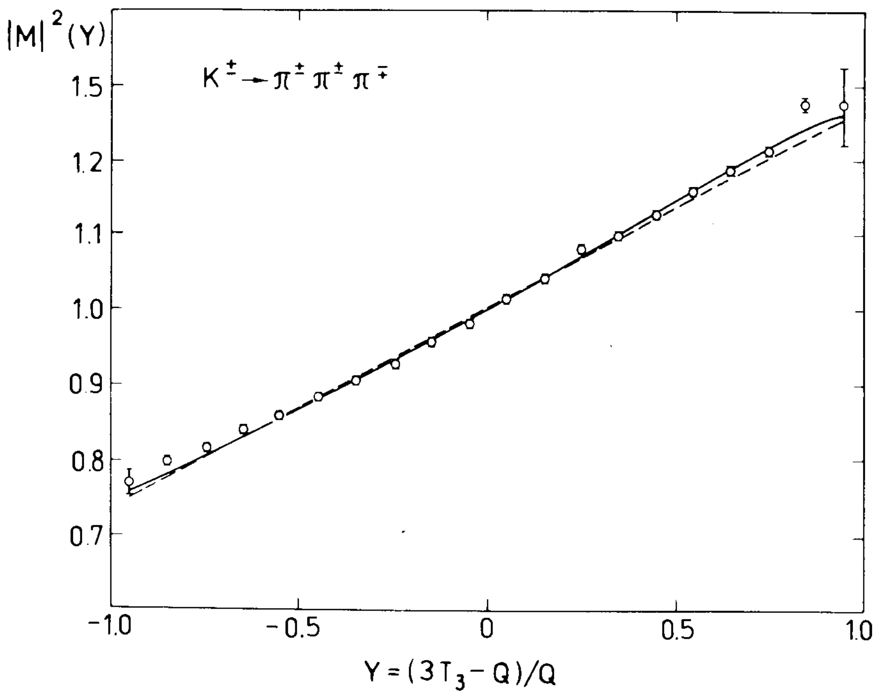

Fig. 1. Dalitz-Fabri Y variable distribution for the τ-decay. Dots are the experimental distribution of ref. /6/. Only statistical errors are given. Dot-and-dashes are the $|M|^2 \sim 1 + cY$ fit. Solid curve is the fit on formula (6).

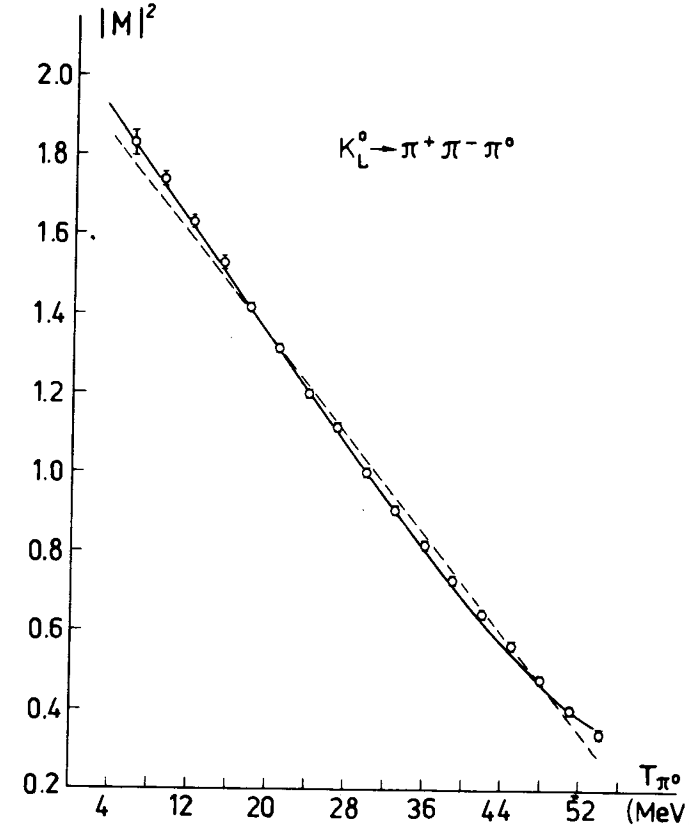

Fig. 2. π^0-meson energy distribution in $K^0_L \to \pi^+\pi^-\pi^0$ decay. Dots is the experimental distribution of ref. /7/. Dot-and-dashes are $|M|^2 \sim 1 + C(k_{12}^2 - k_0^2)$ fit. Solid curve is the fit on formula (6).

Thus, experimental data on $K\to 3\pi$ decays are described satisfactorily by matrix elements to an accuracy of cubic terms by taking into account the peculiarities in the amplitudes due to pion rescattering.

In refs.[6,7] experimental data on modes (1) and (2) separately have been described only by the analytical terms (under the assumption, that $\pi\pi$- scattering lengths $a_0 = a_2 = 0$); rather large terms of the fourth order had to be introduced in the matrix elements of these reactions. It is worth noting, however, that when describing these data on different modes separately it is not clear what transitions are responsible for so large fourth order terms.

To perform a complete analysis to the accuracy of the fourth order terms inclusive, one must take into account both analytical and nonanalytical terms. In this case the terms appear in the amplitude which along with $\pi\pi$ -scattering lengths a_0 and a_2 depends on the effective radii r_0 and r_2 of $\pi\pi$ -interaction. Refs.[5,13] give the calculations of all the isotopic amplitudes $A_{1,2\Delta I}$ to an accuracy of the fourth order terms. However, due to the lack of experimental data especially on decay modes (3) and (4) this analysis cannot be performed.

If the analytical terms of the fourth order are not anomalously large (comparing to the square of the terms $C_1 k^2, C_2 k^2$), then their effect on the accuracy of scattering length determination can be taken into account by including the $C_1^2 k^4$, $C_2^2 k^4$ terms appearing in the square matrix elements when the amplitudes are squared. Such a procedure alters the values of a_0 presented in Table 1, by ± 0.2, while the values of a_2 are altered by ± 0.1.

Thus, the evaluations show that the scattering length values (Table 1) obtained in the cubic approximation can vary due to the 4th order terms within the following limits:

in solution I	in solution II
$a_0 = 0.59 \pm 0.20$	$a_0 = -0.43 \pm 0.20$
$a_2 = -0.20 \pm 0.10$	$a_2 = 0.10 \pm 0.10$

The summarizing of the results of the analysis of the $K\to 3\pi$ decays in order to determine pion-pion scattering lengths allows the following conclusions:

1. It is hardly probable to obtain data on $\pi\pi$ -scattering lengths from the analysis of one of the $K\to 3\pi$ decay channels.
2. It is necessary to perform the combined analysis of the $K\to 3\pi$ decays in order to determine safely $\pi\pi$ -scattering lengths. It is also necessary to take into consideration both the main $|\Delta I| = 1/2$ transition and the contribution of at least transition $|\Delta I| = 3/2$.
3. The majority of experimental evaluations of pion-pion scattering lengths a_0 and a_2 is at present within 1W and 3W, where 1W is the values of $a_0 = 0.2\lambda_\pi$ and $a = -0.06\lambda_\pi$ predicted by Weinberg[18] on the basis of current algebra.

It follows from the analysis of data on $K\to 3\pi$ decays that Weinberg's predictions may prove correct only with the presence of the anomalously large analytical terms of the fourth order in the amplitudes of the $K\to 3\pi$ decays.

4. The combined analysis of data on $K\to 3\pi$ decays performed to an accuracy of the third order terms inclusive provides the value of a_0 equal to 3W (the first solution of Table 1). This is in agreement with the results of refs.[19,20].

One can hope that the still richer and the more detailed experimental information on the $K\to 3\pi$ decays will allow to solve the problem on the value of fourth order analytical terms and to determine more accurately pion-pion scattering lengths.

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